1.

Method 1

Let $g(x) = e^{-2\pi i 5000x} f(x)$. Note $G(s) = F(s+5000)$ and thus $G(s) = 0$ for $|s| > 5000$. Hence $g \in B_{5000}$ (that is, g is $5000Hz$-band limited). Shannon's sampling formula yields

$$g(x) = \sum_n g(nT) sinc(\frac{x - nT}{T}) \quad , \quad T = \frac{1}{10000} = 100\mu s$$

But $g(nT) = e^{-\pi i n} f(nT) = (-1)^n f(nT)$. Thus

$$f(x) = e^{10000\pi i x} g(x) = \sum_n (-1)^n f(nT) e^{10000\pi i x} sinc(\frac{x - nT}{T})$$

Method 2

Use:

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{2\pi i s x} ds = \int_{2000}^{12000} e^{2\pi i s x} F(s) ds$$

But

$$F(s) = \sum_n c_n e^{2\pi i n s/(10000)} = \sum_n c_n e^{2\pi i n T s}$$

Now

$$c_n = \frac{1}{10000} \int_{2000}^{12000} F(s) e^{-2\pi i n s/10000} ds = T f(-nT)$$

Thus

$$f(x) = \sum_n T f(-nT) \int_{2000}^{12000} e^{2\pi i s x + 2\pi i s n T} ds = \sum_n T f(nT) \int_{2000}^{12000} e^{2\pi i s (x - nT)} ds =$$

$$= \sum_n T f(nT) \frac{1}{2\pi i (x - nT)} \left(e^{2\pi i 10000 (x - nT)} - 1 \right) = \sum_n f(nT) \frac{e^{\pi i \frac{x-nT}{T}}}{2\pi i \frac{x-nT}{T}} \left(e^{\pi i \frac{x-nT}{T}} - e^{-\pi i \frac{x-nT}{T}} \right) =$$

$$= \sum_n f(nT) e^{-\pi i n} e^{10000\pi i x} sinc\left(\frac{x - nT}{T}\right) = \sum_n f(nT) (-1)^n e^{10000\pi i x} sinc\left(\frac{x - nT}{T}\right)$$

2.

The maximum sampling period is $T = \frac{1}{2 \cdot 50000} = \frac{1}{100000} s = 10^{-5} s = 10\mu s$.

3.

The sampling period is $T = \frac{1}{2 \cdot 10000} s = 50\mu s$ (Nyquist rate). Hence the Shannon's formula becomes:

$$f(x) = \sum_n f(nT) sinc(\frac{x - nT}{T} = f(-24T) sinc(\frac{x + 1.2 \cdot 10^{-3}}{5 \cdot 10^{-5}}) + f(4T) sinc(\frac{x - 0.2 \cdot 10^{-3}}{5 \cdot 10^{-5}}) = -2 sinc(20000x + 24) + sin$$

Thus

$$f(0) = 0$$

and

$$f(10^{-6}) = -2 sinc(0.02 + 24) + sinc(0.02 - 4) = -2 sinc(24.02) - sinc(3.98)$$

4.

The maximal reconstruction error is given by

$$|f(x) - \sum_n f(nT) sinc(\frac{x - nT}{T})| \le 2 \int_{|\omega| \ge \frac{1}{2T}}^{\infty} e^{-|\omega|} d\omega = 4 \int_{500}^{\infty} e^{-\omega} d\omega = 4e^{-500}$$