Thursday Practice 1 | **Name:** ____________________
MATH 241 (Spring 2023) | **Section:** 0112 (8AM-9AM) / 0122 (9:30AM-10:20AM)
02/02/2023 | **TA:** Shin Eui Song

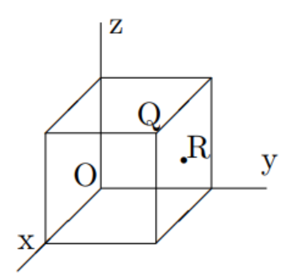
Problem 1. Find the perimeter of the triangle with vertices $(1, 0, 3), (0, -2, 1)$ and $(3, 2, 1)$.

Problem 2. Find an inequality satisfied by all points that belong to the closed ball with radius 6 and center $(-2, 0, 7)$.

Problem 3. A unit cube lies in the first octant, with a vertex at the origin (see figure).

(a) Express the vectors $\overrightarrow{OQ}$ (a diagonal of the cube) and $\overrightarrow{OR}$

(b) Find the cosine of the angle between $\overrightarrow{OQ}$ and $\overrightarrow{OR}$.



We illustrate why it is natural to look at dot products. You are **not** required to learn the following during this course. In short, we wanted to define a function that inputs two vectors and outputs a number with *properties* that a nice product should have, and there is only one such product.

Problem 4. Consider **a product** $*$ (we don't know what product it is at the moment) satisfying the following properties:

(i) $\boldsymbol{v} * \boldsymbol{w} = \boldsymbol{w} * \boldsymbol{v}$,

(ii) $(a\boldsymbol{v}) * \boldsymbol{w} = a(\boldsymbol{v} * \boldsymbol{w})$,

(iii) $\boldsymbol{v} * \boldsymbol{v} = ||\boldsymbol{v}||^2$,

(iv) $(\boldsymbol{v}_1 + \boldsymbol{v}_2) * \boldsymbol{w} = (\boldsymbol{v}_1 * \boldsymbol{w}) + (\boldsymbol{v}_2 * \boldsymbol{w})$,

(v) $\boldsymbol{v} * (\boldsymbol{w}_1 + \boldsymbol{w}_2) = (\boldsymbol{v} * \boldsymbol{w}_1) + (\boldsymbol{v} * \boldsymbol{w}_2)$.

for vectors $\boldsymbol{v} = (a_1, b_1, c_1)$, $\boldsymbol{w} = (a_2, b_2, c_2)$, and a scalar a. Then

(a) Using the (i) and (iv), show (v).

(b) Show that $||\boldsymbol{v} + \boldsymbol{w}||^2 = ||\boldsymbol{v}||^2 + ||\boldsymbol{w}||^2 + 2(\boldsymbol{v} * \boldsymbol{w})$.
(Hint: Observe that $||\boldsymbol{v} + \boldsymbol{w}||^2 = (\boldsymbol{v} + \boldsymbol{w}) * (\boldsymbol{v} + \boldsymbol{w})$ by (iii). Use (iv) and (v) to expand it out and organize.)

(c) Show that $\boldsymbol{v} * \boldsymbol{w}$ is the dot product used in our class. In other words, show that

$$\boldsymbol{v} * \boldsymbol{w} = a_1 b_1 + a_2 b_2 + a_3 b_3$$