Homework #7
Due: Tuesday, November 13, 2007

1. (15 pt; 5pts each) Consider $\dot{X} = A(t)X$ on the interval $0 < t < \infty$ where

$$A(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6t^{-3} & -6t^{-2} & 3t^{-1} \end{bmatrix}$$

   (a) Show that

$$\begin{bmatrix} t^3 & t^2 & t \\ 3t^2 & 2t & 1 \\ 6t & 2 & 0 \end{bmatrix}$$

   is a fundamental matrix solution of $\dot{X} = A(t)X$;

   (b) Calculate X(t,s), the principal matrix solution;

   (c) Use X(t,s) to solve the third-order initial-value problem

$$\frac{d^3y}{dt^3} + \frac{3}{t}\frac{d^2y}{dt^2} + \frac{6}{t^2}\frac{dy}{dt} + \frac{6}{t^3}y = 0$$

$$y(1) = 1, \dot{y}(1) = 2, \ddot{y}(1) = 3$$

2. (10 pt) Let p(t), q(t), and g(t) be continuous real-valued functions on an open interval I and suppose $\varphi_1(t)$ and $\varphi_2(t)$ are linearly independent solutions of the second-order scalar equation

$$\ddot{y} + p(t)\dot{y} + q(t)y = 0.$$

   Derive an explicit formula for a particular solution of

$$\ddot{y} + p(t)\dot{y} + q(t)y = g(t).$$

3. (5 pt) Let $\varphi_1, \ldots, \varphi_n$ be linearly independent solutions of the nth order homogeneous linear differential equation

$$x^{(n)} + a_n(t)x^{(n-1)} + a_{n-1}(t)x^{(n-2)} + \cdots + a_1(t)x = 0.$$

   Find necessary and sufficient conditions for the Wronskian $W(\varphi_1, \ldots, \varphi_n)$ to be constant.

Total: 30 pts