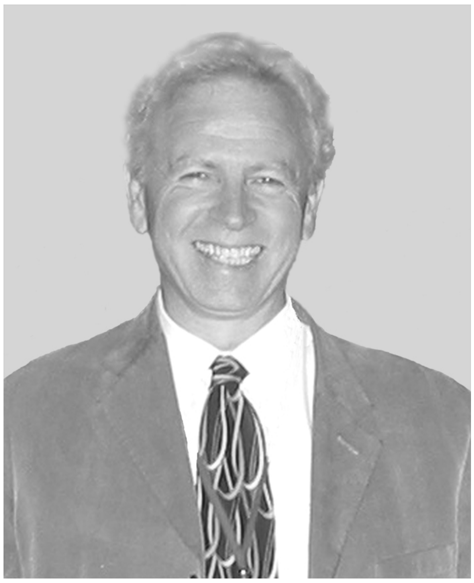
COMPUTATIONAL METHODS IN APPLIED MATHEMATICS, Vol. 4 (2004), No. 3, pp. 265–270

Eitan Tadmor – 50



Professor Eitan Tadmor turned 50 earlier this year. He is one of the most active and influential mathematicians in the area of numerical analysis, general theory of applied PDEs, and scientific computing. He has influenced the field of applied mathematics in many ways: through his deep and broad mathematical research, his strong efforts in advising, training, and mentoring young scientists, and active participation in the scientific life of the international mathematical community by holding important administrative positions and serving on editorial boards of leading mathematical journals. The aim of this article is to briefly summarize some of his major scientific achievements and his contributions to various branches of the field of mathematics.

Eitan Tadmor did his both undergraduate and graduate studies at the Tel Aviv University in Israel. He earned the M.Sc. degree in mathematics in 1975 under the supervision of Professors Gideon Zwas and Moshe Goldberg, and the Ph.D. degree in mathematics in 1979 under the supervision of Professor Saul S. Abarbanel. After completing the dissertation, he held a postdoctoral position at the California Institute of Technology (Cal. Tech.) and at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, and afterward he had regular faculty positions at the Tel Aviv University (1983–1998) and at the University of California, Los Angeles (UCLA) (1995–present). Currently, Eitan Tadmor is at the University of Maryland where he holds a position of a director of the Center for Scientific Computation and Mathematical Modeling (CSCAMM) and also professor positions at the Institute for Physical Science and Technology (IPST) and at the Department of Mathematics.

During his fruitful career, Eitan Tadmor has had many scientific collaborators (among them are G.-Q. Chen, D. Gottlieb, J. Goodman, T. Hou, P.-L. Lions, Y. Maday, S. Osher, B. Perthame, C.-W. Shu, and other excellent scientists), graduate students (among them are T. Tassa, H, Nessyahu, A. Kurganov, D. Levy, C.-T. Lin, J. Tanner, S. Nezzar, J. Balbás), and post-docs. Eitan also actively serves the mathematical community by being an editor in several leading journals in applied and computational mathematics, including SIAM Journal on Numerical Analysis, Numerische Mathematik, M^2AN Mathematical Modelling and Numerical Analysis, Journal of Hyperbolic Differential Equations, IMA Journal of Numerical Analysis, and others. One of Tadmor's biggest administrative achievements was a successful bid of UCLA for the site of the third national NSF institute for mathematical research — Institute for Pure and Allied Mathematics (IPAM), where he served as a director in the period 2000–2001.

Most of Tadmor's early works are related in one way or another to linear PDEs. His very first works were on the numerical radius of linear bounded operators in finite- and infinite-dimensional Hilbert spaces. A main result was the application of the methods to the stability theory of the Lax-Wendroff scheme for linear multidimensional hyperbolic problems [6]. In his remarkable work [37], Tadmor sharpened the known results regarding the equivalence between the resolvent condition and the L_2-stability of a family of $N \times N$ matrices as stated by the Kreiss matrix theorem. Specifically, he showed that the power-boundedness depends, at most, linearly on the dimension N. This result improved the best known bound at the time which scaled like N^2 (compared with the original $(r(A))^{N^N}$). The optimal result was proved by Spijker confirming the linear bound and placing the constant e in front of N.

