Model-Predicitive Control Strategies for Agent-Based Systems

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joint work
on best-reply with P. Degond, J-G. Liu.

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Notation and topic of talk for particle system of N particles

$$X = (x_i)_{i=1}^N, \qquad X_{-j} = (x_i)_{i=1, i\neq j}^N, \quad m_X^N = \frac{1}{N}\sum_{j=1}^N \delta(x - x_i) \in \mathcal{P}(\mathbb{R})$$

- Particle games with control $\frac{d}{dt} x_i = f_i(X) + u_i, x_i(0) = \bar{x}_i$
- State x_i of particle i will be in $\mathbb{R}$ (but results are not limited to this case)
- Each particle i shows its own control u_i (hard case, compared with a single control u for all particles)

$$u_i = \text{ argmin } J_i(X, U) \text{ subject to particle dynamics}$$

- Interest: $N \to \infty$ in the associated control problem
- Many contributions and applications: Lasry/Lions et al (meanfield games), Piccoli/Fornasier (sparse controls), H./Pareschi/Albi (MPC), Degond/Liu/Ringhofer (best-reply),

Setting of the problem $i = 1, \ldots, N$

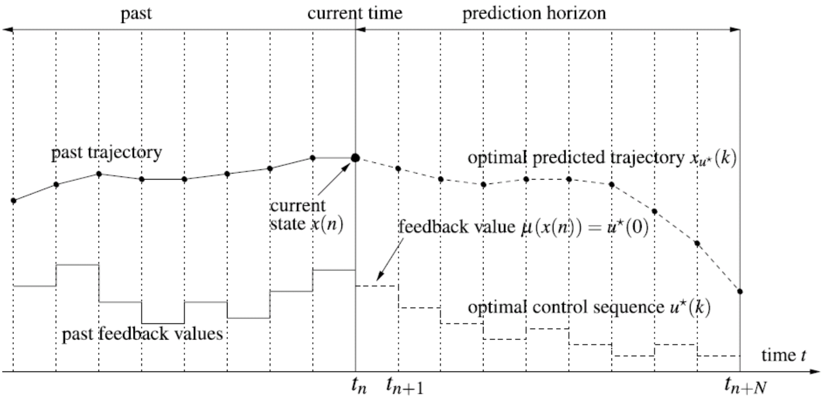
$$(P) \quad \frac{d}{ds} x_i = f_i(X) + u_i, u_i = \text{ argmin }_{\tilde{u}} \int_0^T \frac{\nu}{2} \tilde{u}(s)^2 + h_i(X(s)))ds$$

- Particle system of N interacting particles each having its own control
- Discussion restricted to quadratic cost in objective functional and linear in dynamics (as in Lasry/Lions), integral costs
- Requires $\nu > 0$ for well–posedness ($\nu >> 1$ corresponds to uncontrolled dynamics)
- (P) are N *coupled* optimal control problems to be solved simultaneously
- Crucial assumption for meanfield limit: symmetry of f_i and h_i in $N - 1$ variables for any N

$$(A) \quad f_i(X) = f(x_i, X_{-i}), \quad f(x_i, X_{-i}) = f(x_i, (x_{\sigma(j)})_{j=1, j \neq i}^N)$$

MPC = Receding horizon control on short time horizon

$L \to R$



MPC = Receding horizon control on short time horizon

$L \to R$

$$(P) \quad \frac{d}{ds}x_i = f_i(X) + u_i, \; u_i = \text{ argmin }_{\tilde{u}} \int_0^T \left(\frac{\nu}{2}\tilde{u}^2 + h_i(X)) \right) ds$$

- Assume u_i is piecewise constant on time intervals of length Δt
- At time interval $(t, t + \Delta t)$ consider the discretized problem as approximation to (P)

$$(MPC) \quad x_i(t + \Delta t) = x_i(t) + \Delta t \left(f_i(X(t)) + u_i \right),$$
$$u_i = \text{ argmin }_{\tilde{u}} \Delta t \left(\frac{\Delta t \nu}{2} \tilde{u}^2 + h_i(X(t + \Delta t)) \right)$$

- Up to $O(\Delta t)$ we have $u_i = -\frac{1}{\nu}\partial_{x_i} h_i(X(t))$
- Solution u_i is independent of the choice of u_j for $j \neq i$
- Scaling of the with ν necessary to have $u_i = O(1)$ in (IC)
- u_i is suboptimal compared with (P)

Meanfield limit of controlled particle dynamics $T \to B$

$$(MPC) \quad x_i(t+\Delta t) = x_i(t) + \Delta t\,(f_i(X(t)) + u_i)\,,$$

$$u_i = \text{ argmin }_{\tilde{u}} \Delta t \left(\frac{\Delta t\ \nu}{2} \tilde{u}^2 + h_i(X(t+\Delta t)) \right)$$

