High order semi-implicit schemes for kinetic equations

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Madison, 4th-9th May 2015

Outline of the Talk

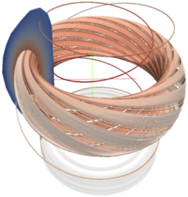
1. Part I: Physical Context
2. Part II : Modeling issues
   - Basic Properties of the guiding center model
   - Derivation if the 4D drift kinetic & guiding center models
   - Basic Properties of the 4D drift kinetic model
3. Part III: IMEX schemes
   - General semi-linear approach
   - Applications of semi-implicit schemes
4. Part IV: Numerical approximation in an arbitrary domain
   - Flow around an airfoil in 2D
   - D shape Simulation
   - Toward plasma physics applications

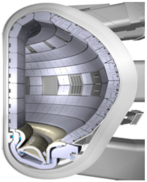
Physical Context : Controled Fusion Energy

Controled fusion energy is one of the major prospects for a long term source of energy.

Magnetic fusion

the plasma is confined in tokamaks using a large external magnetic field. The international project ITER is based on this idea and aims to build a new tokamak which could demonstrate the feasibility of the concept.





We assume that electrons are adiabatic and study the motion of electrons

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + \frac{e}{m}\left[E + v \times \frac{B_{\text{ext}}}{\varepsilon}\right] \cdot \nabla_v f = 0.$$

coupled with Maxwell's or Poisson equations for electromagnetic fields.

The 2D guiding center model

It gives the 2D guiding center model in the transverse plane of a Tokamak.

$$\begin{cases} \frac{\partial \rho}{\partial t} + \mathbf{U} \cdot \nabla_{\mathbf{x}_\perp} \rho = 0, \\ \mathbf{U} = \mathbf{E}^\perp, \\ -\Delta_{\mathbf{x}_\perp} \phi = \rho. \end{cases}$$

Boundary condition :

$$\phi(\mathbf{x}_\perp) = 0, \quad \mathbf{x}_\perp \in \partial D,$$

where ∂D can be arbitrary boundary.

If f is smooth, we have

(1) Maximum principle : $0 \le \rho(t, \mathbf{x}_\perp) \le \max_{\mathbf{x}_\perp \in D}(\rho(0, \mathbf{x}_\perp))$.

(2) L^p norm conservation : $\frac{d}{dt}\left(\int_D (\rho(t, \mathbf{x}_\perp))^p d\mathbf{x}_\perp\right) = 0$.

(3) Energy conservation : $\frac{d}{dt}\left(\int_D |\nabla \phi|^2 d\mathbf{x}_\perp\right) = 0$.

Towards reduced kinetic models

We assume

- the magnetic field is uniform $\mathbf{B}_{\text{ext}} = B\, e_z$, where e_z stands for the unit vector in the toroidal direction,
- the ratio between orthogonal and longitudinal characteristic lenght is $L_\perp / L_z = \varepsilon \ll 1$,
- f is vanishing at infinity of velocity field and periodic boundary condition is taken in z direction.
- we are interesting by the long time asymptotic

To derive the Drift-Kinetic model, we formally follow the same ideas as for the guiding center model and split the variables as

$$\mathbf{x} = (\mathbf{x}_\perp, x_\parallel)$$

with $x_\parallel = z$ and $\mathbf{x}_\perp = (x, y)$.

4D drift kinetic & guiding center models

For the Poisson equation, setting that $L_\perp / L_z = \varepsilon$, it leads to

$$-\Delta_\perp \phi - \varepsilon^2 \partial_{zz}\phi = n(t, \mathbf{x}_\perp, z) - n_0.$$

We split $\mathbf{E}$ into components along $\mathbf{B}_{\rm ext}$ and perpendicular to $\mathbf{B}_{\rm ext}$: it gives

$$\mathbf{E} = \mathbf{E}_\perp + \varepsilon E_\parallel e_z.$$

Assuming that $B = O(1/\varepsilon)$ and substituting this expression in the Vlasov equation, it yields

$$\varepsilon \frac{\partial f}{\partial t} + \mathbf{v}_\perp \cdot \nabla_{\mathbf{x}_\perp} f + \varepsilon v_z \partial_z f + \left(\mathbf{E}_\perp + \frac{\mathbf{v}_\perp^\perp}{\varepsilon} \right) \cdot \nabla_{\mathbf{v}_\perp} f + \varepsilon E_z \partial_{v_z} f = 0.$$

Then we integrate with respect to $\mathbf{v}_\perp = (v_x, v_y)$, we get formally an equation for

$$\tilde{f} = \int_{\mathbb{R}^2} f \, d\mathbf{v}_\perp.$$

4D drift kinetic & guiding center model

It yields to the $3D \times 1D$ drift kinetic system

$$\begin{cases} \dfrac{\partial \tilde{f}}{\partial t} + \mathbf{U}_\perp \cdot \nabla_{\mathbf{x}_\perp} \tilde{f} + v_z \partial_z \tilde{f} + E_z \partial_{v_z} \tilde{f} = 0. \\ -\Delta_\perp \phi = \int_{\mathbb{R}} \tilde{f} dv_z - n_0 \end{cases}$$

with $\mathbf{U}_\perp = (\partial_y \phi, -\partial_x \phi)$ and $E_z = -\partial_z \varphi$.

