# Homework #10
# Due: Tuesday, December 11, 2007

1. (10 pt) Consider $\dot{x} = Ax$ for a real *d*x*d* matrix. Show that the origin is neither positively nor negatively stable when $\lambda^k - \alpha$ divides $Det[A - \lambda I]$ for some k≥3 and real α≠0.

2. (10 pt) Graph the equation $\Delta A = 0$ in a plane with axes Tr *A* and Det *A*, and label where the different types of phase portraits occur for a 2x2 real matrix.

3. (10pt) Suppose $X(t)$ is a fundamental matrix solution for $\dot{X} = A(t)X$ on the interval $a<t<\infty$, and suppose the entries of $X(t)$ are rational functions of the real variable t. In other words, $x_{ij}(t)=p_{ij}(t)/q_{ij}(t)$ where both $q_{ij}(t)$ and $p_{ij}(t)$ are polynomials with real coefficients and $q_{ij}(t)$ is not the zero polynomial.

    a. (5pt) Prove that $\dot{x} = A(t)x$ is Liapunov stable if and only if the degree of $p_{ij}$ is less than or equal to the degree of $q_{ij}$ for all i and j.
    b. (5pt) Prove that $\dot{x} = A(t)x$ is Liapunov asymptotically stable if and only if the degree of $p_{ij}$ is strictly less than the degree of $q_{ij}$ for all i and j.

Total: 30 pts