An Interacting Particle Model for Animal Migration

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CSCAMM FRG Workshop

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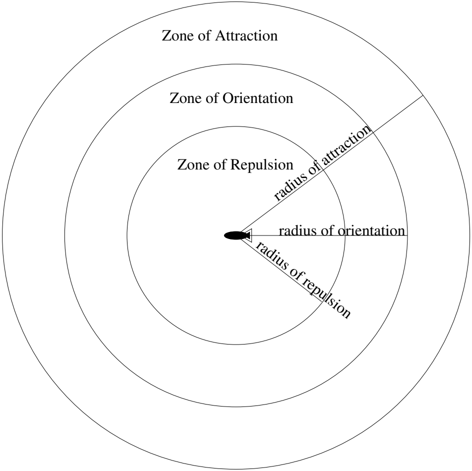
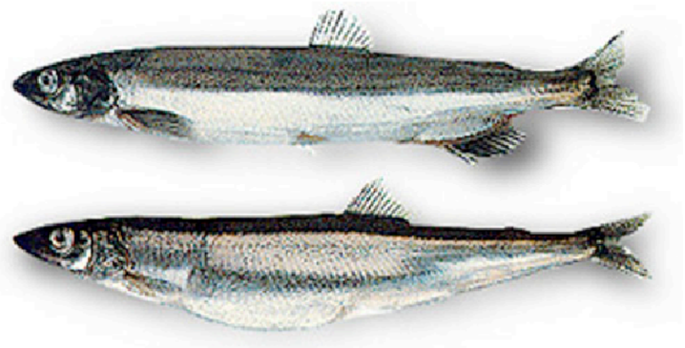
This research was done jointly with Björn Birnir, Baldvin Einarsson, and the Marine Research Institute of Iceland

Interactions in this model





The Icelandic stock of capelin

Our model

$$\begin{pmatrix} x_k(t+\Delta t) \\ y_k(t+\Delta t) \end{pmatrix} = \begin{pmatrix} x_k(t) \\ y_k(t) \end{pmatrix} + \Delta t \cdot v_k(t+\Delta t) \frac{\mathbf{D}_k(t+\Delta t)}{\| \mathbf{D}_k(t+\Delta t) \|} + \mathbf{C}(\tilde{\mathbf{p}}_k(t))$$

- Here, $\mathbf{D}_k$ is the directional heading of particle k
- Δt is the timestep
- Particle k's position in the plane is $\mathbf{p}_k$
- $\tilde{\mathbf{p}}_k$ is the nearest gridpoint to $\mathbf{p}_k$
- $\mathbf{C}(\tilde{\mathbf{p}}_k)$ is the current at $\tilde{\mathbf{p}}_k$

The directional heading of the particles (based on interactions among particles) is determined as follows:

$$\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix} = \frac{\mathbf{d}_k(t+\Delta t)}{\| \mathbf{d}_k(t+\Delta t) \|}$$

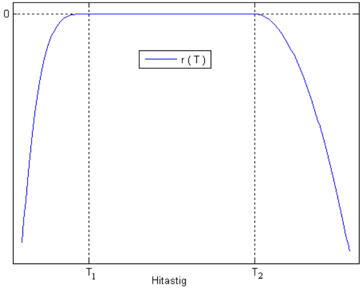
where

$$\mathbf{d}_k(t+\Delta t) := \frac{1}{|R_k| + |O_k| + |A_k|} \Bigg(\sum_{r \in R_k} \frac{\mathbf{p}_k(t) - \mathbf{p}_r(t)}{\| \mathbf{p}_k(t) - \mathbf{p}_r(t) \|} + \sum_{o \in O_k} \begin{pmatrix} \cos(\phi_o(t)) \\ \sin(\phi_o(t)) \end{pmatrix} + \sum_{a \in A_k} \frac{\mathbf{p}_a(t) - \mathbf{p}_k(t)}{\| \mathbf{p}_a(t) - \mathbf{p}_k(t) \|} \Bigg).$$

