

Quiz 4

Math 241: Spring 2023

Answer Key

Problem 1. (5 points) Assume $x(t)$ and $y(t)$ are functions in t , and $f(x, y) = x^2 \sin y$. Write $\frac{d}{dt}f(x(t), y(t))$ in terms of $x(t)$, $y(t)$, $x'(t)$ and $y'(t)$.

$$\begin{aligned}\frac{d}{dt}f(x(t), y(t)) &= \frac{\partial f}{\partial x}x'(t) + \frac{\partial f}{\partial y}y'(t) = (2x \sin y)x'(t) + (x^2 \cos y)y'(t) \\ &= \boxed{(2x(t) \sin y(t))x'(t) + (x(t)^2 \cos y(t))y'(t)}.\end{aligned}$$

Problem 2. (5 points) Assume $x = f(t)$ and $y = g(t, s)$. If $z = \sin x \cos y$, compute $\frac{\partial z}{\partial t}$.

By the Chain Rule, $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$. We expand this as follows:

$$\begin{aligned}\frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} &= (\cos x \cos y) \frac{\partial f}{\partial t} + (-\sin x \sin y) \frac{\partial g}{\partial t} \\ &= (\cos f(t) \cos g(t, s)) \frac{\partial f}{\partial t} + (-\sin f(t) \sin g(t, s)) \frac{\partial g}{\partial t} = \boxed{\cos f(t) \cos g(t, s)f'(t) - \sin f(t) \sin g(t, s)g_t(t, s)}.\end{aligned}$$