Introduction to

NUMERICAL ANALYSIS

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Undergraduate course – Applied Mathematics

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Contents

References

GENERAL TEXTBOOKS

[CdB] S. Conte & C. deBoor, *ELEMENTARY NUMERICAL ANALYSIS*, McGraw-Hill, User friendly; Shows how 'it' works; Proofs, exercises and notes

[DB] G. Dahlquist & A. Bjorck, *NUMERICAL METHODS*, Prentice-Hall, User friendly; Shows how 'it' works; Exercises

[IK] E. Isaacson & H. Keller, *ANALYSIS of NUMERICAL METHODS*, Wiley, The 'First'; Proofs; out-dated in certain aspects; Encrypted message in Preface

[RR] A. Ralston & P. Rabinowitz, *FIRST COURSE in NUMERICAL ANALYSIS*, 2nd ed., McGraw-hill, Detailed; Scholarly written; Comprehensive; Proofs exercises and notes

[SB] J. Stoer & R. Bulrisch, *INTRODUCTION TO NUMERICAL ANALYSIS*, Springer-Verlag, detailed account on approximation, linear solvers & eigensolvers, ODE solvers,..

[W] B. Wendroff, *THEORETICAL NUMERICAL ANALYSIS*, Academic Press, 1966, Only the 'Proofs'; elegant presentation

APPROXIMATION THEORY

[C] Cheney, *INTRODUCTION TO APPROXIMATION THEORY*, Classical

[D] P. Davis, *INTERPOLATION & APPROXIMATION*, Dover very readable

[R] Rivlin, *AN INTRODUCTION to the APPROXIMATION of FUNCTIONS*, Classical

NUMERICAL INTEGRATION

[DR] F. Davis & P. Rabinowitz, *NUMERICAL INTEGRATION* Everything

1 Approximation Theory

On the choice of norm: L^∞ vs. L^2
Weirstrass' density theorem, Bernstein polynomials,

1.1 Least squares approximations

Gramm mass matrix, ill-conditioning of monomials in L^2

1.1.1 Fourier expansion

Bessel, Parseval, ...

1.1.2 Orthogonal polynomials

Examples: Legendre, Chebyshev,...
3-term recurssion formulae, Sturm sequence
more examples: Jacobi, Hermite,..

1.1.3 Trigonometric polynomials

Complex & real representations; Chebyshev transformation,..

1.2 Interpolation

1.2.1 Algebraic interpolation

1.2.2 ... Lagrange interpolant

1.2.3 ... Newton interpolant

Synthetic calculus with the translation operator, Forward backward and centered formulae,

1.2.4 ... Error estimates

Runge effect, region of analyticity

1.2.5 Interpolation with derivatives

1.2.6 ... Hermite interpolation

1.2.7 ... Splines

mass matrix, variational characterization, error estimates,

1.2.8 ... Error estimates

1.2.9 Trigonometric interpolation

FFT, truncation + aliasing, error estimates,
applications: polynomial approximation (Chebyshev interpolation), elliptic solvers, fast summations (- discrete convolution), conformal mappings,...

1.3 MinMax approximation

1.4 Smoothness & error estimates

2 Numerical Differentiation

2.1 The method of undetermined coefficients

2.1.1 The differentiated algebraic interpolant

2.1.2 The differentiated spline interpolant

2.1.3 Error estimates

2.2 Equidistant differentiation

2.2.1 The synthetic approach – operational calculus

2.2.2 Richardson extrapolation

2.2.3 Trigonometric differentiation

2.2.4 Spline differentiation

3 Numerical Integration – Quadratures

3.1 Equidistant points

3.1.1 Newton-Cotes formulae

3.1.2 Composite Simpson's rule

3.1.3 Romberg & adaptive integration

3.2 Gauss quadratures