Temperature function

Function $r(T)$ determines the reaction of the particles to the temperature field.

$$r(T) := \begin{cases} -(T - T_1)^4 & \text{if} \quad T \leq T_1 \\ 0 & \text{if} \quad T_1 \leq T \leq T_2 \\ -(T - T_2)^2 & \text{if} \quad T_2 \leq T \end{cases}$$

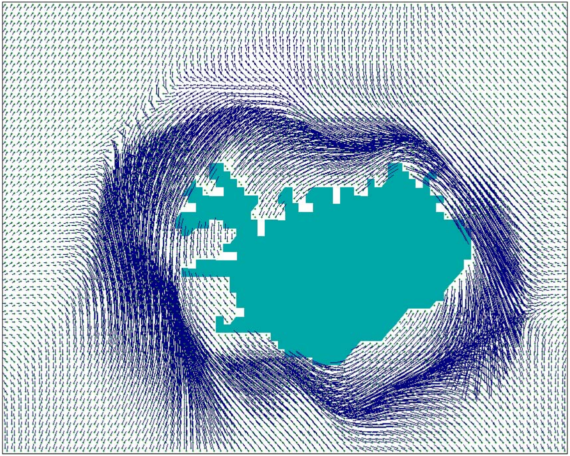


Putting it all together, the directional heading is determined as follows:

$$\mathbf{D}_k(t+\Delta t) := \left(\alpha \underbrace{\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix}}_{\text{interaction term}} + \beta \underbrace{\frac{\nabla r\big(T(\mathbf{p}_k(t))\big)}{\| \nabla r\big(T(\mathbf{p}_k(t))\big) \|}}_{\text{temperature term}} \right)$$

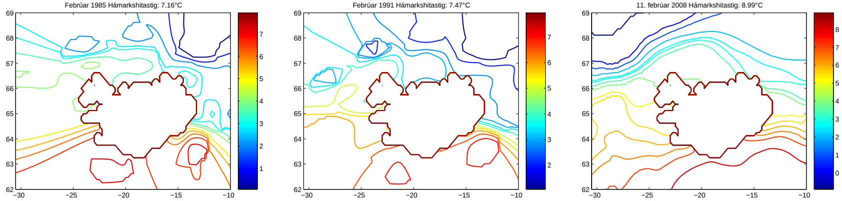
where $\alpha + \beta = 1$.

The currents



The maximum speed is 15 km/day

Temperature fields used in the simulations



February, 1985 (left), February 1991 (middle), February 2008[1] (right)

[1] http://www.wetterzentrale.de/topkarten/fsfaxsem.html

Acoustic data from 1984–1985

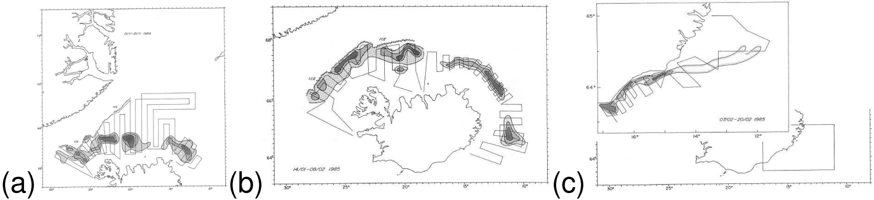
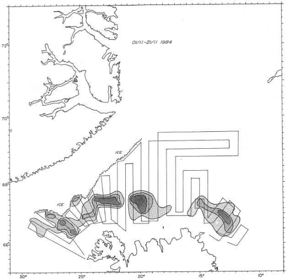


Figure: The distribution of capelin during the spawning migration of 1984 – –1985. a) Acoustic data from November 1 to November 21. b) Acoustic data from January 14 to February 8. (c) Close up of the distribution of capelin from February 7 to February 20 of 1985.

