Stat 410 Sample Problems for Test 1

Instructions. The test could consist of 4 or 5 problems of the level of difficulty and rough extent of coverage of these. The answers (e.g., for (IV)) need not be given as decimal numbers but must be written out to the point where only simple arithmetic would be needed to get from your expression to a numerical answer.

(I). *Probability axioms and set operations.* Suppose that a probability P defined on a sample space S assigns probabilities

$$P(A) \;=\; P(B) \;=\; P(C) \;=\; \frac{2}{5}$$

to three events $A, B, C \subset S$. Using a Venn diagram or any other method, find the smallest and largest possible values (subject to the information given and P being a Probability) for $P(A \cap B)$ if

$$B \cap C \;=\; \emptyset \quad \text{and} \quad P(A \cap C) \;=\; \frac{1}{10}$$

(II). *Independence and Conditional Probability.* An airplane has two engines on each wing. In order for it to fly without crashing, at least one engine on each wing must be functioning. If each of the four engines functions or not independently of the others, and if each engine fails with probability p, then find a formula (in terms of p) for the probability that the airplane can fly.

(III). *Sample Space & Bayes Rule.* I have four coins in my pocket. Two are fair (i.e., have probability $1/2$ of coming up heads when I flip them); one is loaded in favor of heads, with probability $3/4$ of coming up heads on each toss; and the last has heads probability $1/4$ on each toss. I first pick a coin at random from my pocket and toss it independently two times.

(a) Carefully define a sample space S whose elements specify the fully detailed results of this experiment, and exhibit as a subset of S the event that the tosses result in 1 Head and 1 Tail (in either order).

(b) Find the probability $P(1 \text{ Head } \& \text{ } 1 \text{ Tail})$ and the conditional probability $P(\text{Coin favoring H} \,|\, 1H\&1T)$.

(IV). *Counting Methods.* What is the probability that of three cards dealt from a well-shuffled deck of 52, at least 2 are either picture cards (denomination J, Q, or K) or from the Heart suit ?

(V). *Probability Distribution of a Discrete Random Variable.* A deck of 12 cards — four Aces, four Kings, and four Queens — is shuffled carefully and all 12 cards are dealt out to four people sitting in a row.

(a). Find the probability distribution for the random variable X defined as the largest number of Aces dealt to any of the 4 people. (*This number is* 1 *if each person receives one ace,* 2 *if either two people each are dealt two Aces or if one person receives two Aces and two other people receive one Ace, etc.*).

(b). What is the equiprobable sample space that you used in solving part (a) ?

(c). Find the expectation of this random variable.

(VI). *Random variables based on repeated trials.* A fair die is tossed repeatedly and independently until the first time that 2 or 4 dots appears. Let X denote the random variable equal to the number of tosses until 2 or 4 dots appears, and Y be the random variable equal to the number of dots appearing on the X'th toss.

(a) Find $E(X)$, $E(Y)$, and $E(X+Y)$.

(b) Are the events $[X < 10]$ and $[Y = 4]$ independent ?

(VII). *Expectation of a function of a random variable.* A fair die is tossed twice, independently. Let the random variable Z be defined as the number of dots seen on the first toss minus the number of dots seen on the second toss. Find the expected value of Z^2.