MATH 241 Calculus III Spring 2023

Groupwork 8: Sections 14.8-14.9

You should work on and discuss this worksheet with members of your group. Your TA will assist as needed. Turn in your solutions either on this sheet or a separate sheet of paper. Be sure to include your name!

1. Evaluate $\iint_R xy^3 \, \mathrm{d}A$ where R is the region bounded by the hyperbolas $xy = 1$, $xy = 3$, $y = 2$, and $y = 6$. Do this by using a change of variables that transforms R into a rectangular domain in the uv-plane.

2. Give any vector parametrization for the following surfaces. Be sure to include the parameter ranges.

   (a) The portion of the plane $6x + y + 3z = 9$ in the first octant.

   (b) The part of the circular paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 9$.

3. In statistics and probability, a classic integral that appears in connection with Gaussian distributions is

$$\int_0^\infty e^{-x^2} \, \mathrm{d}x.$$

   It is well-known that e^{-x^2} doesn't have an antiderivative expressible in terms of elementary functions. However, this particular definite integral can be evaluated with multivariable calculus techniques.

   (a) First show that

$$\left(\int_0^\infty e^{-x^2} \, \mathrm{d}x \right)^2 = \int_0^\infty \int_0^\infty e^{-x^2-y^2} \, \mathrm{d}x\mathrm{d}y.$$

   [Hint: Make a dummy variable change in one of the integrals on the left.]

   (b) Next, make a change of variable in the iterated integral so it is in polar coordinates.

   (c) Evaluate the iterated integral, and conclude that

$$\int_0^\infty e^{-x^2} \, \mathrm{d}x = \frac{\sqrt{\pi}}{2}$$