High-Resolution Methods for Wave Propagation in Heterogeneous Media

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Wave propagation problems in heterogeneous media are modeled by hyperbolic systems of equations with spatially-varying coefficients or flux functions. These problems arise in numerous applications involving the propagation of acoustic, elastic, or electromagnetic waves, for example. I will discuss the use of high-resolution finite volume methods for such problems, and the development of approximate Riemann solvers for nonlinear examples. I will also present some results on wave propagation in nonlinear periodicly layered media, where dispersive effects arising from the heterogeneity lead to the appearance of solitary waves and perhaps solitons.

Finite volume methods based on Riemann solvers are a natural choice for such problems. Each grid cell is assumed to consist of a single material and the Riemann problem at the interface between two grid cells is solved by determining the waves propagating into each cell, based on the consituitive relations describing the two materials. This Riemann solution can be directly interpreted in terms of reflection and transmission of propagating waves. These methods, when combined with appropriate wave limiters, can yield high-resolution results on problems with sharp interfaces. Multidimensional generalizations can also deal with problems where interfaces are not aligned with the grid.

For nonlinear problems (e.g., nonlinear elasticity in a heterogeneous medium), the exact Riemann solution may not be easy to compute. I will discuss a general approach to developing approximate Riemann solutions for generalized Riemann problems having a discontinuity in the flux function as well as in the data. This general approach to solving conservation laws with spatially varying flux functions is based on decomposing the flux difference into eigenvectors of an approximate Jacobian matrix. This approach can also be related to a generalization of relaxation schemes.

These methods have been applied to nonlinear elasticity equations in a rapidly-varying periodic layered medium and yield some surprising results. For some choices of nonlinear material properties, a pulse breaks up into solitary waves that appear to interact as solitons.