I.

a.
$$\int_{-\infty}^{\infty} \delta(x-2)e^{-\pi x^2}dx = \int_{-\infty}^{\infty} \delta(y)e^{-\pi(y+2)^2}dy = e^{-4\pi}.$$

b.
$$\int_{-\infty}^{\infty} \delta'(x-1)e^{-\pi x^2}dx = \int_{-\infty}^{\infty} \delta'(y)e^{-\pi(y+1)^2}dy = -\left(\frac{d}{dy}e^{-\pi(y+1)^2}\right)|_{y=0} = 2\pi e^{-\pi}.$$

c.
$$\int_{-\infty}^{\infty} \delta''(x)e^{-\pi x^2}dx = \left(\frac{d^2}{dx^2}e^{-\pi x^2}\right)|_{x=0} = -2\pi.$$

d.
$$\int_{-\infty}^{\infty} [cos(\pi x)\delta(x)]e^{-\pi x^2}dx = \int_{-\infty}^{\infty} \delta(x)cos(\pi x)e^{-\pi x^2}dx = \left(cos(\pi x)e^{-\pi x^2}\right)|_{x=0} = 1.$$

e.
$$\int_{-\infty}^{\infty} [sin(\pi x)\delta'(x)]e^{-\pi x^2}dx = -\left(\frac{d}{dx}[sin(\pi x)e^{-\pi x^2}]\right)|_{x=0} = -\pi.$$

II.

a.
$$f\{\Phi\} = C_2(D\delta)\{\Phi\} = C_2\delta'\{\Phi\}$$

where D is the differential operator, and C_2 is the contraction by a factor of 2. Since $C_2\alpha\{\Phi\} = \alpha\{\Psi\}$ where $\Psi(x) = \frac{1}{2}\Phi(\frac{x}{2})$ we obtain

$$f\{\Phi\} = \delta'\{\Psi\} = -\delta\{\Psi'\} = -\Psi'(0) = -\frac{1}{4}\Phi'(0)$$

b.
$$f\{\Phi\} = -\Pi\{\Phi'\} = -\int_{-0.5}^{0.5} \Phi'(x)dx = \Phi(-0.5) - \Phi(0.5)$$

c.
$$f\{\Phi\} = \delta''\{\Phi(\cdot+5)\} = \Phi''(5)$$

d. Note first that $f(x) = 2p_{-1}(x+4) + 3p_{-2}(x+4) - 2p_{-1}(x+1) + 3p_{-2}(x+1)$. Thus

$$f\{\Phi\} = 2p_{-1}\{\Phi_4\} + 3p_{-2}\{\Phi_4\} - 2p_{-1}\{\Phi_1\} + 3p_{-2}\{\Phi_1\}$$

where $\Phi_4(x) = \Phi(x-4)$ and $\Phi_1(x) = \Phi(x-1)$. Thus

$$f\{\Phi\} = \int_{-\infty}^{\infty}\left[2\frac{\Phi(x-4)-\Phi(-4)}{x} + 3\frac{\Phi(x-4)-\Phi(-4)-x\Phi'(-4)}{x^2} - 2\frac{\Phi(x-1)-\Phi(-1)}{x} + \right.$$
$$\left. 3\frac{\Phi(x-1)-\Phi(-1)-x\Phi'(-1)}{x^2}\right]dx$$

The expression can be simplified further (optionally):

$$f\{\Phi\} = \int_{-\infty}^{\infty}\left[\frac{27\Phi(x)}{(x^2+5x+4)^2} - \frac{2\Phi(-4)}{x+4} - \frac{3(\Phi(-4)+(x+4)\Phi'(-4))}{(x+4)^2} + \frac{2\Phi(-1)}{x+1} - \frac{3(\Phi(-1)+(x+1)\Phi'(-1))}{(x+1)^2}\right]dx$$

e. Let $\Psi = \Phi^{\wedge\vee}$, that is
$$\Psi(s) = \hat{\Phi}(-s) = \int_{-\infty}^{\infty} e^{2\pi isx}\Phi(x)dx \quad .$$

Note the Fourier transform of Ψ is Φ, that is $\Phi = \Psi^{\wedge}$.

$$f\{\Phi\} = f^{\wedge}\{\Psi\} = \Psi''(0) = \hat{\Phi}''(0) = -4\pi^2\int_{-\infty}^{\infty} x^2\Phi(x)dx$$