Homework #9
Due: Tuesday, December 4, 2007

1. (10 pt) Given $\dot{x} = f(x)$ on Ω an open subset of $\mathbf{R}^d$, suppose there exists a compact set K contained in Ω and a point ξ such that $x(t,\xi)\in K$ for all $t\geq 0$. Prove that $\omega(\xi)$ is connected, that is, show that it is impossible to find open sets U_1 and U_2 such that $\omega(\xi)\subset U_1\cup U_2$ and $\omega(\xi)\cap U_j\neq\emptyset$ for j=1,2.

2. (10 pt) Consider the following system of differential equations on $\mathbf{R}^2$:
$$\dot{x} = y - \alpha x y^2$$
$$\dot{y} = -x - \beta y$$
where $\alpha,\beta>0$ are positive real constants. Show that the origin is positively asymptotically stable and determine the omega limit set for every point in the plane.

Total: 20 pts