Remark integrating on the longitudinal direction in space and velocity, we recover the guiding center model:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \mathbf{U}_\perp \cdot \nabla_{\mathbf{x}_\perp} \rho = 0, \\ -\Delta_\perp \phi = \int_{\mathbb{R}} \tilde{f} dv_z - n_0 \end{cases}$$

4D Drift-Kinetic Model

Normalized Drift-Kinetic model reads (cf. Grandgirard *et al.*)

$$\begin{cases} \frac{\partial f}{\partial t} + \mathbf{U}_\perp \cdot \nabla_{\mathbf{x}_\perp} f + v_\| \partial_z f + E_\| \partial_{v_\|} f = 0, \\ -\nabla_\perp \cdot \left(\frac{\rho_0(\mathbf{x}_\perp)}{B} \nabla_\perp \phi \right) + \frac{\rho_0(\mathbf{x}_\perp)}{T_e(\mathbf{x}_\perp)} (\phi - \bar{\phi}) = \rho. \end{cases}$$

In the following simulation, we consider a cylinder domain

$$\Omega = \{(x, y, z) \in \mathbb{R}^3 : (x, y) \in D, 0 \le z \le L_z\}.$$

Boundary condition :

- $\phi(\mathbf{x}) = 0$ on $\partial D \times [0, L_z]$.
- Periodic boundary condition in z-direction.

If f is smooth, we have

(1) Maximum principle : $0 \le f(t, \mathbf{x}, v_\|) \le \|f(0)\|_\infty$.

(2) L^p norm conservation : $\frac{d}{dt}\left(\int_{\mathbb{R}} \int_{\Omega_\mathbf{x}} (f(t, \mathbf{x}, v_\|))^p d\mathbf{x} dv_\| \right) = 0$.

(3) Kinetic entropy conservation : $\frac{d}{dt}\left(\int_{\mathbb{R}} \int_{\Omega_\mathbf{x}} f \ln|f| d\mathbf{x} dv_\| \right) = 0$.

(4) Energy conservation

Difficulty for the Numerical Simulations

- High dimension of the problem. Kinetic equations are set in phase space $(x, v) \in \mathbb{R}^3 \times \mathbb{R}^3$.
- Various instability occurs : microscopic phenomena (like two stream instability), macroscopic phenomena (fluid like instability Raleight-Taylor, Kelvin-Helmholtz instability in fluid mechanics).
- Nonlinearities
- Effect of collisions (not take into account here)
- multi-species plasma and quasineutral with large mass ratio
- Describe bounadry effects when they occur or the effect of the geometry (tokamak in the poloidal plane).

IMEX schemes : additive and partitioned form

Here we shall use finite difference discretization in space for simplicity, and concentrate on time discretization, so we can see the problem as a system of ODES:

$$\frac{dy}{dt} = \underbrace{f(y)}_{\text{Explicit}} + \underbrace{\frac{1}{\varepsilon}g(y)}_{\text{Implicit}}, \tag{1}$$

- The stiffness is associated to one of the terms on the RHS. We say that in this case the stiffness is additive.
- In other cases the stiffness can be associated to a variable, e.g.

$$\frac{du}{dt} = F(u,v), \qquad \frac{dv}{dt} = \frac{1}{\varepsilon}G(u,v) \tag{2}$$

We say that the system is partitioned.

Let us emphasize that setting $y = (u,v)^T$, $f = (F,0)^T$, $g = (0,G)^T$, partitioned can be seen as a particular case of additive.

→ A natural choice for all such cases is offered by IMEX methods.

General formulation

In many cases the separation of scales is not additive nor partitioned. We may have a situation of the form

$$\begin{cases} \frac{du}{dt}(t) = \mathcal{H}(t, u(t), u(t)), \quad \forall\, t \geq t_0, \\ u(t_0) = u_0, \end{cases} \tag{3}$$

with $\mathcal{H}\colon \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$ sufficiently regular

- Dependence on the second argument of $\mathcal{H}$ is non stiff.
- Dependence on the third argument is stiff.

This includes partitioned and additive as particular cases.

Strong relation with partitions systems: by setting $y = u$ and $z = u$, system (3) implies

$$\begin{cases} \frac{dy}{dt}(t) = \mathcal{H}(t, y(t), z(t)), \\ \frac{dz}{dt}(t) = \mathcal{H}(t, y(t), z(t)), \end{cases}$$

Doubled system?

By doubling the variables, the systems takes a partitioned form.

