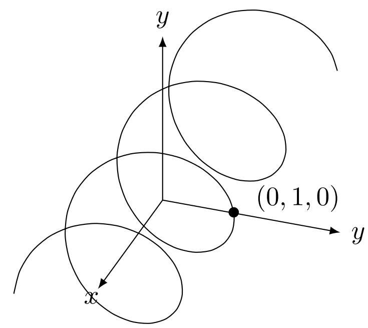
Groupwork 3 Solution
MATH 241 (Spring 2023)
02/14/2023

Problem 1.

(a)



(b)
- velocity: $\boldsymbol{v}(t) = \boldsymbol{r}'(t) = \hat{\boldsymbol{\imath}} - \pi \sin(\pi t)\hat{\boldsymbol{\jmath}} + \pi \cos(\pi t)\hat{\boldsymbol{k}}$,
- acceleration: $\boldsymbol{a}(t) = \boldsymbol{r}''(t) = -\pi^2 \cos(\pi t)\hat{\boldsymbol{\jmath}} - \pi^2 \sin(\pi t)\hat{\boldsymbol{k}}$

Problem 2. The velocity is given by

$$\boldsymbol{v}(t) = \int \boldsymbol{a}(t)\,dt = (3t^2 + c_1, 4t^3 + c_2, -3t^2 + c_3)$$

Since $\boldsymbol{v}(0) = (1, -1, 3)$, one has $c_1 = 1, c_2 = -1, c_3 = 3$. To conclude

$$\boldsymbol{v}(t) = (3t^2 + 1, 4t^3 - 1, -3t^2 + 3)$$

Next,

$$\boldsymbol{p}(t) = \int \boldsymbol{v}(t)\,dt = (t^3 + t + c_1, t^4 - t + c_2, -t^3 + 3t + c_3)$$

Since it starts at the origin,

$$\boldsymbol{p}(t) = (t^3 + t, t^4 - t, -t^3 + 3t)$$

Problem 3. In both cases, the two curves intersect when $t = 0$. (**Warning**. In general, you might have to use different t-values for each curve.) Therefore, we find two direction vectors at $t = 0$. We can find this by taking the derivative of both

$$\boldsymbol{r}_1(t) = (\cos t, \sin t, t) \Rightarrow \boldsymbol{r}_1'(t) = (-\sin t, \cos t, 1) \Rightarrow \boldsymbol{r}_1'(0) = (0, 1, 1)$$
$$\boldsymbol{r}_2(t) = (1 + t, t^2, t^3) \Rightarrow \boldsymbol{r}_2'(t) = (1, 2t, 3t^2) \Rightarrow \boldsymbol{r}_1'(0) = (1, 0, 0)$$

As $\boldsymbol{r}_1'(0) \cdot \boldsymbol{r}_2'(0) = 0$, we can conclude that the angle of intersection is $\frac{\pi}{2}$.

Problem 4. We use the formula that says

$$L = \int_a^b ||\boldsymbol{r}'(t)||\,dt$$

To figure out a and b, we need to find t-values that gives $(1, 0, 0)$ and $(1, 0, 2\pi)$. But this is easy since $z = t$, so $a = 0$ and $b = 2\pi$. The derivative is $\boldsymbol{r}'(t) = (-\sin t, \cos t, 1)$, so we have

$$||\boldsymbol{r}'(t)|| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

Then

$$L = \int_0^{2\pi} \sqrt{2}\,dt = 2\sqrt{2}\pi$$

This is not closed because if t is different, then the z-value is different, so you cannot have two different t-values that give the same point. However, it is smooth because $\boldsymbol{r}'(t)$ are composed of well-known continuous function ($-\sin t, \cos t$, and a constant function), and $\boldsymbol{r}'(t) \neq \vec{0}$ as $z = 1$ for all t.

Problem 5. We have

$$x = t \text{ and } y = 0 \text{ and } z = 2t - t^2$$

Once we plug this into the equation of the paraboloid, we solve

$$2t - t^2 = t^2 \quad \Rightarrow \quad 2t - 2t^2 = 0 \quad \Rightarrow \quad 2t(t-1) = 0$$

(**Warning**. You cannot divide by t because t can be equal to 0 as above.) Hence $t = 0$ or 1. Then two points of intersection are then $(0, 0, 0)$ and $(1, 0, 1)$.

Problem 6.

(a) The range of $\boldsymbol{r}(t) = t^3\hat{\boldsymbol{\imath}} + 2t^3\hat{\boldsymbol{\jmath}} + 3t^3\hat{\boldsymbol{k}}$ is same as the $\boldsymbol{s}(t) = t\hat{\boldsymbol{\imath}} + 2t\hat{\boldsymbol{\jmath}} + 3t^3\hat{\boldsymbol{k}}$ which is a line. Therefore, curve with vector equation $\boldsymbol{r}(t)$ is a line.

(b) Let $\boldsymbol{r}(t) = (t, t, t)$ be a parametrization of a curve. Then $-\boldsymbol{r}(t)$ is also a parametrization of a curve because the ranges of $\boldsymbol{r}$ and $-\boldsymbol{r}$ are the same. However, $\boldsymbol{r}'(t) = (1, 1, 1) \neq (-1, -1, -1) = -\boldsymbol{r}'(t)$. Hence **False**.

(c) Consider a curve on a circle on the xy-plane that travels the circle at a different speed at different t. Since it is on the circle, $||\boldsymbol{r}(t)|| = 1$ for all t, but the velocity $||\boldsymbol{r}(t)||$ won't be constant. More precisely, take $\boldsymbol{r}(t) = (\cos(t^2), \sin(t^2), 0)$ as an example. Then $||\boldsymbol{r}(t)|| = \sqrt{\cos^2(t^2) + \sin^2(t^2)} = 1$. However,

$$||\boldsymbol{r}'(t)|| = \sqrt{(-2t\sin(t^2))^2 + (2t\cos(t^2))^2} = 2|t|$$

which is not a constant.