Worksheet 23	**Name:**	____________________
MATH 240 (Spring 2024)	**Section:**	0111(12PM-1PM) / 0121(1PM-2PM)
Apr 25th, 2024	**TA:**	Shin Eui Song

1. **Suggested Problems**

Problem 1 (6.4.5*). Let

$$A = \begin{bmatrix} 1 & 7 \\ -4 & -7 \\ 0 & -4 \\ 1 & 1 \end{bmatrix}$$

and let $W = \mathrm{Col}(A)$.

(a) Use the Gram-Schmidt process to produce an orthogonal basis for W.

(b) Find a basis of $W^{\perp}$.

Problem 2 (6.4.11). Find an orthogonal basis for the column space of

$$\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$$

Problem 3 (6.4.13). The columns of Q were obtained by applying the Gram-Schmidt process to the columns of A. Find an upper triangular matrix R such that $A = QR$.

$$A = \begin{bmatrix} 5 & 9 \\ 1 & 7 \\ -3 & -5 \\ 1 & 5 \end{bmatrix}, \quad Q = \begin{bmatrix} 5/6 & -1/6 \\ 1/6 & 5/6 \\ -3/6 & 1/6 \\ 1/6 & 3/6 \end{bmatrix}$$

Problem 4 (6.4.15). Find a QR factorization of the matrix in Problem 2.

Problem 5 (6.4.17-22). All vectors and subspaces are in $\mathbb{R}^n$. Mark each statement **(T/F)**.

(a) **(T/F)** If $\{v_1, v_2, v_3\}$ is an orthogonal basis for W, then multiplying v_3 by a scalar c gives a new orthogonal basis $\{v_1, v_2, cv_3\}$.

(b) **(T/F)** If $W = \mathrm{Span}\{x_1, x_2, x_3\}$ with $\{x_1, x_2, x_3\}$ linearly independent, and if $\{v_1, v_2, v_3\}$ is an orthogonal set in W, then $\{v_1, v_2, v_3\}$ is a basis of W.

(c) **(T/F)** The Gram-Schmidt process produces from a linearly independent set $\{x_1, \ldots, x_p\}$ an orthogonal set $\{v_1, \ldots, v_p\}$ with the property that for each k, the vectors $v_1, \ldots, v_k$ span the same subspace as that spanned by $x_1, \ldots, x_k$.

(d) **(T/F)** If x is not in a subspace W, then $x - \mathrm{proj}_W x$ is not zero.

(e) **(T/F)** If $A = QR$, where Q has orthonormal columns, then $R = Q^T A$.

(f) **(T/F)** In a QR factorization, say $A = QR$ (when A has linearly independent columns), the columns of Q form an orthonormal basis for the column space of A.