MATH 241 Calculus III Spring 2023
Groupwork 1: Sections 11.1-11.3

You should work on and discuss this worksheet with members of your group. Your TA will assist as needed. Turn in your solutions either on this sheet or a separate sheet of paper. Be sure to include your name!

1. Describe in words (and sketch if possible) the region of $\mathbb{R}^3$ represented by the inequality $(x-1)^2 + (y-1)^2 \leq 9$. [Hint: First sketch this inequality for points in $\mathbb{R}^2$.]

2. Write an inequality describing the open region containing all points between (but not on) the spheres of radius r and R centered at the origin. (Suppose $r < R$.)

3. Of the following expressions, explain which are meaningful, and which are meaningless. (For example $\dfrac{1}{\mathbf{a}}$ is meaningless because we do not define division of vectors.) For the ones that make sense, evaluate them using $\mathbf{a} = \hat{\boldsymbol{\imath}}$, $\mathbf{b} = 2\hat{\boldsymbol{\imath}}$, and $\mathbf{c} = \hat{\boldsymbol{k}}$.

   (a) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$

   (b) $\|\mathbf{a}\|(\mathbf{b} \cdot \mathbf{c})$

   (c) $(\mathbf{a} \cdot \mathbf{b}) + \mathbf{c}$

   (d) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

   (e) $\|\mathbf{a}\| \cdot (\mathbf{b} + \mathbf{c})$

4. Suppose a quarterback throws a football with angle of elevation $\pi/6$ radians, and with speed 50 feet per second. Find the horizontal and vertical components of the football's velocity vector $\mathbf{v}$ at the time of release, and write $\mathbf{v}$ in the form $\mathbf{v} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}}$ (i.e. determine v_x and v_y). Note that *speed* is a scalar quantity. It is the magnitude or norm of the velocity vector.

5. Is the triangle defined by the vertices $P(1, -3, -2)$, $Q(2, 0, -4)$, and $R(6, -2, -5)$ a right-triangle? (Hint: First find three vectors that define the sides of the triangle.)

6. Let $\mathbf{a} = (1, 1)$ and $\mathbf{b} = (-4, 0)$.

   (a) Are these vectors perpendicular? If so, explain why. If they are not perpendicular, find the angle between them. Recall angles between two vectors are defined to take values in $0 \leq \theta \leq \pi$.

   (b) Find the projection of $\mathbf{b}$ onto $\mathbf{a}$, i.e. find the vector $\mathbf{p} = \mathrm{proj}_{\mathbf{a}}\mathbf{b}$.

   (c) Sketch the vectors $\mathbf{b}$ and $\mathbf{p}$ on the same graph. Label them. Then use the geometric version of vector addition/subtraction (i.e. use arrows), to find a formula for a vector $\mathbf{q}$ perpendicular to $\mathbf{a}$.

   (d) Use your formula to explicitly compute the value of $\mathbf{q}$, and then verify it is perpendicular to $\mathbf{a}$.