Partitioned methods: apply two different R-K methods, *i.e.*

$$\begin{array}{c|c} \hat{c} & \hat{A} \\ \hline & \hat{b}^T \end{array} \qquad \begin{array}{c|c} c & A \\ \hline & b^T \end{array} \tag{4}$$

treat y with the method on the left, and z with the one on the right.
Then one has, for the stage fluxes:

$$k_i = \mathcal{H}\left(t^n + \hat{c}_i \Delta t, Y_i, Z_i\right), \quad \ell_i = \mathcal{H}\left(t^n + c_i \Delta t, Y_i, Z_i\right), \quad 1 \le i \le s,$$

with

$$Y_i = y^n + \Delta t \sum_{j=1}^{s} \hat{a}_{i,j}\, k_j, \quad Z_i = y^n + \Delta t \sum_{j=1}^{s} a_{ij}\, \ell_j, \quad 1 \le i \le s,$$

and the numerical solutions at the next time step are

$$y^{n+1} = y^n + \Delta t \sum_{i=1}^{s} \hat{b}_i\, k_i, \quad z^{n+1} = y^n + \Delta t \sum_{i=1}^{s} b_i\, \ell_i.$$

How to avoid doubling the number of variables

Remark 1. If $\hat{c} = c$ then $k = \ell \Rightarrow \mathcal{H}$ has to be computed only once per stage.

Remark 2. Furthermore,

- if $\hat{b} = b \Rightarrow y^{n+1} = z^{n+1}$,
- if $\hat{b} \neq b$ and $y^n = z^n \Rightarrow y^{n+1} \neq z^{n+1}$, however if both schemes are consistent to order p once can choose any one of the two, say the one to compute y^{n+1}, and then set $n \leftarrow n+1$, and $z^n = y^n$

Remark 3. If $\hat{c} = c$ and the two schemes have different orders, then the difference $y^{n+1} - z^{n+1}$ can be used to estimate the local error $\Rightarrow$ time step control.

In all such cases, no duplication of variables is needed!

Is it possible to construct such a scheme?

- For autonomous problems, it is all right!
- Up to second order, two stages schemes it is easy since we can impose that

$$\sum_j \hat{a}_{i,j} \neq \hat{c}_i, \quad \text{and} \quad \sum_j a_{ij} \neq c_i, \quad \text{for} \quad 1 \leq i \leq s. \tag{5}$$

The IMEX-SSP2(2,2,2) L-stable scheme

We choose $b_2 = 1/2$, $\hat{c} = 1$ and $\gamma = 1 - 1/\sqrt{2}$, *i.e.* the corresponding Butcher tableau is given by

$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1 & 0 \\ \hline & 1/2 & 1/2 \end{array} \qquad \begin{array}{c|cc} \gamma & \gamma & 0 \\ 1-\gamma & 1-2\gamma & \gamma \\ \hline & 1/2 & 1/2 \end{array}$$

Solution : Replace $(\hat{c}_1, \hat{c}_2) = (0, 1)$ by $(\hat{c}_1, \hat{c}_2) = (\gamma, 1-\gamma)$

Third order conditions and scheme

The semi-implicit Runge-Kutta method is of order three, if it satisfies the conditions

$$\sum_i b_i = 1, \quad \sum_i b_i\, c_i = 1/2, \quad \sum_i b_i\, \hat{c}_i = 1/2.$$

and the implicit part satisfies the classical third order conditions

$$\sum_i b_i c_i^2 = 1/3, \quad \sum_{i,j} b_i\, a_{ij}\, c_j = 1/6,$$

the explicit part satisfies the classical third order conditions

$$\sum_i b_i\, \hat{c}_i^2 = 1/3, \quad \sum_{i,j} b_i\, \hat{a}_{ij}\, \hat{c}_j = 1/6,$$

and moreover the additional coupling conditions

$$\sum_i b_i\, \hat{c}_i\, c_i = 1/3, \quad \sum_{i,j} b_i\, a_{ij}\, \hat{c}_j = 1/6, \quad \sum_{i,j} b_i\, \hat{a}_{ij}\, c_j = 1/6.$$

are satisfied.

Third order conditions and scheme

A possible choice satisfying these properties is given by the IMEX-SSP3(4,3,3) L-stable scheme, *i.e.*

$$
\begin{array}{c|cccc}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 \\
1/2 & 0 & 1/4 & 1/4 & 0 \\
\hline
 & 0 & 1/6 & 1/6 & 2/3
\end{array}
\qquad
\begin{array}{c|cccc}
\alpha & \alpha & 0 & 0 & 0 \\
0 & -\alpha & \alpha & 0 & 0 \\
1 & 0 & 1-\alpha & \alpha & 0 \\
1/2 & \beta & \eta & 1/2-\beta-\eta-\alpha & \alpha \\
\hline
 & 0 & 1/6 & 1/6 & 2/3
\end{array}
$$

with $\alpha = 0.24169426078821$, $\beta = \alpha/4$ and $\eta = 0.12915286960590$.

What about fourth order schemes?

