

LI Tatsien (LI Daqian)

Fudan University, Shanghai 200433, China

dqli@fudan.edu.cn

Abstract

By means of introducing the concept of weak linear degeneracy and the method of normalized coordinates, a complete result is presented on the global existence and the blow-up mechanism of C^1 solution to the Cauchy problem for general first order quasilinear hyperbolic systems in 1-D case with small and decaying initial data as follows:

1. The Cauchy problem admits a unique global C^1 solution for all $t \in \mathbb{R}$ if and only if the system is weakly linear degenerate (WLD).

2. If the system is not WLD and the corresponding index α is an integer ≥ 0 , then for a large class of initial data, the first order derivatives of the C^1 solution to the Cauchy problem must blow up in a finite time with a sharp estimate on the life-span and the formation of singularity is due to the envelope of Characteristics of the same kind.

This result implies all previous results obtained by F. John, T. P. Liu and L. Hörmander and can be applied to the following physical situations: the system of nonlinear elasticity, the system of 1-D gas dynamics without convexity, the system of the motion of elastic strings and the system of finite amplitude plane waves for hyperelastic materials such as Hadamard-Green material, neo-Hookean material, St. Venant-Kirchhoff material and Odgen material etc.

References

- 1 Li Ta-tsien, Zhou Yi, Kong De-xing, *Global classical solutions for general quasilinear hyperbolic systems with decay initial data*, Nonlinear Analysis, 28(1997), 1299-1332.
- 2 Li Ta-tsien, Kong De-xing, *Breakdown of classical solutions to quasilinear hyperbolic systems*, Nonlinear Analysis, 40(2000), 407-437.

3 Li Ta-tsien, *Une remarque sur les coordonnées normalisées et ses applications aux systèmes hyperboliques quasi linéaires*, C. R. Acad. Sci. Paris, 331, Série I (2000), 447-452.

4 Kong De-xing, Li Ta-tsien, *A note on blow-up phenomenon of classical solutions to quasilinear hyperbolic systems*, to appear in Nonlinear Analysis.