Homework #6
Due: Tuesday, November 6, 2007

1. (10pt) Let *x(t)* be a solution of $\dot{x} = A(t)x + h(t)$, where A(t) and h(t) are continuous on an open interval 0<t<∞. Prove that |*x(t)*| is bounded for t≥1, if both

$$\int_1^\infty |A(t)|dt < \infty \quad \text{and} \quad \int_1^\infty |h(t)|dt < \infty .$$

2. (10 pt) Let $A : I \to M_d(R)$ and $B : I \to M_d(R)$ be differentiable functions on the interval *I*, that is, every entry of *A(t)* and *B(t)* is just a real-valued differentiable function on an interval. Prove that

$$\frac{d}{dt}\left[A(t)B(t)\right] = \dot{A}(t)B(t) + A(t)\dot{B}(t)$$

3. (10pt) Consider $\dot{x} = A(t)x + h(t)$, where

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \text{ and } h(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}.$$

Verify that

$$X(t) = \begin{bmatrix} e^{3t} & te^{3t} \\ 0 & e^{3t} \end{bmatrix}$$

is a fundamental matrix solution of $\dot{x} = A(t)x + h(t)$. Find a solution to the initial value problem

$$\dot{x} = A(t)x + h(t) \text{ and } x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

4. (10pt) Suppose $A : (0, \infty) \to M_d(R)$ is continuous. Prove the following: If

$$\int_1^\infty Tr\ A(t)\, dt = \infty$$

Then there exists a solution *x(t)* of $\dot{x} = A(t)x$ such that |*x(t)*| is unbounded for t≥1.

Total: 40 pts