Reaction diffusion problem

We consider the reaction diffusion system $\omega = (\omega_1, \omega_2) : \mathbb{R}^+ \times (0, 2\pi)^2 \mapsto \mathbb{R}^2$

$$\begin{cases} \dfrac{\partial \omega_1}{\partial t} = \Delta \omega_1 - \alpha_1(t)\,\omega_1^2 + \dfrac{9}{2}\omega_1 + \omega_2 + f(t), \\ \dfrac{\partial \omega_2}{\partial t} = \Delta \omega_2 + \dfrac{7}{2}\omega_2\,, \qquad t \geq 0, \end{cases}$$

with $\alpha(t) = 2\,e^{t/2}$ and $f(t) = -2e^{-t/2}$. Initial conditions compatible with exact solution

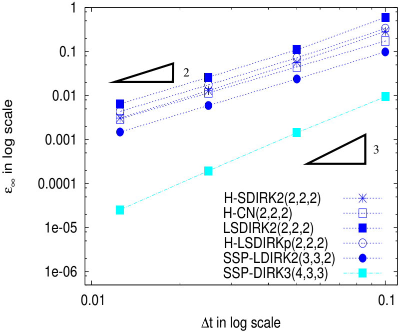
$$\begin{cases} \omega_1(t, x, y) = \exp(-0.5t)\,(1 + \cos(x)), \\ \omega_2(t, x, y) = \exp(-0.5t)\,\cos(2\,x). \end{cases}$$

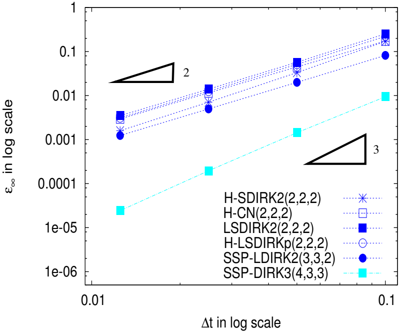
Separate explicit variable $u = (u_1, u_2)$ from implicit $v = (v_1, v_2)$, according to:

$$\mathcal{H}(t, u, v) = \begin{pmatrix} \Delta v_1 - \alpha(t) u_1\, v_1 + \dfrac{9 u_1}{2} + v_2 + f(t) \\ \Delta v_2 + \dfrac{7\, v_2}{2} \end{pmatrix}.$$

Reaction-Diffusion equation: results

- Fourth order accurate space discretization (error is mainly in time discretization).
- Hyperbolic CFL condition $\Delta t = \Delta x/2$.
- Schemes SSP2 and SSP3.





Nonlinear convection-diffusion equation

We consider the convection diffusion equation

$$\begin{cases} \dfrac{\partial \omega}{\partial t} + [V + \mu \nabla \log(\omega)] \cdot \nabla \omega - \mu \Delta \omega = 0, & (t, x) \in \mathbb{R}^+ \times \mathbb{R}^2, \\ \omega_0(t = 0) = e^{-\|x\|^2/2}, \end{cases}$$

where $V = (1, 1)^T$, $\mu = 0.5$. The exact solution is given by

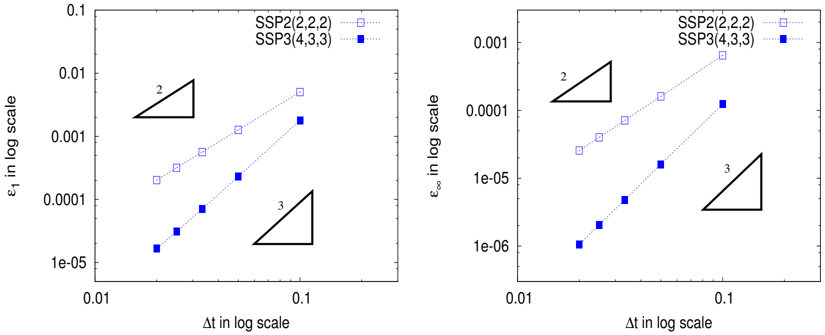
$$\omega(t, x) = \frac{1}{\sqrt{4 \mu t + 1}} \exp\left(- \frac{\|x - V t\|^2}{8 \mu t + 2} \right), \quad t \geq 0, \quad x \in \mathbb{R}^2.$$

We choose $\mathcal{H}$ as follows

$$\mathcal{H}(t, u, v) = -(V + \mu \nabla \log(u)) \cdot \nabla v + \mu \Delta v.$$

Nonlinear convection-diffusion equation: results

We apply the same discretization in space and time with $x \in (-10, 10)^2$. Final time $T = 0.5$.



Surface diffusion flow

We consider the following nonlinear fourth order differential equation

$$\frac{\partial \omega}{\partial t} + \mathrm{div} S(\omega) = 0, \quad x \in \mathbb{R}^2, \quad t \geq 0, \tag{6}$$

where the nonlinear differential operator S is given by

$$S(\omega) := \left(Q(\omega) \left(I - \frac{\nabla \omega \otimes \nabla \omega}{Q^2(\omega)} \right) \nabla N(\omega) \right),$$

where Q is the area element

$$Q(\omega) = \sqrt{1 + |\nabla \omega|^2}$$

and N is the mean curvature of the domain boundary Γ

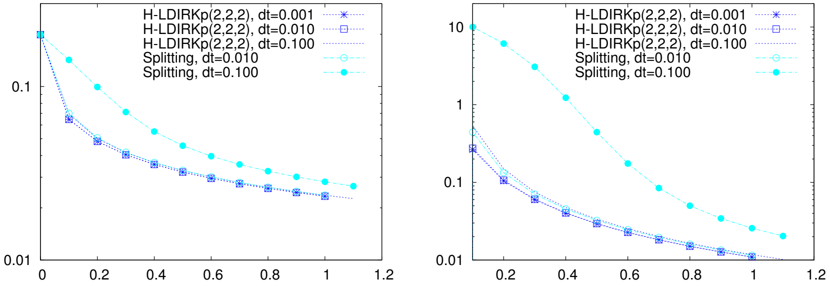
$$N(\omega) := \left(\frac{\nabla \omega}{Q(\omega)} \right).$$

