Worksheet 17	**Name:**	________________________
MATH 240 (Spring 2024)	**Section:**	0111(12PM-1PM) / 0121(1PM-2PM)
Apr 4th, 2024	**TA:**	Shin Eui Song

1. Concept

Definition 1.1. Let A be an $n \times n$-matrix. A nonzero vector $x \in \mathbb{R}^n$ such that $Ax = \lambda x$ for some scalar $\lambda \in \mathbb{R}$ is called an **eigenvector**. If such λ exists, we call λ the **eigenvalue** of x.

Definition 1.2. Let $\lambda \in \mathbb{R}$. The **eigenspace** of λ is the null space $\mathrm{Null}(A - \lambda I)$.

2. Suggested Problems

Problem 1 (5.1.1)**.** Is $\lambda = 2$ an eigenvalue of $\begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}$? Why or why not?

Problem 2 (5.1.3)**.** Is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}$? If so, find the eigenvalue.

Problem 3 (5.1.9, 5.1.10)**.** Find a basis for the eigenspace corresponding to each listed eigenvalue.

(a)

$$A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}, \lambda = 1, 5$$

(b)

$$A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}, \lambda = 4$$

Problem 1. Setting $A = \begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix}$ and $\lambda = 2$, we have

$$A - \lambda I = \begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

Since the linear system associated to $(A - \lambda I)x = 0$ has a free variable, λ is an eigenvalue of A.

Problem 2.

$$\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 29 \end{bmatrix}$$

As $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 29 \end{bmatrix}$ are not scalar multiple of each other, $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ is not a eigenvector.

Problem 3.

(a)

	$A - \lambda I$	rref	basis
$\lambda = 1$	$\begin{bmatrix} 4 & 0 \\ 2 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$
$\lambda = 5$	$\begin{bmatrix} 0 & 0 \\ 2 & -4 \end{bmatrix}$	$\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$	$\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$

(b)

	$A - \lambda I$	ref	basis
$\lambda = 4$	$\begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}$	$\begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix}$	$\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$ or $\left\{ \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} \right\}$