MATH 241 Calculus III Spring 2023 Groupwork 7: Sections 14.4-14.6

You should work on and discuss this worksheet with members of your group. Your TA will assist as needed. Turn in your solutions either on this sheet or a separate sheet of paper. Be sure to include your name!

1. Set up the iterated integral for evaluating

$$\iiint_E z \, \mathrm{d}V,$$

where *E* is the tetrahedron bounded by the four planes x = 0, y = 0, z = 0, and x + y + z = 1.

2. Suppose an object occupies the "ice cream cone"-shaped solid region *E* bounded above by the sphere $x^2+y^2+z^2 = 4$ and below by the portion of the cone $z^2 = x^2+y^2$ above the *xy*-plane. The mass density is f(x, y, z) = z.

Set up, but do not evaluate, iterated triple integrals (with the integrand and all limits of integration written in the appropriate coordinate system) for the mass of the object, i.e. $\iiint_E f \, dV$. Do this in both

- (a) cylindrical coordinates
- (b) spherical coordinates
- 3. The volume of a cylinder of radius r and height h is well-known to be $V = \pi r^2 h$. Derive this formula by evaluating an appropriate triple integral in cylindrical coordinates.
- 4. If you have time, evaluate the integral in Problem 1 (it should be equal to 1/24) and evaluate both integrals in Problem 2 (you should get 2π for the total mass).