Groupwork 2 Solution
MATH 241 (Spring 2023)
02/07/2023

Problem 1.

(a) meaningful,

(b) meaningless as $\boldsymbol{b} \cdot \boldsymbol{c}$ is a scalar,

(c) meaningful,

(d) meaningless as both $\boldsymbol{a} \cdot \boldsymbol{b}$ and $\boldsymbol{c} \cdot \boldsymbol{d}$ are scalars.

Problem 2. A vector parallel to the line is given by $\overrightarrow{L} = (2,4,5) - (6,1,-3) = (-4,3,8)$. Therefore the parametric equation is given by

$$
\begin{aligned}
x &= -4t + 6 \\
y &= 3t + 1 \\
z &= 8t - 3
\end{aligned}
$$

Then the symmetric equation is given by

$$
\frac{x-6}{-4} = \frac{y-1}{3} = \frac{z+3}{8}
$$

Problem 3.

(i) a vector $\boldsymbol{a}$ parallel to the line through the points $(4,1,-1)$ and $(2,5,3)$ is

$$
(2,5,3) - (4,1,-1) = (-2,4,4)
$$

(ii) a vector $\boldsymbol{b}$ parallel to the line through the points $(-3,2,0)$ and $(5,1,4)$ is

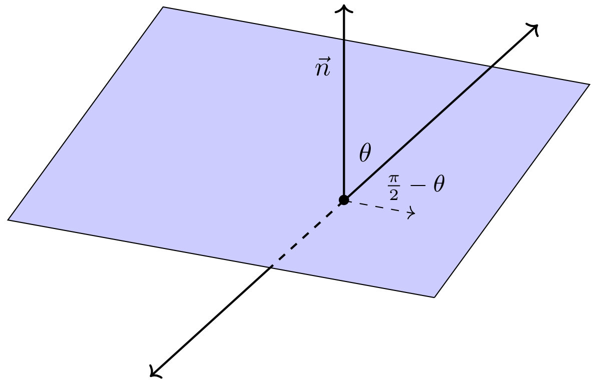
$$
(5,1,4) - (-3,2,0) = (8,-1,4)
$$

Then $\boldsymbol{a} \cdot \boldsymbol{b} = -16 - 4 + 16 = -4 \neq 0$. Therefore, two lines are not perpendicular.

Problem 4. First we, find the point of intersection. We substitute the parametric equation of the line to the the equation of the plane. Then we obtain $(3-t) - (2+t) + 2(5t) = 9$. Then

$$
\begin{aligned}
& (3-t) - (2+t) + 2(5t) = 9 \\
\Rightarrow \quad & 3 - t - 2 - t + 10t = 9 \\
\Rightarrow \quad & 1 + 8t = 9 \\
\Rightarrow \quad & 8t = 8 \\
\Rightarrow \quad & t = 1
\end{aligned}
$$

Hence the point of intersection is $(2,3,5)$ by substituting $t = 1$ to the line equation.



Let θ be the angle between the normal vector of the plane and line. Then the angle between the plane and the line is $\frac{\pi}{2} - \theta$. A vector parallel $\overrightarrow{L}$ to the line is given by $(-1, 1, 5)$, and a normal vector $\vec{n}$ of the plane is $(1, -1, 2)$. If θ is the angle between the plane and the line, then one has

$$\cos\theta = \frac{\vec{n} \cdot \overrightarrow{L}}{||\vec{n}|| \cdot ||\overrightarrow{L}||} = \frac{(-1,1,5) \cdot (1,-1,2)}{||(-1,1,5)|| \cdot ||(1,-1,2)||} = \frac{8}{3\sqrt{3} \cdot \sqrt{6}} = \frac{8}{9\sqrt{2}}$$

Then $\theta \approx 0.89112$ and the angle is $\frac{\pi}{2} - \theta \approx 0.67967$.

Problem 5. Let $P_1 = (1, -2, 4)$. This is not on the plane because $3 \cdot 1 + 2 \cdot (-2) + 6 \cdot 4 = 3 - 4 + 24 = 23 \neq 5$. We choose a point $P_0 = (1, 1, 0)$ on the plane. Then $\overrightarrow{P_0P_1} = (1, -2, 4) - (1, 1, 0) = (0, -3, 4)$. The normal vector $\vec{n}$ of the plane is $(3, 2, 6)$, so we have that the distance is

$$D = \frac{|\vec{n} \cdot \overrightarrow{P_0P_1}|}{||\vec{n}||} = \frac{|(3,2,6) \cdot (0,-3,4)|}{\sqrt{3^2 + 2^2 + 6^2}} = \frac{|-6+24|}{\sqrt{49}} = \frac{18}{7}$$

Problem 6. It suffice to see whether the normal vectors of these planes are parallel, perpendicular, or neither. The normals vectors are $(2, -3, 4)$ and $(1, 6, 4)$. Then the dot product is

$$(2, -3, 4) \cdot (1, 6, 4) = 2 - 18 + 16 = 0$$

Hence the two planes are perpendicular.