Thursday Practice 4 Solutions
MATH 241 (Spring 2023)
03/02/2023

Problem 1.

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$$

Then

$$\begin{aligned} \frac{\partial w}{\partial s} &= (y\sin(z^2)) \cdot 1 + (x\sin(z^2)) \cdot (2s) \\ &= s^2\sin(t^4) + 2s(s-t)\sin(t^4) \end{aligned}$$

Problem 2. Taking partial x on both sides, we get

$$1 - y\frac{\partial z}{\partial x} - \sin(xyz)\frac{\partial}{\partial x}(xyz) = 0$$

Product rule says that

$$\frac{\partial}{\partial x}(xyz) = yz + xy\frac{\partial z}{\partial x}$$

Hence we get

$$1 - y\frac{\partial z}{\partial x} - \sin(xyz)\left(yz + xy\frac{\partial z}{\partial x}\right) = 0$$

After expanding out and reordering we obtain

$$1 - yz\sin(xyz) = (y + xy\sin(xyz))\frac{\partial z}{\partial x}$$

Hence one has

$$\frac{\partial z}{\partial x} = \frac{y + xy\sin(xyz)}{1 - yz\sin(xyz)}$$

Problem 3. Chain rule says

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$

Notice here that we used d for dw/dt since z can be considered as a single-variable function of t. But we used ∂ for w as a function of x and y because there are two variables. Then $\frac{dx}{dt} = x'(t)$ and $\frac{dy}{dt} = y'(t)$. Hence

$$\frac{dw}{dt} = \cos(y(t)^2) \cdot x'(t) - 2x(t)y(t)\sin(y(t)^2) \cdot y'(t)$$

Problem 4. If we take partial with respect to x of $F(x, y, z(x, y)) = 0$, we get

$$\frac{\partial F}{\partial x}\cancelto{1}{\frac{dx}{dx}} + \frac{\partial F}{\partial y}\cancelto{0}{\frac{dy}{dx}} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x} = 0$$

Then we get

$$F_z(x,y,z)\frac{\partial z(x,y)}{\partial x} = -F_x(x,y,z) \Rightarrow \frac{\partial z(x,y)}{\partial x} = -\frac{F_x(x,y,z)}{F_z(x,y,z)}$$

Problem 5. Let $u = tx$ and $v = ty$. Then

$$\frac{d}{dt}f(u,v) = \frac{\partial f}{\partial u}\frac{du}{dt} + \frac{\partial f}{\partial v}\frac{dv}{dt} = f_u x + f_v y$$

$$\frac{d}{dt}t^n f(x,y) = nt^{n-1}f(x,y)$$

If $t = 1$, then $u = x$ and $v = y$. Hence we obtain

$$xf_x(x,y) + yf_y(x,y) = nf(x,y)$$