MATH 241 Calculus III Spring 2023

Groupwork 9: Sections 15.1-15.2

You should work on and discuss this worksheet with members of your group. Your TA will assist as needed. Turn in your solutions either on this sheet or a separate sheet of paper. Be sure to include your name!

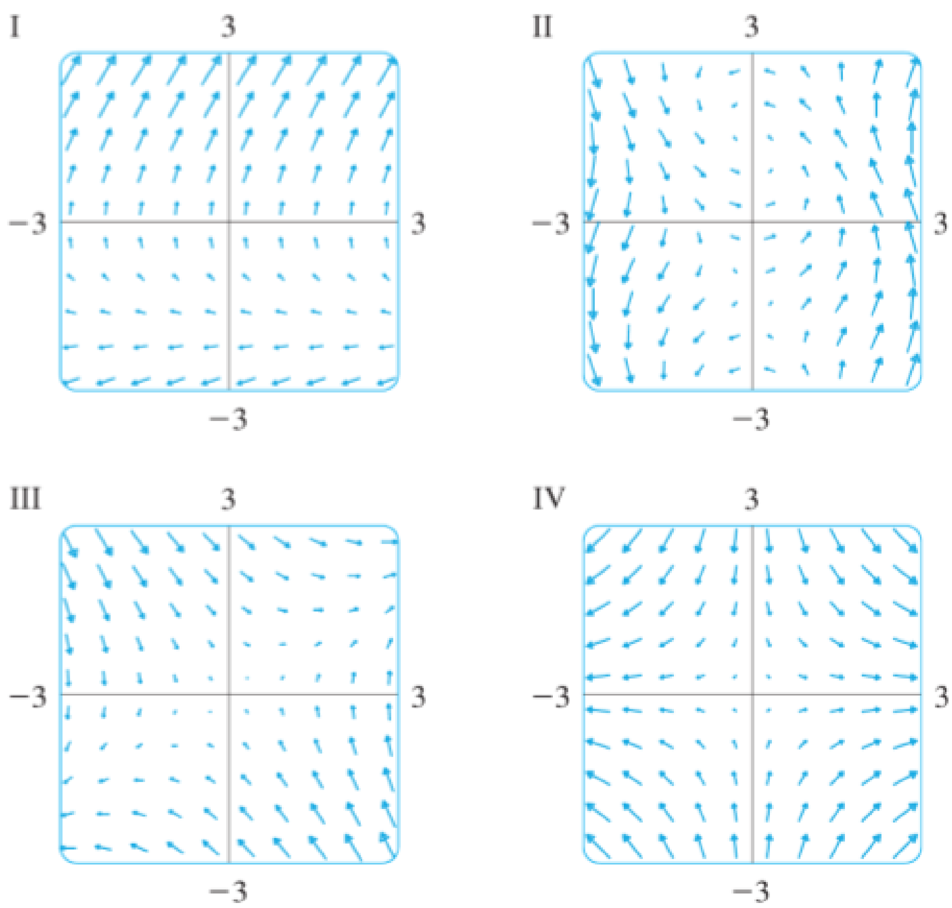
1. (Conceptual Check) Match the vector field $\mathbf{F}$ with the correct plot.

   (a) $\mathbf{F}(x, y) = (x, -y)$

   (b) $\mathbf{F}(x, y) = (y, x - y)$

   (c) $\mathbf{F}(x, y) = (y, y + 2)$

   (d) $\mathbf{F}(x, y) = (\cos(x + y), x)$



2. Which of the following quantities are vector-valued, which are scalar-valued, and which are undefined? (No computation needed.)

   (a) $\nabla(fg)$

   (b) $\text{curl}(\nabla f)$

   (c) $\nabla(\text{div}\, \mathbf{F})$

   (d) $\text{curl}(\text{curl}\, \mathbf{F})$

   (e) $\text{div}(\nabla f)$

   (f) $\text{curl}(\text{div}\, \mathbf{F})$

   (g) $\text{div}(\text{div}\, \mathbf{F})$

3. Find the curl of $\mathbf{F}(x, y, z) = (3 + 2xy)\hat{\mathbf{i}} + (x^2 - 3y^2)\hat{\mathbf{j}}$. Is this vector field conservative? If it is, find a function f such that $\mathrm{F} = \nabla f$. Otherwise explain why it is not conservative.

4. A thin wire bent in the shape of a curve C has mass density $\rho(x, y) = 2x + 1$ where C consists of the arc C_1 of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$, followed by the vertical line segment C_2 from $(1, 1)$ to $(1, 2)$. First draw a picture of the curve C. Then find $\displaystyle\int_C \rho(x, y)\, \mathrm{d}s$, the total mass of the wire.

5. Find the work done by the force field $\mathbf{F}(x, y) = (x^2, -xy)$ on a particle moving along the quarter circle in the first quadrant with radius 2, starting from the point $(2, 0)$ (i.e. compute the line integral $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$). Also find the work done if the particle travels along the entire circle (i.e. starting and ending at the point $(2, 0)$).

6. (Extra Conceptual Check) Consider Figures I and II from Problem 1 again. For both, do you expect $\text{div}\, \mathbf{F}(-2, 2)$ to be positive, negative, or zero? Also do you expect $\text{curl}\, \mathbf{F}(-2, 2)$ to point in the direction $\hat{\mathbf{k}}$, in the direction $-\hat{\mathbf{k}}$, or be equal to $\mathbf{0}$? If you have time, verify your guesses by computing the divergence and curl at $(-2, 2)$ for each of the vector fields in Figures I and II.