1984–1985

(a) 

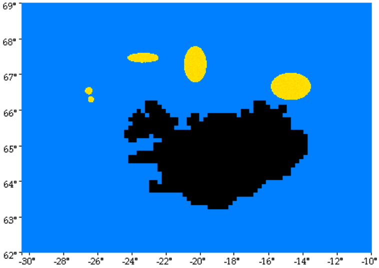
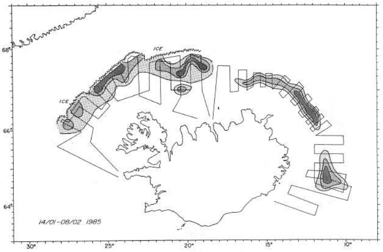
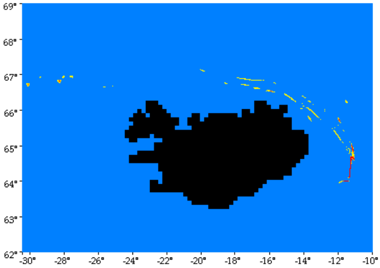
(b) 

Figure: The distribution of capelin in November of 1984. a) Acoustic data from November 1 to November 21. b) Initial distribution for the simulation.

1984–1985



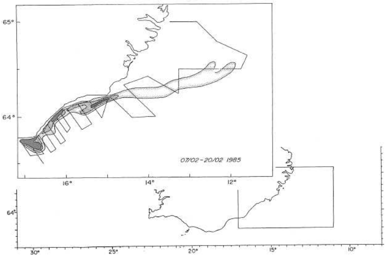
(a)



(b)

Figure: The distribution of capelin in mid-January to early February of 1985. a) Acoustic data from January 14 to February 8. b) Simulated distribution in mid-January, day 65 of the simulation.

1984–1985

(a) 

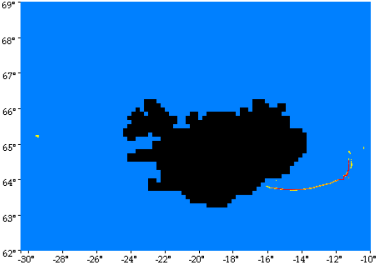
(b) 

Figure: a) Close up of the distribution of capelin from February 7 to February 20 of 1985. b) Simulated distribution in late February, day 109.

Acoustic data from 1990–1991

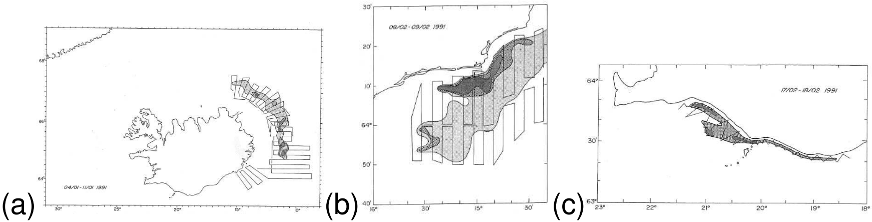
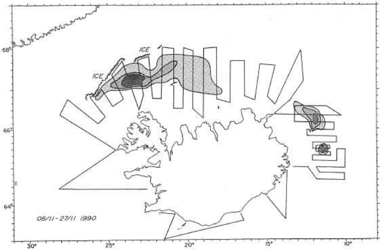


Figure: The distribution of capelin during the spawning migration of 1991. a) Acoustic data from January 4 to January 11. b) Close up of the distribution of capelin southeast of Iceland from February 8 to February 9 of 1991. (c) Acoustic data from February 17 to February 18.