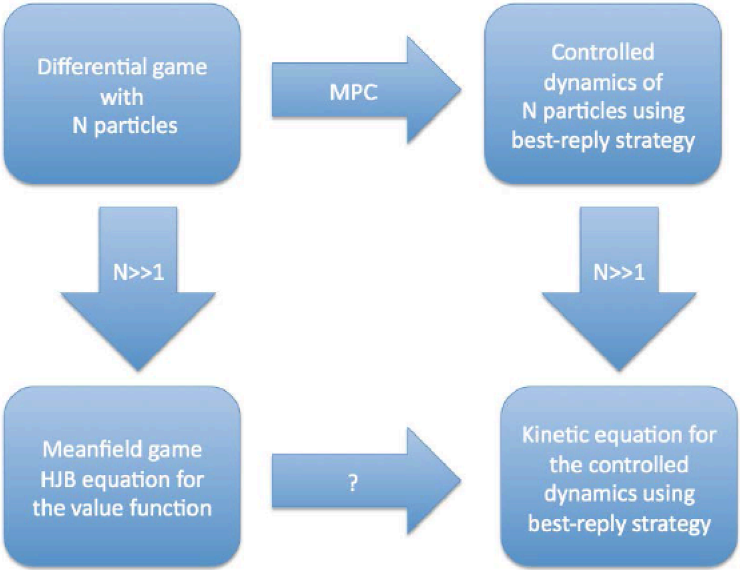
- Feedback formulation $u_i(t) = -\frac{1}{\nu}\partial_{x_i} h_i(X(t))) + O(\Delta t)$
- Substituting to continuous dynamics $\frac{d}{ds} x_i = f(x_i, X_{-i}) - \frac{1}{\nu}\partial_{x_i} h(x_i, X_{-i})$ and to the kinetic equation for $m = m(t,x)$ as

$$\partial_t m + \partial_x \left(\left(\mathbf{f}(x,m) - \frac{1}{\nu}\partial_x \mathbf{h}(x,m) \right) m \right) = 0.$$

- Toy example.

$$\partial_t m + \partial_x \left(\left(\int P(y,x)(y-x) - \frac{1}{\nu}\partial_x \phi(y,x) m dy \right) m \right) = 0.$$

Meanfield games and model predictive control (MPC)



Hamilton–Jacobi Bellmann (HJB) Equation $\quad T \to B$

$$(P) \quad \frac{d}{ds} x_i = f_i(X) + u_i, \; u_i = \text{ argmin }_{\tilde u} \int_0^T \left(\frac{\nu}{2} \tilde u^2 + h_i(X)) \right) ds$$

- Pontryagins maximum principle gives existence of co–states ϕ^i_j for $(i,j) = 1, \dots, N$ such that the optimal control u_i and corresponding optimal trajectory X fulfills

$$-\frac{d}{ds}\phi^i_j - \sum_{k=1}^N \phi^i_k(\partial_{x_j}) f_k(X) = \partial_{x_j} h_i(X), \; \phi^i_j(T) = 0,$$
$$\nu u_i = -\phi^i_i.$$

- Value function for particle i starting at time t with initial data $X(t) = Y$ is

$$V_i(t, Y) = \int_t^T \left(\frac{\nu}{2} u_i^2 + h_i(X)) \right) ds$$

- PMP are characteristics for $(s, X) \to V_i(s, X)$

Toy example and formal computation to highlight main ideas (1/2)

$$(Dynamics) x_i' = u_i,\; u_i^* = argmin_u \int_0^T g(\frac{1}{N}\sum_j x_j) + \frac{u^2}{2} ds$$

$$(Nash) \nabla L_i(u_i, X, (\lambda_j^i)_j ; (u_j^*)_j) = 0 :$$

$$x_i' = u_i, \lambda_i^i = u_i, (\lambda_j^i)' = g'(\frac{1}{N}\sum_j x_j)\frac{1}{N}, \lambda_j^i(T) = 0$$

$$(Value) V_i(\tau, Y) = \int_\tau^T g(\frac{1}{N}\sum_j \int_\tau^t u_j^* ds + y_j) + \frac{(u_i^*)^2}{2} dt$$

$$(HJB)(V_i)_{y_k} = -\lambda_k^i,\; (V_i)_{y_i} = u_i^*,$$

$$(V_i)_\tau = -\frac{(V_i)_{y_i}^2}{2} - g(\frac{1}{N}\sum_j x_j) - \int_\tau^T (\lambda_j^i)' dt \sum_k \lambda_k^k(\tau)$$

