From Ghost-cell to Conservative Front Tracking

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Abstract

Front tracking method combines Lagrangian particle propagation of the interface and Eulerian finite difference solvers for fluid in subdomains separated by an interface. By separating different finite difference solver in each subdomain and avoiding interpolation of physical quantities across a tracked interface, it maintains a sharp discontinuity of the physical quantities across the interface. In many physical problems, such discontinuity plays an important role in obtaining correct dynamical measurement. A good example is the simulation of the two and three dimensional Rayleigh-Taylor instability with randomly perturbed interface. The prevention of numerical diffusion across the fluid interface has retained the correct buoyancy acceleration and gives a much closer agreement with the experiment.

To solve the conservation laws in fluid physics

$$u_t + f(u)_x = 0,$$

the front tracking method consists of three key components. They are: (1) the Riemann solution for the interface propagation, (2) the finite difference solution in each subdomain bounded by the interface, and (3) the coupling of the Riemann solution and the finite difference solution at the interface. The Riemann solution not only calculates the normal velocity of the interface but also advances the states on both sides of the interface which satisfy the Rankine-Hugoniot condition. For many years since 1980, the interface-interior coupling in the front tracking method has used ghost-cells whose states are extrapolated from the Riemann solution at the interface. For example, suppose the position of the interface is between two cells j and $j+1$, to update the state of the cell j which is in the left of the interface, a second order Godunov-type scheme has

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x}\left(F_{j+1/2,L}^n - F_{j-1/2}^n\right).$$

Since the cells $j+1$ and $j+2$ are in the right side of the interface, the extrapolated states $\overline{U_{j+1}^n}$ and $\overline{U_{j+1}^{n+1}}$ must be used to calculate

$$F_{j+1/2,L}^n = f(U_{j-1}^n, U_j^n, \overline{U_{j+1}^n}, \overline{U_{j+1}^{n+1}})$$

while in the right side of the interface, the flux at the same location $F_{j+1/2,R}^n$ must be calculated through

$$F_{j+1/2,R}^n = f(\overline{U_{j-1}^n}, \overline{U_j^n}, U_{j+1}^n, U_{j+1}^{n+1})$$

Since $F_{j+1/2,L}^n \neq F_{j+1/2,R}^n$, conservation cannot be achieved near the interface.

Our new scheme for cells near the tracked interface is based on the integral form of Euler's equation over a cell with both fixed and moving boundaries.

$$\frac{\partial}{\partial t}\int_V u dV + \int_{S_F} f_n(u) dS + \int_{S_M} (f_n(u) - uv_n) dS = 0 \quad .$$

where v_n is the normal velocity of the moving boundary. We define $f_n(u) - uv_n$ as the dynamic flux in contrast to $f_n(u)$, the stationary flux. To update the states of the two cells j and $j+1$ near the interface, we use

$$\overline{\Delta x}_j^{n+1} U_j^{n+1} = \overline{\Delta x}_j^n U_j^n - \Delta t\left(F_I^{n,L} - F_{j-1/2}^n\right)$$

and

$$\overline{\Delta x}_{j+1}^{n+1} U_{j+1}^{n+1} = \overline{\Delta x}_{j+1}^n U_{j+1}^n - \Delta t\left(F_{j+3/2}^n - F_I^{n,R})\right)$$

here $F_I^{n,L} = f(u_L^n) - s^n u_L^n$ and $F_I^{n,R} = f(u_R^n) - s^n u_R^n$ are dynamic fluxes on the left and right sides of the interface, respectively. Conservation is achieved because $F_I^{n,L} = F_I^{n,R}$ due to Rankine-Hugoniot condition. When the interface crosses the cell center during a time step, cell merge is needed to ensure numerical stability.

The extension of the conservative front tracking into higher dimensional spaces requires the construction of time-space cells and the merge of neighboring finite volume cells at the old and new time steps. In this presentation we will give a detailed account on the implementation of the conservative algorithm in two dimensional space.

1 Presented by Xiaolin Li