Surface diffusion flow

For this aplication we choose

$$\mathcal{H}(u,v) := \left(Q(u) \left(I - \frac{\nabla u \otimes \nabla u}{Q^2(u)} \right) \nabla \mathbb{N}(u,v) \right),$$

Hyperbolic CFL condition is used on the time step.



Towards plasma physics : one single particle motion

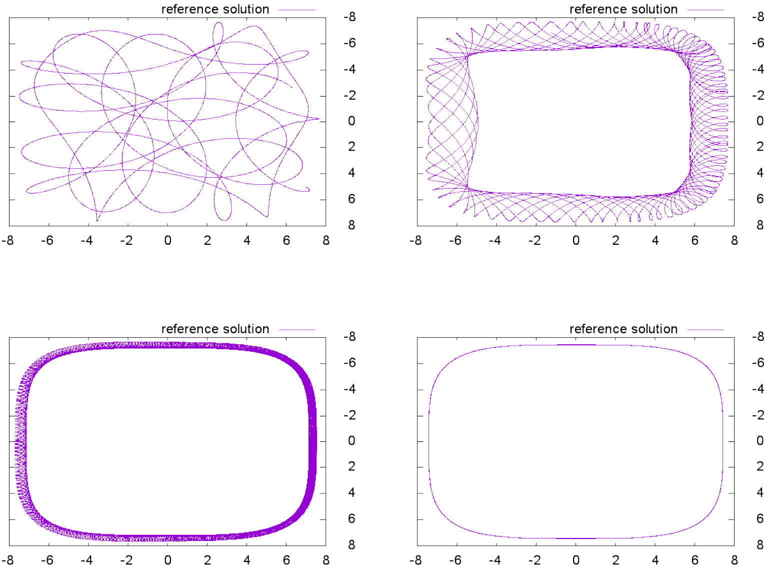
Let us conside $\mathbf{X}(t) = (x(t), y(t))$ and $\mathbf{V}(t) = (v_x(t), v_y(t))$ with

$$\begin{cases} \dfrac{d\mathbf{X}}{dt} = \dfrac{1}{\varepsilon}\mathbf{V} \\ \\ \dfrac{d\mathbf{V}}{dt} = \dfrac{1}{\varepsilon}\left(\mathbf{E}(\mathbf{X}) + B(\mathbf{X})\dfrac{\mathbf{V}^{\perp}}{\varepsilon}\right) \end{cases}$$

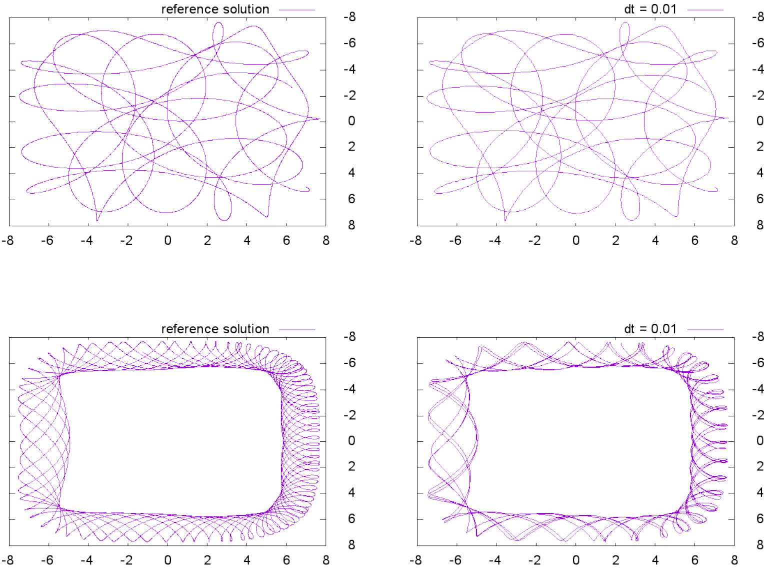
with $B(\mathbf{X}) = (1 + 0.1\, y)$ and

$$\mathbf{E}(\mathbf{X}) = -0.1\left(X + \begin{pmatrix} x^3(t) \\ y^3(t) \end{pmatrix}\right)$$

