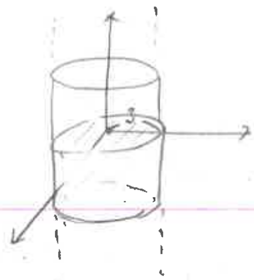
1\. (a) In the xy-plane, $x^2+y^2=9$ is a circle. Hence



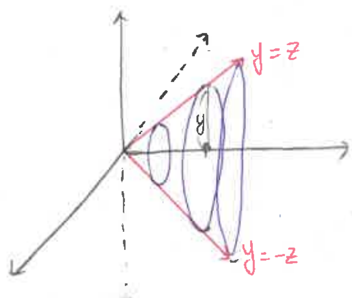
the surface is an (infinite) cylinder whose projection to the xy-plane is the circle of radius 3 centered at the origin.

OR

simply cylinder

(b) If $x=0$, $y=\sqrt{z^2} \Rightarrow y^2=z^2$

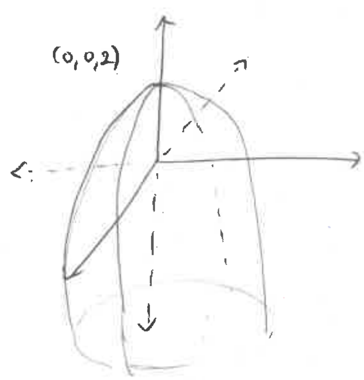
$\Rightarrow y=\pm z$ (given in red)



(given in blue)

Fixing a y gives a circle of radius y. Hence the surface is a cone that opens toward the positive y-axis. Observe that $y=\sqrt{x^2+z^2} \geq 0$ is always non-negative.

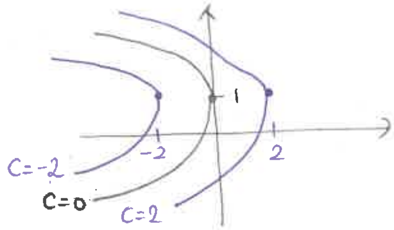
(c)



$z=-x^2-y^2+2$

2. (a) $C = x + (y-1)^2$

$\Rightarrow x = -(y-1)^2 + C$



First, sketch when $C=0$

Then $C = \pm 2$ is just a translate of the first

(b) We have

$\vec{r}(t) = (x(t), y(t)) = (t^2, t^3)$ location function

$f(x,y) = x^2 + 2y$ temperature function

Want to find $\left.\dfrac{df}{dt}\right|_{(x,y)=(4,8)}$

By chain rule, $$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = (2x+y)\cdot 2t + x\cdot(3t^2).$$

What is t for $(x(t), y(t)) = (t^2, t^3) = (4,8)$? $t = 2$.

Hence $$\left.\frac{df}{dt}\right|_{(x,y)=(4,8)} = 16\cdot 4 + 4\cdot 12 = \boxed{112}.$$

3. (a) $$D_{\vec{u}} f(2,-1) = \nabla f(2,-1)\cdot\vec{u}$$

$= \|\nabla f(2,-1)\|\cdot\|\vec{u}\|\cos\theta$ where θ = angle between $\vec{u}$ and $\nabla f(2,-1) = \frac{\pi}{3}$

$= \|\nabla f(2,-1)\|\cos\frac{\pi}{3}$ as $\|\vec{u}\|=1$

$= \frac{1}{2}\|\nabla f(2,-1)\| = \frac{1}{2}\sqrt{180} = \boxed{\sqrt{45}}$

as $\nabla f(x,y) = (3x^2y,\ x^3+2y) \Rightarrow \nabla f(2,-1) = (-12, 6)$

(b) We use the function $f(x,y) = x\sqrt{y}$, then $\nabla f(x,y) = \left(\sqrt{y}, \frac{x}{2\sqrt{y}}\right)$. $\Rightarrow \nabla f(5,9) = \left(3, \frac{5}{6}\right)$.

Equation of the tangent plane at $(5,9)$ is $z = f(5,9) + f_x(5,9)(x-5) + f_y(5,9)(y-9)$

$= 15 + 3(x-5) + \frac{5}{6}(y-9)$

Then $5.01\sqrt{8.9} \approx \boxed{15 + 3\cdot(0.01) - \frac{5}{6}\cdot(0.1)}$

4. $h_x(x,y) = 2xy - 4x = 0 \Rightarrow 2x(y-2) = 0 \Rightarrow x = 0$ or $y = 2$.

$h_y(x,y) = x^2 - 2y = 0 \Rightarrow x^2 = 2y \cdots ①$

If $x=0$, then $y=0$ by ①

If $y=2$, then $x^2=4$ by ① $\Rightarrow x = \pm 2$

Hence the critical points are $(0,0)$, $(2,2)$, or $(-2,2)$.

$$\left.\begin{aligned} h_{xx} &= 2y-4 \\ h_{xy} = h_{yx} &= 2x \\ h_{yy} &= -2 \end{aligned}\right\} \quad D = -4y+8-4x^2$$

	D	f_{xx}	
$(0,0)$	$8>0$	$-4<0$	relative maximum
$(2,2)$	$-16<0$		Saddle point
$(-2,2)$	$-16<0$		Saddle point

5. $f(x,y) = y + xy$

$g(x,y) = x^2+y^2$

$$\begin{cases} f_x = g_x \lambda \\ f_y = g_y \lambda \end{cases} \Rightarrow \begin{cases} y = 2x\lambda & \cdots ① \\ 1+x = 2y\lambda & \cdots ② \\ x^2+y^2 = 1 & \cdots ③ \end{cases}$$

$$\left.\begin{aligned} ① \times y &\Rightarrow y^2 = 2xy\lambda \\ ② \times x &\Rightarrow x + x^2 = 2xy\lambda \end{aligned}\right\} \Rightarrow x + x^2 = y^2$$

$$③ \Rightarrow y^2 = 1 - x^2$$

$\Rightarrow x + x^2 = 1 - x^2 \Rightarrow 2x^2 + x - 1 = 0$

$\Rightarrow (2x-1)(x+1) = 0$

$\Rightarrow x = \frac{1}{2}$ or $x = -1$

Case 1: $x=-1$

③ $\Rightarrow (-1)^2 + y^2 = 1 \Rightarrow y^2 = 0 \Rightarrow y = 0. \rightsquigarrow (-1,0)$.

Case 2: $x = \frac{1}{2}$

③ $\Rightarrow (\frac{1}{2})^2 + y^2 = 1 \Rightarrow y^2 = \frac{3}{4} \Rightarrow y = \pm\frac{\sqrt{3}}{2} \rightsquigarrow (\frac{1}{2}, \frac{\sqrt{3}}{2}), (\frac{1}{2}, -\frac{\sqrt{3}}{2})$.

$f(-1,0) = 0$

$f(\frac{1}{2}, \frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}$ maximum

$f(\frac{1}{2}, -\frac{\sqrt{3}}{2}) = -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}$ minimum