MATH 241 Calculus III Spring 2023

Groupwork 5: Sections 13.5-13.8

You should work on and discuss this worksheet with members of your group. Your TA will assist as needed. Turn in your solutions either on this sheet or a separate sheet of paper. Be sure to include your name!

1. Find the tangent plane at the point $(1, 0, 1)$ that best approximates the function $f(x, y) = xe^{xy}$. Use your linearization of f (i.e. use the tangent plane equation), to approximate the value of $f(1.1, -0.1)$.

2. Find the point on the paraboloid $z = 9 - 4x^2 - y^2$ at which the tangent plane is parallel to the plane $z = 4y$.

3. Is the directional derivative a vector? Explain.

4. In which direction does $f(x, y, z) = ze^{xy}$ increase most rapidly at the point $(0, 1, 2)$? What is the maximum rate of increase?

5. Find the critical points of the function $f(x, y) = x^4 + y^4 - 4xy + 1$. For each point, explain whether f has a relative maximum, relative minimum, or saddle point. Otherwise explain why the 2nd Partial Derivative Test fails.

6. (physics application) Suppose a force field $\mathbf{F}$ is defined as $\mathbf{F} = -\nabla V(x, y, z)$ where V is a function called the *potential function*. Suppose an object with mass m moves along the trajectory parametrized by $\mathbf{r}(t)$, and satisfies Newton's 2nd Law $m\mathbf{a} = \mathbf{F}$, where $\mathbf{a}$ is the object's acceleration. Use the chain rule to prove the Conservation of Energy for all times t:

$$E(t) = \frac{1}{2}m\|\mathbf{v}(t)\|^2 + V(\mathbf{r}(t)) = \text{constant}$$