MATH 241 Calculus III Spring 2023

Groupwork 10: Sections 15.3-15.5

You should work on and discuss this worksheet with members of your group. Your TA will assist as needed. Turn in your solutions either on this sheet or a separate sheet of paper. Be sure to include your name!

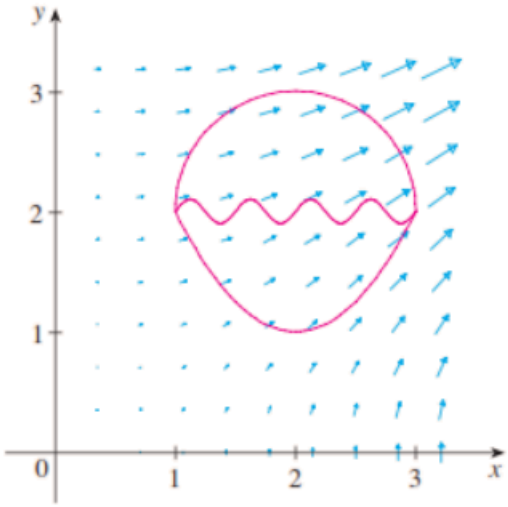
1. Use Green's Theorem to evaluate $\int_C x^2 y\,\mathrm{d}x - xy^2\mathrm{d}y$, where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

2. Evaluate $\int_C x^4\,\mathrm{d}x + xy\mathrm{d}y$, where C is the triangular curve consisting of the line segments from $(0,0)$ to $(1,0)$, from $(1,0)$ to $(0,1)$, and from $(0,1)$ to $(0,0)$. Note the implied direction of C! [You can try using three separate line integrals, though it is much easier to use Green's Theorem.]

3. The figure below shows a vector field $\mathbf{F}(x,y) = 2xy\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$, and three curves beginning at $(1,2)$ and ending at $(3,2)$.

   (a) Explain why $\int_C \mathbf{F}\cdot\mathrm{d}\mathbf{r}$ has the same value for all three curves.

   (b) What is this common value?



4. A thin cone has shape $z = \sqrt{x^2 + y^2}$, $1 \le z \le 4$. If the density of the cone material is $\rho(x,y,z) = 10 - z$ (in grams per cm^2), find the total mass of the cone.