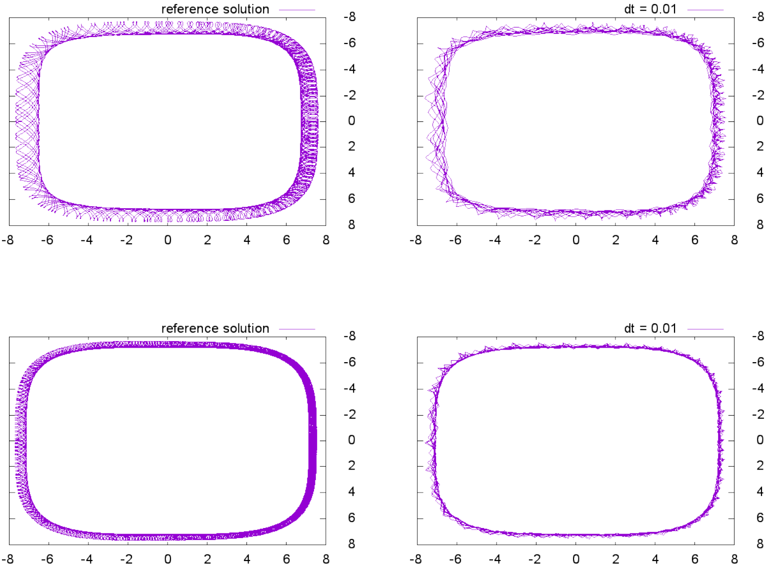
Towards plasma physics : one single particle motion



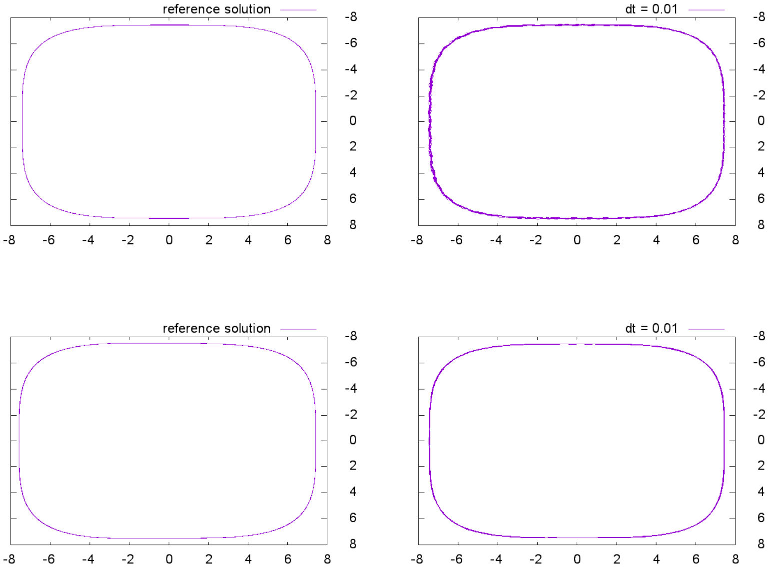
Comparison with semi-implicit schemes with large time steps $\Delta t = 0.01$



Comparison with semi-implicit schemes with large time steps $\Delta t = 0.01$



Comparison with semi-implicit schemes with large time steps $\Delta t = 0.01$



Part II : Treatment of boundary conditions

Solve numerically kinetic type equation on complex geometry.

Some algorithms based on Cartesian meshes

⋆ Immersed boundary method (IBM) of Peskin, Lai and etc
- popular in fluid mechanics applications,
- add a singular source term to fluid mechanics equations to take into account boundary effects
- poor accuracy

⋆ Cartesian cut-cell method (D. Ingram, D. Causon and C. Mingham)
- reconstruct the domain around the boundary
- apply a finite volume scheme on the new control volume

⋆ Inverse Lax-Wendroff (ILW) procedure (finite difference method or whatever)

S. TAN AND C.-W. SHU, *Inverse Lax-Wendroff procedure for numerical boundary conditions of conservation laws*, Journal of Computational Physics, 229 (2010), 8144–8166.

ILW Procedure in 2D Case

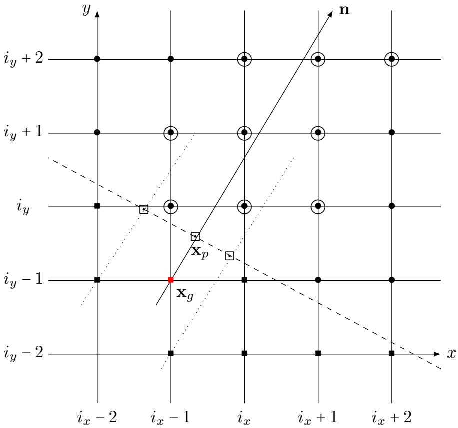


Figure: Spatially 2D Cartesian mesh. • is interior point, ▪ is ghost point, ⊡ is the point at the boundary, ◯ is the point for extrapolation, the dashed line is the boundary.

