Quiz 7 **Name:** _____

Math 241: Spring 2023 **Section:** _____

Problem 1. Consider the solid region bounded by the plane $2x+y+z=2$ and the three coordinate planes.

(a) (3 points) Set up the integral for computing the volume of this region (express the bounds for y as functions of x, and the bounds for z as functions of x, y).

(b) (2 points) Evaluate the integral.

(a):

$$\int_0^1 \int_0^{2-2x} \int_0^{2-2x-y} 1 \, dz \, dy \, dx$$

(b):

$$\int_0^1 \int_0^{2-2x} \int_0^{2-2x-y} 1 \, dz \, dy \, dx = \int_0^1 \int_0^{2-2x} 2-2x-y dy dx = \int_0^1 (2-2x)^2 - \frac{(2-2x)^2}{2} dx =$$
$$= \frac{1}{2}\int_0^1 (2-2x)^2 dx = \frac{1}{12}(2^3-0^3) = \frac{2}{3}$$

Problem 2. (5 points) Let D be the solid region in the first octant bounded by the spheres $x^2 + y^2 + z^2 = 4$ and $x^2+y^2+z^2=16$. Set up the integral $\int\int\int_D (x^2+z+1)\, dV$ in spherical coordinates (You don't need to evaluate it).

$$0 \le \theta \le \pi/2, \ 0 \le \phi \le \pi/2, \ 2 \le \rho \le 4$$
$$x = \rho sin\phi \, cos\theta \Rightarrow x^2 = \rho^2 \, sin^2\phi \, cos^2\theta$$
$$z = \rho cos\phi$$
$$\Rightarrow \int\int\int_D (x^2+z+1) dV = \int_0^{\pi/2} \int_0^{\pi/2} \int_2^4 (\rho^2 \, sin^2\phi \, cos^2\theta + \rho cos\phi + 1)\rho^2 sin\phi d\rho d\phi d\theta$$