The work that emerged from Tadmor's Ph.D. dissertation can be characterized as "beyond GKS". In 1972, Gustafsson, Kreiss, and Sundström published a stability criterion for numerical approximations of initial-boundary value problems. In a series of works, most of which were co-authored with Moshe Goldberg, Eitan has gradually improved the stability criterion for general classes of approximations (see [7, 8] and the references therein). An important account of stability results for various approximations of solutions to linear initial value hyperbolic problems is presented in [40]. There, Tadmor provided a unified framework for the stability analysis of finite-difference, pseudo-spectral, and Fourier-Galerkin methods.

More general questions of stability were dealt with in connection with ODE solvers. The work of Tadmor and Levy on the stability of Runge-Kutta schemes [20] established the stability of high-order discretizations in time provided that the first-order method is stable. This is valid, for example, for coercive operators (that can be viewed as dissipative operators in the sense of Kreiss). When put into the framework of specific numerical approximations, this result enables one to extend the results of Tadmor and Gottlieb [12] on the stability of pseudo-spectral spatial discretizations of linear hyperbolic equations with variable coefficients with a forward Euler temporal discretization to higher-order discretizations in time. Strong-stability preserving high-order time discretizations for semidiscrete methods of lines approximations of PDEs were presented in [13]. These schemes extend the class of schemes of Shu and Osher, previously termed total variation diminishing (TVD) Runge-Kutta methods.

In the mid-1980s, Tadmor's research was focused on nonlinear finite-difference approximations, their total variation, and entropy stability. In [38], Tadmor introduced the notions of a numerical viscosity coefficient and a viscosity form of finite-difference schemes. He proved that a general conservative $(2p+1)$-point scheme is TVD provided its viscosity coefficients satisfy certain lower and upper bounds (this introduced a new framework for total variation stability analysis of finite-difference schemes). It was also demonstrated in [38] that the entropy stability of the schemes can also be expressed in terms of the corresponding bounds on the numerical viscosity coefficients. In [32], Tadmor and Osher utilized the numerical viscosity framework to prove convergence of second-order TVD schemes for scalar conservation laws with convex fluxes by enforcing a single discrete entropy inequality. Second-order entropy conservative schemes for one-dimensional systems of conservation laws were introduced by Tadmor in [39]. These schemes were used to show that conservative schemes are entropy stable if and — for three-points schemes — only if they contain more viscosity than that present in the entropy conservative schemes. This framework has been recently extended in Tadmor's milestone work [43], where a detailed study of entropy stability for a host of first- and second-order schemes for both scalar problems and systems of conservation laws.

Another area of Tadmor's interests is the theory of spectral methods. It is well known that spectral projections are high order accurate provided the data is globally smooth but they exhibit spurious Gibbs type oscillations near jump discontinuities of the underlying function. This deteriorates their efficiency to first order for piecewise smooth data. Tadmor and his collaborators investigated the stability of spectral and pseudospectral approximation for linear scalar equations and linear systems of conservation laws in [9–12]. Unfortunately, the oscillations in spectral methods lead to their instability when applied to nonlinear conservation laws. To overcome this fundamental difficulty, Tadmor proposed the spectral viscosity (SV) method and proved the convergence of such SV-approximations via compactness arguments in [41]. The resulting SV-approximation is stable without sacrificing spectral accuracy but the method is at best first order as a result of the formation of shock discontinuities in weak solution. This raised the important questions of accurate edge detection (also of independent interest) and elimination of the oscillations in the SV-approximation to underlying piecewise smooth solution. In a sequence of papers with Gelb, Tadmor developed the theory of edge detectors and applied it to identify discontinuities and post-process the SV-approximations [3–5]. The result was the enhanced SV-method which recovers the exact entropy solutions with high resolution and eliminates the oscillations present in the regular SV-solution. Recently, Tadmor and Tanner developed a new class of spectral mollifiers that gives optimal recovery of piecewise smooth data from its spectral information [36].

