

## MATH 464 – Mid-Term Exam March 29, 2011

1. (20pts) Find the inverse Fourier transform of the function  $F: \mathbb{R} \rightarrow \mathbb{R}$

$$F(s) = \frac{1}{2} \left( \operatorname{sinc} \left( \frac{s}{2} \right) \right)^2$$

$$\text{where } \operatorname{sinc}(s) = \begin{cases} \frac{\sin(\pi s)}{\pi s} & s \neq 0 \\ 1 & s = 0 \end{cases}$$

2. (20pts) A 1-periodic function on  $\mathbb{R}$  having the Fourier series  $f(x) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi i k x}$  takes the values

$$f(x) = \begin{cases} 1 & \text{if } 0 < x < 1/10 \\ 0 & \text{if } 1/10 < x < 1 \end{cases}$$

Draw the graph of  $f$  on  $[0,1]$  and of the function  $g$  represented by the following Fourier series

$$g(x) = \sum_{k=-\infty}^{\infty} \{(c_{k-5} - c_{k+5}) / (2i)\} e^{2\pi i k x}$$

NOTE: You DO NOT need to compute the Fourier coefficients  $(c_k)_k$  of  $f$ .

3. A signal  $f$  has its Fourier transform supported in the frequency band  $[-10\text{KHz}, 20\text{KHz}]$ .

- a) (10pts) What is the largest sampling period  $T$  so the signal  $f(x)$  can be reconstructed from the entire sequence of samples  $\{f(nT); n = \dots, -2, -1, 0, 1, 2, \dots\}$
- b) (10pts) Assume the signal is sampled with a sampling frequency of 35KHz. Write down an interpolation formula for  $f(x)$  from the set of samples  $\{f(nT_1); n = \dots, -2, -1, 0, 1, 2, \dots\}$  where  $T_1 = 1/35000$ .

4. (10pts) Compute the Fourier transform of the function  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x e^{-|x|}$ .

5. (30pts) For fixed  $a > 0$ , find a continuous function  $f: \mathbb{R} \rightarrow \mathbb{R}$  that satisfies

$$f'(x) + af(x) = \begin{cases} 2ae^{ax} & , x < 0 \\ 0 & , x > 0 \end{cases}$$

Total: 100pts.