1990–1991

(a)



(b)

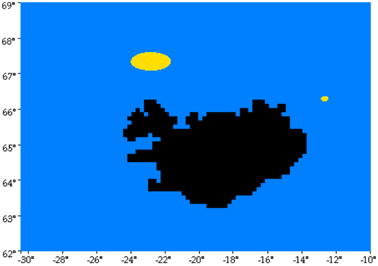
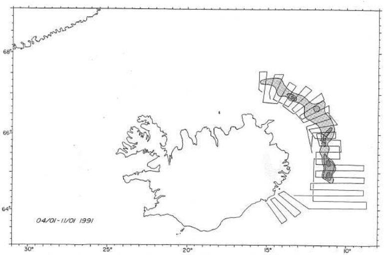


Figure: The distribution of capelin in November of 1990. a) Acoustic data from November 8 to November 27. b) Initial distribution for the simulation.

1990–1991

(a)



(b)

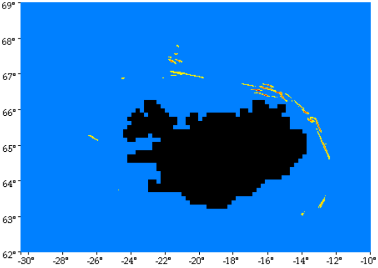


Figure: The distribution of capelin in January of 1991. a) Acoustic data from January 4 to January 11. b) Simulated distribution in early January, day 44 of the simulation.

1990–1991

(a) (b)

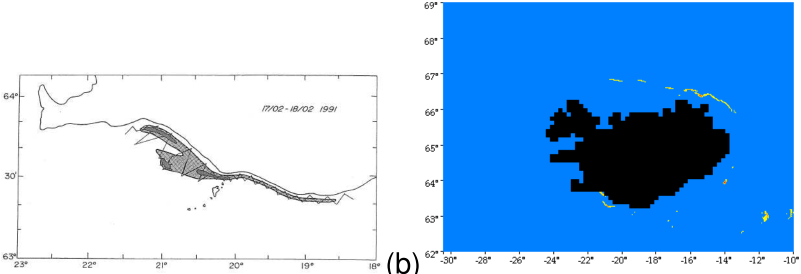


Figure: The distribution of capelin southwest of Iceland in February of 1991. a) Acoustic data from February 17 to February 18. b) Simulated distribution in mid-February, day 72.

2008

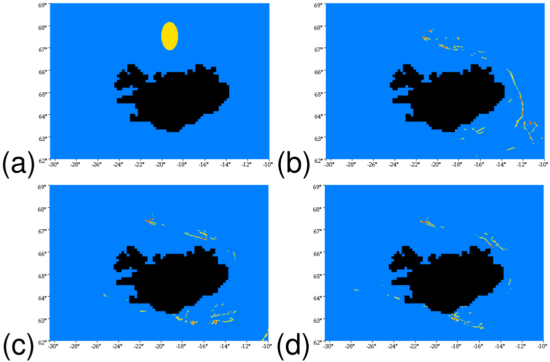


Figure: Simulation of the 2007-2008 spawning migration. a) Early January, day 0 b) Mid-February, day 47 c) Late February, day 59 d) Early March, day 65.

Acoustic data from 2008

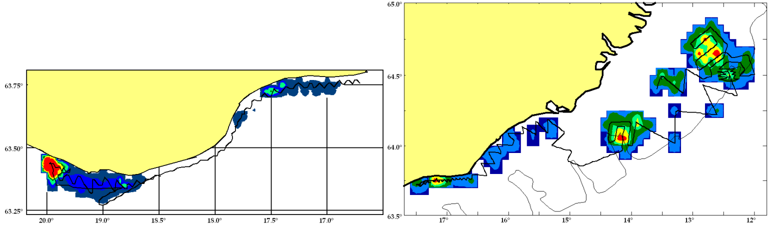


Figure: a) Measured distribution of capelin near the south coast of Iceland from February 26 to February 27 of 2008. b) Measured distribution of capelin near the southeast coast of Iceland from February 29 to March 3 of 2008.

