Homework #5
Due: Tuesday, October 23, 2007

1. (10pt) Let *f(x)* be a locally Lipschitz function on $\mathbf{R}^d$ satisfying
$$\| f(x)\| \le e^{-\|x\|}$$
Show that the solution x(t,ξ) of $\dot{x} = f(x)$ is defined for all t and satisfies
$$\|x(t,\xi)\| \le \log(|t| + e^{\|\xi\|})$$

2. (30pts, 10 pt each item) Let f:$\mathbf{R}^{d+1} \to \mathbf{R}^d$ be a continuous vector-valued function, and let β:$\mathbf{R} \to \mathbf{R}$ be a continuous non-negative function such that
$$\int_0^\infty \beta(s)ds < \infty$$
Prove that, if
$$\| f(t,x)\| \le \beta(t)\|x\|$$
Then the solution of $\dot{x} = f(t,x)$ have the following properties:
   a. The solution x(t,τ,ξ) is defined for all t≥τ.
   b. Given τ and ξ, there exists a constant C such that $\|x(t,\tau,\xi)\| \le C$.
   c. The limit as t goes to infinity of x(t,τ,ξ) exists.

Total: 40 pts