Hyperbolic Models of Traffic Flow.

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In this talk, I will first briefly describe a model introduced a few years ago with A. Aw ([1]: "Resurrection of ...", SIAM J. Appl. Math., 2000) in which we replaced the Payne-Whitham (PW) class of "second order" (= 2 equations) continuous models of traffic flow based on the gas dynamics system by a new system. In the PW model, the way the drivers adjust their velocity depends on the x-derivative of the pressure $P(\rho)$, whereas in our model it depends on the Lagangian time-derivative of $P(\rho)$, and we have shown in [1]that this modification suppresses *all* the severe inconsistencies of the PW model, such as cars going backward in some cases !!!

$$\begin{aligned} \partial_t \rho + \partial_x(\rho v) &= 0 \\ \partial_t(\rho w) + \partial_x(v \rho w) &= 0\ ,\ w := v + P(\rho). \end{aligned}$$

I will then describe - joint paper with Aw, Klar and Materne - how this heuristic model can be discretized either in Eulerian coordinates with Lagrangian cells or, see also J. Greenberg, in Lagrangian mass coordinates by the Godunov scheme, call it (God), with - surprisingly - uniform BV-estimates, due to the very special nature both of the system, which admits coinciding shocks and rarefaction waves, and of the discretization. Moreover, this Godunov approximation turns out to be the natural explicit time-discretization of the microscopic "Follow-the-Leader" models.

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= C\frac{v_{i+1} - v_i}{(x_{i+1} - x_i)^{\gamma+1}}, \end{aligned} \tag{1}$$

where $x_i(t), v_i(t), i = 1, \ldots,$ are location and speed of the vehicles at time t.
In fact, the model (1) introduced in [1] can be *rigorously* viewed as the fluid limit of this microscopic model, with an appropriate function P. The precise relations between the three levels are the following :

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(i) Start with the fully discrete system, i.e. the Godunov approximation of the Lagrangian version (1') of (1). In the natural "hyperbolic" scaling, i.e. when we "make a zoom" $(x', t') := \varepsilon(x, t)$, where x is the Lagrangian "mass"" coordinate, then the solution constructed by the Godunov scheme converges to an entropy weak solution of (1'), with uniform BV-estimates.

(ii) Now, starting again with the fully discrete system (God), make the same scaling, but only in time, with a fixed Δx. Then the solution converges to the solution of the "Follow-the-leader" model (3), which therefore inherits the same uniform BV and L^∞ estimates, and turns out to be the semi-discretization of (1').

(iii) In turn, start now with system (3), and let Δx tend to 0. Then, at least for a sub-sequence (see other talks in this Conference for uniqueness results, the solution of (3) converges to an (the) entropy solution of (1'), which is therefore the fluid limit of (2), without passing through any kinetic description.

Finally, I will describe a recent work with P. Bagnerini, in which we consider an extension of (1) or of the equivalent system (1'), with oscillating initial data, and write down the homogenized macrosopic model. Other recent attempts to describe oscillations in experimental data include a joint preprint with J. Greenberg and A. Klar.