1.
$$\int_0^\infty \frac{1}{x^2+1}dx = \int_0^{\pi/2} \frac{1}{\tan^2(\theta)+1}\sec^2(\theta)d\theta = \int_0^{\pi/2} \frac{\sec^2(\theta)}{\sec^2(\theta)} = \int_0^{\pi/2} d\theta = \pi/2$$

2.
$$\int_{-\infty}^\infty \frac{1}{x^2+a^2}dx = 2\int_0^\infty \frac{1}{x^2+a^2}dx = \frac{2}{a^2}\int_0^\infty \frac{1}{x^2/a^2+1}dx = \frac{2}{|a|}\int_0^\infty \frac{1}{y^2+1}dy = \frac{2}{|a|}\cdot\pi/2 = \frac{\pi}{a}$$

3. If $a \neq 1$, we have that
$$\frac{1}{(x^2+1)(x^2+a^2)} = \frac{1}{a^2-1}\left[\frac{1}{x^2+1} - \frac{1}{x^2+a^2}\right],$$
and the integral is then $\frac{\pi}{a(a+1)}$. If $a = 1$, the integral is $\frac{\pi}{2}$.

4. If $a > 0$, we have
$$\int_0^\infty e^{-ax}dx = \frac{1}{a}\int_{-\infty}^0 e^y dy = \frac{1}{a}\lim_{y\to-\infty}(1-e^y) = \frac{1}{a}.$$

5. Using integration by parts, the integral becomes
$$\lim_{x\to\infty} -(x^2+2x+2)e^{-x} + 2 = 2.$$

6. Denote the value of these integrals by I_n for $n > 0$. Integration by parts yields the recursion $I_n = n \cdot I_{n-1}$, so we conclude that $I_n = n!$.

7. If $\omega = 0$, then the integral is 1. Otherwise, integration by parts yields
$$\begin{aligned}\int_0^t e^{-x}\cos(\omega x)dx &= -e^{-x}\cos(\omega x) - \omega\int e^{-x}\sin(\omega x)dx\Big|_0^t \\ &= -e^{-x}\cos(\omega x) - \omega\left[-e^{-x}\sin(\omega x) + \omega\int e^{-x}\cos(\omega x)dx\right]\Big|_0^t.\end{aligned}$$
Thus,
$$(1+\omega^2)\int_0^t e^{-x}\cos(\omega x)dx = -e^{-x}\cos(\omega x) + e^{-x}\omega\sin(\omega x)\big|_0^t,$$
and hence
$$\begin{aligned}\int_0^\infty e^{-x}\cos(\omega x)dx &= \lim_{x\to\infty}\frac{e^{-x}}{1+\omega^2}\left[\omega\sin(\omega x) - \cos(\omega x)\right] + \frac{1}{1+\omega^2} \\ &= \frac{1}{1+\omega^2}.\end{aligned}$$

8. Similarly, this integral is $\frac{\omega}{1+\omega^2}$.

9. Note that $e^{-x-i\omega x} = e^{-x}(\cos(\omega x) - i\sin(\omega x))$. Thus, the value of the integral is
$$\frac{1}{1+\omega^2} - i\frac{\omega}{1+\omega^2}.$$

10. Breaking up the integration domain into $(-\infty, 0)$ and $(0,\infty)$ and then substituting $x \mapsto -x$ into the first integral, yields a value of $\frac{2}{1+\omega^2}$.

11. We have that
$$\int_{-\infty}^\infty e^{-x^2}dx\int_{-\infty}^\infty e^{-y^2}dy = \int_{-\infty}^\infty\int_{-\infty}^\infty e^{-x^2-y^2}dxdy = \int_0^{2\pi}\int_0^\infty e^{-r^2}rdrd\theta = \pi,$$
so the desired integral has a value of $\sqrt{\pi}$.