We consider 2D model

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = \frac{1}{\varepsilon} \mathcal{Q}(f),$$

Compute f at ghost point x_g:

1. Extrapolation of f for the outflow
   - compute $f(\mathbf{x}_p, \mathbf{v} \cdot \mathbf{n} < 0)$ and $f(\mathbf{x}_g, \mathbf{v} \cdot \mathbf{n} < 0)$ by WENO type extrapolation

ILW Procedure in 2D Case

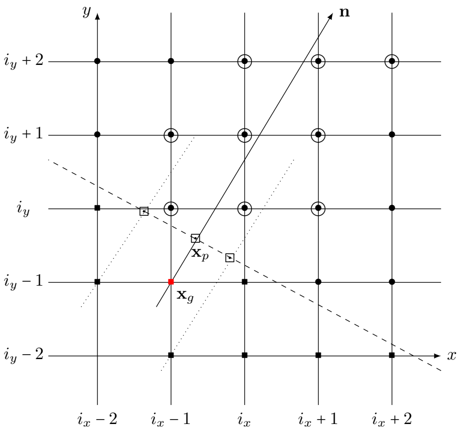


Figure: Spatially 2D Cartesian mesh. • is interior point, ▪ is ghost point, ⊡ is the point at the boundary, ◯ is the point for extrapolation, the dashed line is the boundary.

We consider 2D model

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = \frac{1}{\varepsilon}\mathcal{Q}(f),$$

Compute f at ghost point x_g:

1. Extrapolation of f for the outflow
2. Compute B.C. at the boundary
   - ⋆ $\mathcal{R}[f(\mathbf{x}_p, \mathbf{v})] = f(\mathbf{x}_p, \mathbf{v} - 2(\mathbf{v}\cdot\mathbf{n})\mathbf{n}), \quad \mathbf{v}\cdot\mathbf{n} > 0$
   - ⋆ interpolate f on $(\mathbf{x}_p, \mathbf{v} - 2(\mathbf{v}\cdot\mathbf{n})\mathbf{n})$
   - ⋆ $\mathcal{M}[f(\mathbf{x}_p, \mathbf{v})] = \mu(\mathbf{x}_p)\exp\left(-\frac{\mathbf{v}^2}{2T_p}\right), \quad \mathbf{v}\cdot\mathbf{n} > 0$

ILW Procedure in 2D Case

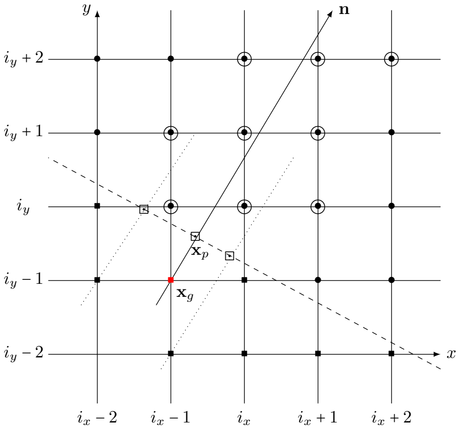


Figure: Spatially 2D Cartesian mesh. • is interior point, ▪ is ghost point, ⊡ is the point at the boundary, ◯ is the point for extrapolation, the dashed line is the boundary.

We consider 2D model

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = \frac{1}{\varepsilon} \mathcal{Q}(f),$$

Compute f at ghost point x_g:

1. Extrapolation of f for the outflow
2. Compute B.C. at the boundary
3. Approximation of f for inflow
   - ⋆ local coordinate system $\mathbf{x} \to \hat{\mathbf{x}}$
   - ⋆ $\frac{\partial \hat{f}}{\partial \hat{x}}(\hat{\mathbf{x}}_p, \mathbf{v}) = -\frac{1}{\hat{v}_x} \left(\frac{\partial \hat{f}}{\partial t} + \hat{v}_y \frac{\partial \hat{f}}{\partial \hat{y}} - \frac{1}{\varepsilon} \mathcal{Q}(\hat{f}) \right)\Big|_{\hat{\mathbf{x}}=\hat{\mathbf{x}}_p}$
   - ⋆ $f(\mathbf{x}_g, \mathbf{v}) \approxeq \hat{f}(\hat{\mathbf{x}}_p, \mathbf{v}) + (\hat{x}_g - \hat{x}_p) \frac{\partial \hat{f}}{\partial \hat{x}}(\hat{\mathbf{x}}_p, \mathbf{v})$

Flow around an airfoil in 2D

Solve the time evolution Boltzmann equation $(x, v) \in \Omega \times \mathbb{R}^3_v$, with $\Omega \subset \mathbb{R}^2$.

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f = \frac{1}{Kn} \mathcal{Q}(f).$$

We consider a Mach number $Ma = 0.3$ and a Reynolds number $Re = 3000$. The Mach, Reynolds and Knudsen numbers relation is given by:

$$Kn = \frac{Ma}{Re}\sqrt{\frac{\gamma\pi}{2}}, \qquad \gamma = 1.4$$

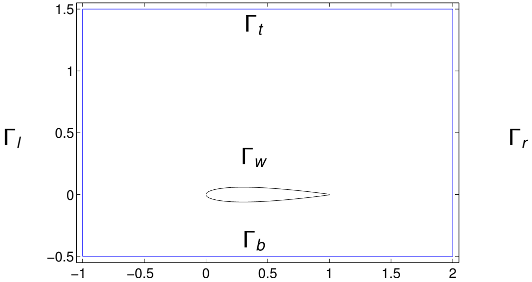
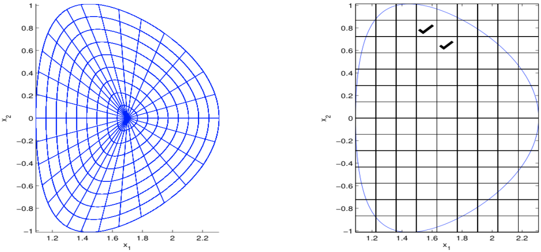


Figure: Flow around an object. *Domain including an airfoil.*

Flow around an airfoil in 2D

D shape Simulation

We still consider the guiding center model but now in a D shape geometry.