Motivating scaling laws

- Because it is not reasonable to simulate all of the fish in the migration, we treat each particle as a *superindividual*, allowing it to represent multiple fish
- Think of each superindividual as a subschool
- We want to ensure that the behavior and spatial patterns be preserved when we change the number of particles in a simulation.
- We therefore need to derive rules for how the various parameters scale with respect to one another

Scaling notation

To this end, let:

- F denote the number of fish undertaking the migration in a given year
- N denote the number of particles in the simulation
- N^s denote the number of fish per particle, so $N^s = F/N$
- Δt still represents the timestep
- Δq denote the distance a fish travels in a timestep and, as such, defines the spatial scale
- M denote the number of particles in a Δq by Δq square
- r_r, r_o, r_a denote the radii of, respectively, repulsion, orientation, and attraction

Scaling argument

- Since $\Delta q = v \cdot \Delta t$, $\Delta q \propto \Delta t$
- r_r, r_o, r_a should all scale linearly with Δq, and thus with Δt
- M should remain constant as parameters change in order to ensure consistency in the simulations
- Assume uniform density over the area of the simulation for the sake of analysis. Then the density of actual fish over a square of area $(\Delta q)^2$ is $\frac{M \cdot N^s}{(\Delta q)^2}$.
  - This density should remain constant regardless of the spatial resolution of the simulation.
  - Thus $\frac{M \cdot N^s}{(\Delta q)^2} = \frac{M \cdot N_0^s}{(\Delta q_0)^2}$
  - Recalling that $N^s := F/N$, $\frac{M \cdot \frac{F}{N}}{(\Delta q)^2} = \frac{M \cdot \frac{F}{N_0}}{(\Delta q_0)^2} \Rightarrow$ $\Delta q = \left(\Delta q_0 \sqrt{N_0}\right) \cdot \frac{1}{\sqrt{N}}$.

Scaling laws

$$\boxed{\Delta t \quad \propto \quad r_r \quad \propto \quad r_o \quad \propto \quad r_a \quad \propto \quad \Delta q \quad \propto \quad \frac{1}{\sqrt{N}} = \sqrt{\frac{N^s}{F}}}$$

The parameters in our simulations are as follows:

- $\Delta t = 0.05$ days
- Initial speed $v_k \simeq 4 - 8$ km/day
- $r_r = 0.01$ or about ~ 120 m
- $r_o = r_a = 0.1$ or about ~ 1.2 km
- $\alpha = 0.99$ and $\beta = 0.01$
- $T_1 = 3\,^\circ$C and $T_2 \sim 6.5\,^\circ$C

Scaling down to an individual level

How do the particles scale as we take N^s to 1? A rough estimate for the total number of fish in a migration is $F \simeq 5 \cdot 10^{10}$.

- $N_0 \simeq 5 \cdot 10^4$, so $N_0^s \simeq 1 \cdot 10^6$ fish, and $N_1 \simeq 5 \cdot 10^{10}$ where $N_1^s = 1$ fish
- $\frac{\Delta q_0}{\frac{1}{\sqrt{N_0}}} = \frac{\Delta q_1}{\frac{1}{\sqrt{N_1}}}$ and $\Delta q_0 \simeq 1.2$ km $\Rightarrow \Delta q_1 \simeq 1.2$ meters
- $\Delta t_0 = 0.05$ days and $\frac{\Delta t_0}{\Delta q_0} = \frac{\Delta t_1}{\Delta q_1} \Rightarrow \Delta t_1 = 4.32$ seconds
- Radii scale with Δq, so
  - $r_{r_0} \simeq 120$ meters $\Rightarrow r_{r_1} \simeq 12$cm
  - $r_{o_0} = r_{a_0} \simeq 1.2$ km $\Rightarrow r_{o_1} = r_{a_1} \simeq 1.2$m

These are all biologically reasonable!

Future work

- We are working on numerically verifying the scaling laws we derived for the system
- There is work in progress on incorporating the bioenergetics into the migration
  - Use more biological data
  - Incorporate bioenergetics into our model to trigger different temperature preferences
  - Allow the maturity of the roe to change the speed of the fish
- Try to improve the data about the current which we incorporate into our model
  - Incorporating the work of Kai Logeman
- Use a larger temperature map and simulate the feeding migration
- Parallelize the full model, including the environment

Thanks!