Another important direction of Tadmor's work is related to the development of high resolution numerical methods for (systems of) multidimensional conservation laws and Hamilton-Jacobi (HJ) equations. He was the first to develop high-resolution nonoscillatory central schemes, which became a simple, robust, and universal alternative to more complex and problem-oriented numerical methods based on upwinding. Since no (approximate) Riemann problem solvers or characteristic decomposition are involved, central schemes can be used as a "black-box" solver for various applied problems, including very complicated ones. Tadmor's pioneering joint work with Nessyahu had been a corner stone in the design of such schemes for conservation laws, where the approximate Riemann problem solvers are avoided by integrating over the local Riemann fans. The one-dimensional second-order Nessyahu-Tadmor scheme [30] was extended to third order in [25] and to higher number of spatial dimensions in [14]. A more accurate estimate of the local speed of propagation of the waves generated at the local Riemann fans helps to reduce the amount of numerical dissipation present in central schemes. Tadmor and Kurganov utilized this idea in [16], where a new class of semidiscrete central schemes for multidimensional (systems of) conservation laws was introduced. This type of central schemes is especially advantageous when small time steps are enforced or when a long time integration or steady state computations are to be performed. The central schemes have been successfully applied to a number of complex models, including compressible and incompressible Euler equations and the MHD equations in [1, 15, 18, 19]. Further, Tadmor and Lin extended the Nessyahu-Tadmor scheme to multidimensional HJ equations [21]. They developed the second-order Godunov-type central scheme based on global projection operators and also studied its L^1 stability and error estimates for the Cauchy problem associated with the multidimensional HJ equation [22]. The semidiscrete central schemes for multidimensional HJ equations with reduced numerical dissipation have been developed by Tadmor and Kurganov in [17].

Another avenue of Tadmor's research is the convergence rate estimates for approximate solutions of scalar conservation laws. Tadmor was the first to recognize the importance of the one-sided Lipschitz condition (OSLC) in deriving convergence and error estimates for conservation laws. In his pioneering work [42], Tadmor developed the theory of the Lip^+ stability and used the dual Lip' approach to derive local error estimates for viscosity approximations of scalar conservation laws. This duality approach was later used by Tadmor and his collaborators to derive error estimates, including pointwise estimates away from the discontinuities, for Godunov-type schemes, SV-approximations and relaxation approximations to genuinely nonlinear scalar conservation laws in [29, 31, 34, 35] and for approximate solutions of multidimensional HJ equations in [22].

One of the directions of Tadmor's theoretical work is the investigation of the link between the continuum mechanics models (macroscopic equations, such as Navier-Stokes or Euler equations) and kinetic models arising at a more detailed description of the evolution. More precisely, Tadmor and Perthame [33] studied the hydrodynamical limit of BGK-type models and their relation to the multidimensional scalar conservation laws. Further, in [23] Tadmor, Lions, and Perthame introduced a new kinetic formulation of a general multidimensional scalar conservation law, coupled with the corresponding entropy inequalities, which they have later extended to the case of isentropic gas dynamics and p-systems [24]. This formulation is called kinetic because of its analogy with the classical kinetic models such as the Boltzmann or Vlasov models. It allows the derivation of completely new estimates, compactness and regularity results for solutions of conservation laws in the multidimensional case and new, or extensions of existing, results in the one-dimensional case, such as the celebrated Tartar's result obtained by the study of oscillations and the use of the compensated compactness theory.

Another area of Tadmor's recent interest is critical threshold phenomena in the Euler dynamics. In the break-through paper [2], Tadmor with Engelberg and Liu studied hyperbolic-elliptic Euler-Poisson equations ranging from the one-dimensional case with or without forcing mechanisms to multidimensional isentropic models with geometrical symmetry. A new phenomenon of a critical threshold for global smoothness vs. finite time breakdown was discovered — only waves with sufficiently large initial gradients break down. However, the long time behavior of the solutions is independent of these initial thresholds. These results were extended by Tadmor and Liu to Burgers-type equations with a nonlocal viscous term ([26]) and to the semiclassical limit of the nonlinear Schrödinger-Poisson equations ([27]). For the models with the rotational Coriolis forces, Tadmor and Liu [28] showed that rotation prevents finite-time breakdown. Namely, the rotating two-dimensional Euler equations admit global smooth solutions for a subset of generic initial configurations. For other initial data, the breakdown depends on a critical threshold for the initial vorticity.

It is impossible in such a brief article as this one to comment on all achievements of Professor Eitan Tadmor during his outstanding carrier as a mathematician. We hope that this short note will make some of his major results known to a wider audience and thus, popularize this very important area of applied mathematics.

Many happy returns of the day, good health and all the best.

A. Kurganov, R. Lazarov, D. Levy, G. Petrova, and B. Popov

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