Nonreflecting Boundary Conditions for Wave Propagation in Unbounded Media

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The simulation of waves in unbounded media arises in many applications from acoustics, electromagnetics, or elasticity. Typically, the local phenomenon of interest contains complicated geometric features, inhomogeneity, and possibly nonlinear effects. Modern numerical methods can handle complicated geometry, inhomogeneous media, and nonlinearities. However, they require an artificial boundary $\mathcal{B}$, which truncates the unbounded exterior domain and restricts the region of interest to a finite computational domain Ω. It then becomes necessary to impose a boundary condition at $\mathcal{B}$, which ensures that the solution in Ω coincides with the restriction to Ω of the solution in the unbounded region. Usually various approximate boundary conditions are used, such as the Bayliss-Turkel [1] or Engquist-Majda [2] boundary conditions, which produce some spurious reflection. To eliminate spurious reflection from the artificial boundary, we have devised *exact nonreflecting boundary conditions* for the wave equation [3,4], Maxwell's equations [5], and the elastic wave equation [6,7]. These boundary conditions are *local in time* and involve only first derivatives of the solution. Therefore, they are easy to use with standard finite difference or finite element methods. Numerical examples demonstrate the improvement in accuracy over standard methods.

The accurate simulation of waves at high frequencies or the detailed representation of small scale geometric features requires the use of adaptive mesh strategies. Then, explicit time integrators become prohibitively expensive because of the stringent CFL condition; hence, implicit methods, such as Crank-Nicolson, are typically used, yet they require the solution of a large linear system of equations at every time step. Because of the nonreflecting boundary condition, this linear system is no longer symmetric, unlike the situation in bounded domains. However, it is possible to reformulate the discretized equations by decoupling the additional unknowns needed on the artificial boundary from the interior unknowns [8]. As a consequence the symmetry and positive definiteness of the linear system are restored while the additional computational effort due to the nonreflecting boundary condition becomes negligible.

For multiple scattering problems the use of a single artificial boundary surrounding all scatterers involved becomes prohibitively expensive in memory requirement. Instead, it is necessary to enclose each scatterer within a single separate computational domain. Clearly waves that leave a certain domain, Ω_1, will impinge upon a different domain, Ω_2, at later times; hence they are no longer purely outgoing waves.

To transfer the time-retarded information from Ω_1 to Ω_2 an analytical representation of the solution in the unbounded medium becomes necessary. This analytical representation is inherent to the exact nonreflecting boundary conditions described above and thus naturally leads to *exact transmission boundary conditions* for multiple scattering problems.

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