MATH 241 Calculus III Spring 2023

Groupwork 6: Sections 14.1-14.2

You should work on and discuss this worksheet with members of your group. Your TA will assist as needed. Turn in your solutions either on this sheet or a separate sheet of paper. Be sure to include your name!

1. If the following equality is true, explain why. If false, fix the integral bounds on the right-side so the two iterated integrals are equal.

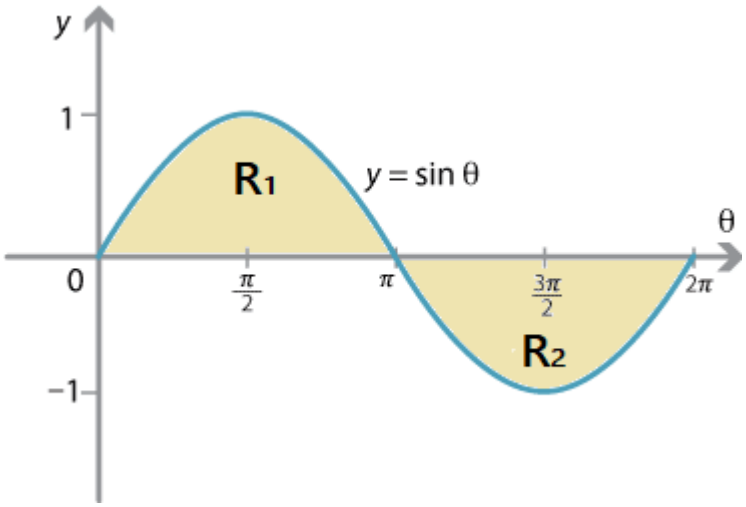
$$\int_0^4 \int_0^{\sqrt{x}} \sqrt{x+y^2}\,\mathrm{d}y\mathrm{d}x = \int_0^{\sqrt{x}} \int_0^4 \sqrt{x+y^2}\,\mathrm{d}x\mathrm{d}y.$$

2. Evaluate the iterated integral

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\mathrm{d}x\mathrm{d}y.$$

3. (a) What is the geometric meaning of $\iint_R f(x,y)\,\mathrm{d}A$ if $f(x,y) \geq 0$? What if f takes both positive and negative values? If f has units of mass density, i.e. mass per unit area, what does this integral represent?

   (b) Consider the region $R = R_1 \cup R_2$ pictured below:



   If $f(\theta, y) = 1$ for all (θ, y) in R_1, and $f(\theta, y) = -1$ for all (θ, y) in R_2, evaluate the integral

$$\iint_R f\,\mathrm{d}A.$$

   Hint: Consider your answer to part (a).

   (c) Write down $\iint_{R_1} f(\theta, y)\,\mathrm{d}A$ as an iterated integral in two different ways (with R_1 as a "vertically simple region" and also as a "horizontally simple region"). Evaluate one of these.

4. Set up the iterated (double) integral that gives the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$.

5. Evaluate the double integral $\iint_R (2x - y)\,\mathrm{d}A$ where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $y = x$.