Thursday Practice 4	**Name:**	______________________
MATH 241 (Spring 2023)	**Section:**	0112 (8AM-9AM) / 0122 (9:30AM-10:20AM)
03/02/2023	**TA:**	Shin Eui Song

Problem 1. Let

$$w = xy\sin(z^2), x = s - t, y = s^2, z = t^2$$

Find $\frac{\partial w}{\partial s}$ by using chain rule.

Problem 2. Let z be implicitly defined function of x and y by the equation

$$x - yz + cos(xyz) = 2$$

Find $\frac{\partial z}{\partial x}$.

Problem 3. Let $w = x\cos(y^2)$, $x = \sin t$, and $y = t^2$. Find $\frac{dw}{dt}$ in terms of $x(t)$, $y(t)$, $x'(t)$ and $y'(t)$.

Problem 4. (Challenge) Let F be a differentiable function of x, y, and z. Suppose $z = z(x, y)$ is a function of x, y such that $F(x, y, z(x, y)) = 0$. Prove using the chain rule that

$$\frac{\partial z(x,y)}{\partial x} = \frac{-F_x(x,y,z)}{F_z(x,y,z)}$$

Problem 5. (Challenge) Let f be a function such that

$$f(tx, ty) = t^n f(x, y)$$

for all real number t. Show that

$$x f_x(x,y) + y f_y(x,y) = n f(x,y)$$

Hint for Problem 4: Take $\frac{\partial}{\partial x}$ on the equation $F(x, y, z) = 0$.
Hint for Problem 5: Let $u = tx$ and $v = ty$ and differentiate both side with respect to t. Set $t = 1$.