Quiz 9
MATH 241 **Name:**______________________
Spring 2023 **Section:**______________________

1. (6 pts) Use Green's theorem to compute the line integral

$$\int_C xydx + (x^2 + y)dy$$

where C is the circle $x^2 + y^2 = 9$.
Solution: Setting $P = xy$ and $Q = x^2 + y$ the curl is given by

$$\begin{vmatrix} \partial_x & \partial_y \\ xy & x^2 + y \end{vmatrix} = 2x - x = x$$

Then, by Green's theorem and by converting to polar coordinates

$$\begin{aligned} \int_C \ldots &= \int_0^{2\pi} \int_0^3 r\cos(\theta) r dr d\theta \\ &= \int_0^{2\pi} \frac{r^3}{3}\Big|_0^3 \cos(\theta) d\theta \\ &= \int_0^{2\pi} 9\cos(\theta) d\theta = 0. \end{aligned}$$

This can also be checked by symmetry.

2. (4 pts) Use a surface integral to compute the surface area of the plane $z = 2 - 2x - 2y$ contained in the box with corners $(0, 0), (1, 0), (1, 1)$, and $(0, 1)$ in the xy plane.
Solution: The scaling in the dS term is

$$\begin{aligned} dS &= \sqrt{z_x^2 + z_y^2 + 1} dA \\ &= \sqrt{4 + 4 + 1} dA = 3dA. \end{aligned}$$

Thus the surface area is simply

$$\int_0^1 \int_0^1 3dxdy = 3.$$