Quiz 5
MATH 241
Spring 2023

Name:______________________
Section:______________________

1. Let $f = x^2y + x$

   (a) (3 pts) Compute $\nabla f(1,3)$
   Solution:

   $$\nabla f(x,y) = (2xy+1)\boldsymbol{i} + x^2\boldsymbol{j}$$
   $$\nabla f(1,3) = 7\boldsymbol{i} + \boldsymbol{j}$$

   (b) (3 pts) Compute the directional derivative of f at $(1,3)$ in the direction of $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j}$
   Solution:

   $$\boldsymbol{u} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|} = \frac{1}{\sqrt{5}}\boldsymbol{i} + \frac{2}{\sqrt{5}}\boldsymbol{j}$$

   $$\begin{aligned} D_{\boldsymbol{u}}f(1,3) &= \boldsymbol{u} \cdot \nabla f(1,3) \\ &= \frac{7}{\sqrt{5}} + \frac{2}{\sqrt{5}} = \frac{9}{\sqrt{5}} \end{aligned}$$

2. (4 pts) Compute the tangent plane to $g(x,y) = x^2 + 4y^2 - y$ at $(2,2)$.
   Solution: The normal vector of the plane at some point (x,y) is $(g_x, g_y, -1)$. The partial derivatives are

   $$\begin{aligned} g_x(x,y) &= 2x \\ g_y(x,y) &= 8y - 1. \end{aligned}$$

   Thus, the normal vector of the plane at $(2,2)$ is $(4,15,-1)$ and $g(2,2) = 18$. The tangent plane is given by

   $$4(x-2) + 15(y-2) - (z-18) = 0$$

   or

   $$z = 18 + 4(x-2) + 15(y-2)$$