Thank you for inviting me to CSCAMM. I would also like to thank my co-authors: Baldvin Einarsson, Peterson Trethewey, Lamia Youseff, Björn Birnir, John Gilbert, Kirk Taylor, Sven Sigurðsson, and our collaborators at Hafró. Many thanks to Melisa Hendrata and Roger Nisbet for working with me on the DEB theory. I am also grateful for the generous support from the NSF, EAF, Marine Research Institute of Iceland, Háskóli Íslands, and the UCSB math department. This research would not have been possible without this support. Thank you!

- Melisa Hendrata and Björn Birnir.
  The dynamics of myxobacteria life cycle.
  Submitted June 2008.
- R. M. Nisbet, E. B. Muller, K. Lika, and S. A. L. M. Kooijman.
  From molecules to ecosystems through dynamic energy budget models.
  Journal of Animal Ecology, 69:913–926, 2000.
- Henk W. van der Veer, Sebastiaan Kooijman, and Jaap van der Meer.
  Body size scaling relationships in flatfish as predicted by Dynamic Energy Budgets (DEB theory): implications for recruitment.
  Journal of Sea Research, 50:255–270, 2003.
- Hjálmar Vilhjálmsson.
  The Icelandic capelin stock.
  Journal of the Marine Research Institute Reykjavik, XIII(1):281 pp., 1994.
- Lamia Youseff, Alethea Barbaro, Peterson Trethewey, Björn Birnir, and John Gilbert.
  Parallel modeling of fish interaction.
  IEEE.

Fish, Oceans, and the World!

Our model consists of three main classes [5]:

- The *Fish* class stores coordinate and velocity data for a particle,
- The *Ocean* class is meant to represent a single body of water,
- The *World* class is a bigger body of water composed of several connected oceans.

Each Fish stores an x and y coordinate for its location in the world. A Fish stores its velocity as the cosine and sine of its direction angle together with a non-negative speed. The Ocean class has a member variable "fish" which is an array of Fish living in that Ocean.

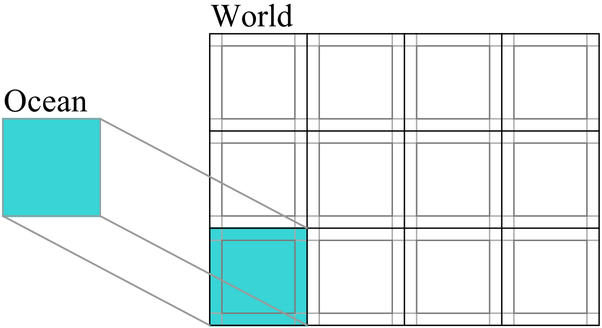


Figure: Oceans are connected in a rectangular grid which resides in a World.

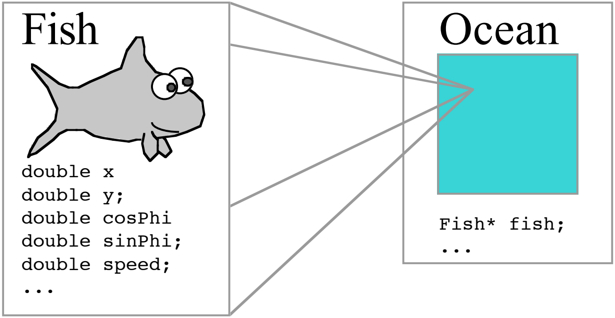


Figure: The Fish class stores coordinate and velocity data for a particle. The Ocean class keeps an array of Fish located in that ocean.