(a) (b)

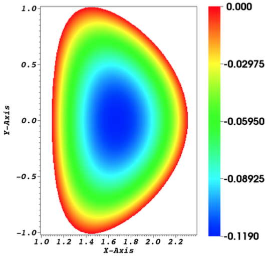
1) We first look for a stationary solution of the guiding center model :

$$\begin{cases} -\nabla_\perp \cdot \left(\frac{\rho_0}{B} \nabla_\perp \phi \right) = \bar{\rho}(\phi) - \rho_0 & \text{in } \Omega, \\ \phi = 0 & \text{on } \partial\Omega. \end{cases} \tag{7}$$

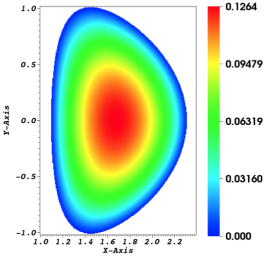
For a suitable function $\bar{\rho}$, we have a unique solution.

D shape Simulation

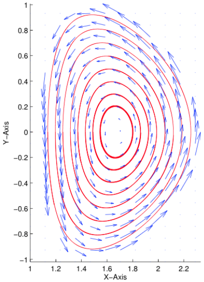
The steady state solution is computed numerically



(a) Potential ϕ_0

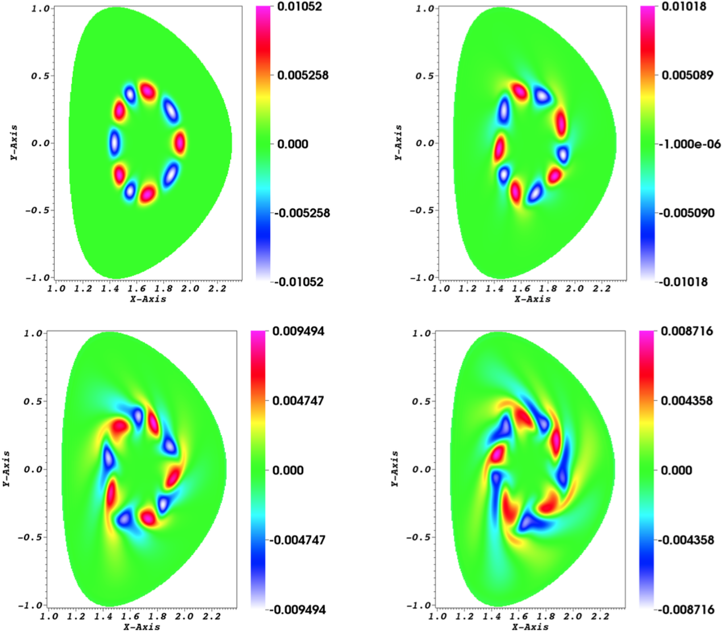


(b) Density ρ_0



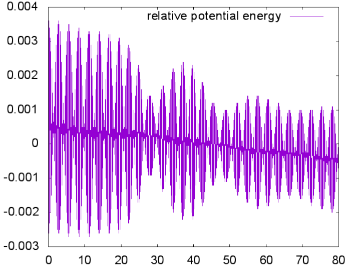
Now we still consider the previous initial data $(\phi_0, \bar{\rho}_0)$ which is a stationary solution of the guiding-center model, but perturb it of magnitude of ε.

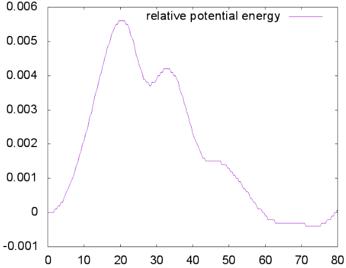
D shape Simulation



Toward plasma physics applications

Let us now consider Particle-In-Cell methods based on semi-implicit schemes in a disk shape domain where the Poisson equation is solved on a cartesian grid (we work in cartesian coordinates here)





Conclusion

Current and future works :

- Applications in plasma physics
  - Joint project with european labs (Eurofusion project) : fusion reaction, plasma confinement using large magnetic fields
  - Dominant term is a magnetics field $\frac{1}{\varepsilon}(v \times B) \cdot \nabla_v f$, no more dissipative effects
  - Inter-disciplinary works : computer science (HPC, large data), physics, engineering
- Applications to collective dynamics and self-interactions
  - there are new kinetic models describing these phenomena (see bacteria motions)
  - the structure of this model is simpler but the operators depends on velocity and space, steady states are not explicitly known
  - construction of hybrid method