Thursday Practice 2	**Name:**	
MATH 241 (Spring 2023)	**Section:**	0112 (8AM-9AM) / 0122 (9:30AM-10:20AM)
02/09/2023	**TA:**	Shin Eui Song

Problem 1.

(a) $\vec{r} = (-3, 5, 1) + t(2, 7, -3)$

(b) $x = -2 + t, \qquad y = 8 - 3t, \qquad z = 13 + 6t$

(c)

$$\frac{x+4}{5} = \frac{y-1}{10} = \frac{z-8}{4}$$

Problem 2. A normal vector of $\mathscr{P}$ is $(1, 1, -1)$.

(a) A normal vector of this plane is $(3, -1, 2)$. The dot product is $(3, -1, 2) \cdot (1, 1, -1) = 3 - 1 - 2 = 0$. Hence this plane is **perpendicular** to $\mathscr{P}$.

(b) If we multiply $-\frac{1}{3}$ on both sides of this equation, we obtain the equation of the plane $\mathscr{P}$. Hence this plane is **identical** to $\mathscr{P}$.

(c) A normal vector of this plane is $(2, 1, -3)$. This is not a multiple of $(1, 1, -1)$ and the dot product $(2, 1, -3) \cdot (1, 1, -1) = 2 + 1 + 3 = 6 \neq 0$. Hence this plane is **neither identical, parallel, nor perpendicular** to $\mathscr{P}$.

(d) A normal vector of this plane is $(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$. We have $\frac{1}{2} \cdot (1, 1, -1) = (\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$. However, they are not identical because $(3, 0, 0)$ is a point on $\mathscr{P}$, but not a point on this plane. Hence this plane is **parallel** to $\mathscr{P}$.

Problem 3. To find an equation of the plane, we need to find a normal vector of the plane. Normal vector is computed using two non-parallel vectors on a plane. One of the vector is a vector parallel to the line which is $(1, 1, 2)$ by reading off the denominators. To find the second vector, note that the line contains a point $(-2, -1, -5)$. The difference $(1, -1, 2) - (-2, -1, -5) = (3, 0, 7)$ gives you the second vector on the plane. We take the cross product to find a vector perpendicular to both.

$$\begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ 1 & 1 & 2 \\ 3 & 0 & 7 \end{vmatrix} = 7\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}} - 3\hat{\boldsymbol{k}}$$

Hence the equation of the plane is

$$\begin{aligned} & 7(x+2) - (y+1) - 3(z+5) = 0 \\ \Rightarrow \quad & 7x - y - 3z = 2 \end{aligned}$$

Problem 4. Since the equation defines a plane, one of a, b, or c is nonzero. This means that one of $(\frac{d}{a}, 0, 0)$, $(0, \frac{d}{b}, 0)$ or $(0, 0, \frac{d}{c})$ is on the plane. Note that these only make sense for a, b, and c that is nonzero. Let us choose one that makes sense, and call the point P. By considering each case, we have $(a, b, c) \cdot \overrightarrow{PO} = -d$.

Let $\vec{n} = (a, b, c)$ be the normal vector of the plane. Then the distance D is

$$\frac{|\vec{n} \cdot \overrightarrow{PO}|}{||\vec{n}||} = \frac{|-d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$$

Problem 5. Any point on the line ℓ has the form $\vec{r}_0 + t\vec{L}$. This means that $\vec{a} = \vec{b} + t_0\vec{L}$ for some real number t_0. (To see this, let $\vec{a} = \vec{r}_0 + t_1\vec{L}$ and $\vec{b} = \vec{r}_0 + t_2\vec{L}$. Then $\vec{a} - \vec{b} = (t_1 - t_2)\vec{L}$. Let $t_0 = t_1 - t_2$, then $\vec{a} = \vec{b} + t_0\vec{L}$.) Then one has

$$\vec{L} \times \vec{a} = \vec{L} \times (\vec{b} + t_0\vec{L}) = \vec{L} \times \vec{b} + \vec{L} \times t_0\vec{L} = \vec{L} \times \vec{b}$$

The last equality follows from the fact that the cross product of two parallel lines is zero.