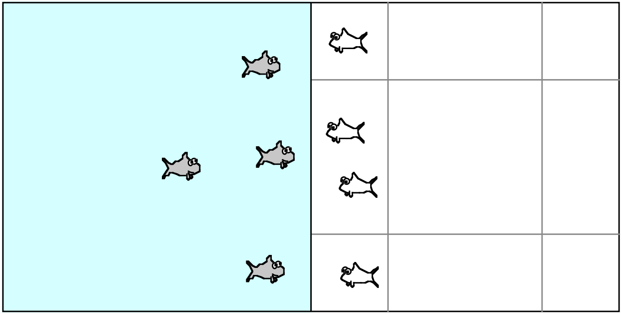


Figure: Fish in an ocean need to interact with sufficiently nearby fish on other oceans. We add fish near the boundary of a neighboring ocean as “ghost fish” to the current ocean.

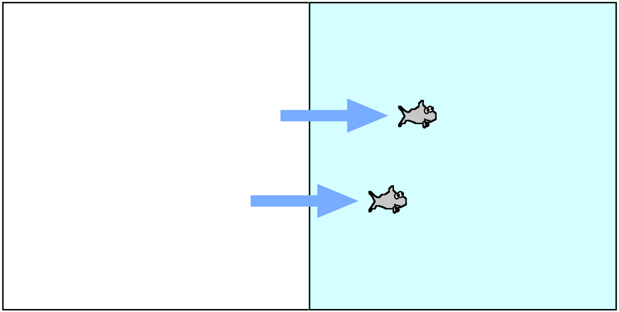


Figure: After the fish move, some fish might have crossed over into the domain of a neighboring ocean. When this happens, those fish need to get copied into that neighboring ocean and removed from the current ocean.

Bioenergetics: an overview

- Bioengergetic model keeps track of:
  - Weight
  - Fat content
  - Roe content
- Allows us to change preferences of the particles based on maturity
  - Preferred temperature
  - Speed
  - α and β

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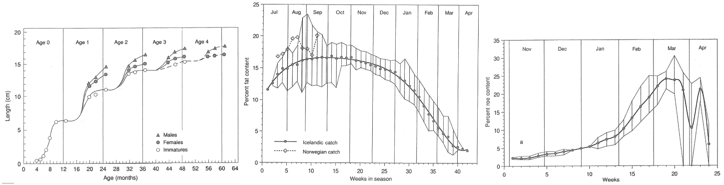


Figure: Data plots of the average (a) length (b) fat percentage and (c) roe percentage of the Icelandic capelin [4].

Bioenergetics

$$\dot{L}(t) = \frac{\tilde{\nu}\,[E(t) - L(t)]}{3L_m\,[g + E(t)]} \qquad (1)$$

$$\dot{E}(t) = \frac{\tilde{\nu}\,[f(t) - E(t)]}{L_m L(t)} \qquad (2)$$

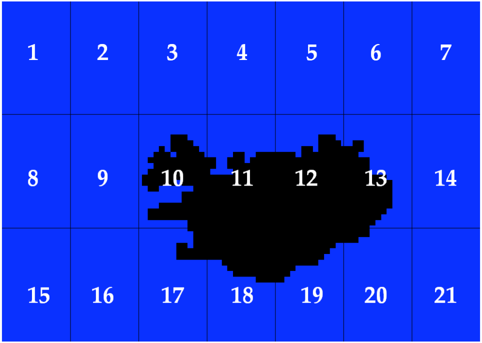
where L is the ratio of the length of the animal to the maximum length and E is the ratio of the weight of internal reserves to the maximum weight of internal reserves. Following [1], derived from [2].

Bioenergetics

$$\dot{R}(t) = cR(t)^{1-b}\left(e^{\left[\frac{T_A}{T}-\frac{T_A}{T_1}\right]}\right) \tag{3}$$

where R is the weight of the roe measured in grams, c and b are constants, T_A is the species-specific Arrhenius temperature (K), T is absolute temperature (K), and T_1 is a chosen reference temperature (K), which we in our model take to be the temperature of the surrounding ocean (K). Derived from [3].

Sensitivity to perturbed parameters



We measure the sensitivity of the system by seeing how the migration route and timing change.