Toy example and formal computation to highlight main ideas (2/2)

$$(HJB)(V_i)_\tau = -\frac{(V_i)^2_{y_i}}{2} - g(\frac{1}{N}\sum_j x_j) - \sum_k (V_k)_{y_k}(V_i)_{y_k},\ (V_k)_{y_k} = u_k^* = x_k'$$

$$(Symm) W(t,\xi,X) = V_i(t,x_1,\dots,\xi,\dots,x_n)$$

$$(HJB-W) W_t = -\frac{W_\xi^2}{2} - g(\frac{1}{N}\sum_j x_j) - \sum_k W_{x_k} W_\xi,$$

$$W_\xi(t,x_i,X_{-i}) = u_i^* \implies x_i' = W_\xi(t,x_i,X_{-i}),$$

$$\frac{d}{dt}W(t,x_i(t),X_{-i}(t)) = W_t + (W_\xi)^2 + \sum_{k,k\neq i} W_{x_k}x_k' = +\frac{1}{2}W_\xi^2 - g(\frac{1}{N}\sum_j x_j)$$

$$(Meanfield) w(t,x) = W(t,x,\rho(t)), \rho(t,\cdot) = \frac{1}{N}\sum_j \delta(\cdot - x_i)$$

$$\partial_t\rho(t) + \partial_\xi(w_x(t,x)\rho(t)) = 0,\ w_t = \frac{1}{2}w_x^2 - g(\int x\rho(t)dx).$$

Hamilton–Jacobi Bellmann (HJB) Equation $T \to B$

$$(P) \quad \frac{d}{ds}x_i = f_i(X) + u_i, \; u_i = \text{ argmin }_{\tilde{u}} \int_0^T \left(\frac{\nu}{2}\tilde{u}^2 + h_i(X) \right) ds$$

- Pontryagins maximum principle gives existence of co–states ϕ^i_j for $(i,j) = 1, \ldots, N$ such that the optimal control u_i and corresponding optimal trajectory X fulfills

$$-\frac{d}{ds}\phi^i_j - \sum_{k=1}^N \phi^i_k(\partial_{x_j}) f_k(X) = \partial_{x_j} h_i(X), \; \phi^i_j(T) = 0,$$
$$\nu u_i = -\phi^i_i.$$

- Value function for particle i starting at time t with initial data $X(t) = Y$ is

$$V_i(t, Y) = \int_t^T \left(\frac{\nu}{2}u_i^2 + h_i(X) \right) ds$$

- PMP are characteristics for $(s, X) \to V_i(s, X)$

Hamilton–Jacobi Bellmann (HJB) Equation $\quad T \to B$

$$(PMP) \quad \frac{d}{ds}x_i = f_i(X) + u_i,\ x_i(t) = y_i,$$

$$-\frac{d}{ds}\phi^i_j - \sum_{k=1}^{N} \phi^i_k(\partial_{x_j})f_k(X) = \partial_{x_j}h_i(X),\ \phi^i_j(T) = 0,$$

$$\nu\ u_i = -\phi^i_i,\ V_i(t,Y) = \int_t^T \left(\frac{\nu}{2}u_i^2 + h_i(X)\right) ds.$$

- Differentiation of V_i with respect to y_k and with respect to t using ϕ^i_j gives HJB at $Y = X(t)$

$$\partial_{y_k} V_i(t,Y) = \phi^i_k(t), \quad -\frac{\nu}{2}u_i^2 - h_i(Y) =$$

$$\frac{d}{dt}V_i(t,Y) = \partial_t V_i(t,Y) + \sum_{k=1}^{N} \partial_{x_k} V_i(t,Y)\left(f_k(Y) + u_k\right).$$

- Substituting u_i by $\phi^i_i = \partial_{y_i} V_i(t,Y)$ leads to the HJB for N particles

HJB Equation for value function V_i of particle i $T \to B$

$$\partial_t V_i + \sum_{k=1, k\neq i}^{N} (\partial_{x_k} V_k)(f_k(X) - \frac{1}{\nu}\partial_{x_k} V_k) + f_i(X)\partial_{x_i} V_i = -h_i(X) + \frac{1}{2\nu}(\partial_{x_i} V_i)^2$$

- Changed $Y \to X$ i.e. $V_i = V_i(t, X)$
- Backwards in time with terminal condition $V_i(T, X) = 0$
- Coupling of N particle dynamics
- Equation might not have a solution (if: Nash equilibrium)
- Retrieve optimal control by

$$u_i(t) = -\frac{1}{\nu}\phi_i^i(t) = -\frac{1}{\nu}\partial_{x_i} V_i(t, X(t))$$

- Backwards implicit Euler discretization leads to

$$V_i(T - \Delta t, X) = h_i(X(t)) + O(\Delta t)$$

Structure of HJB Equations $i = 1, \ldots, N$ $\quad T \to B$

$$\partial_t V_i + \sum_{k=1, k\neq i}^{N} (\partial_{x_k} V_k)(f_k(X) - \frac{1}{\nu}\partial_{x_k} V_k) + f_i(X)\partial_{x_i} V_i = -h_i(X) + \frac{1}{2\nu}(\partial_{x_i} V_i)^2$$

- $f_i(X) = f(X)$ symmetric in all variables, $h_i(X) = h(x_i, X_{-i})$ are symmetric in X_{-i}, let $\mathbb{Z} = (\eta, z_1, \ldots, z_{N-1})$ and $\mathbb{Z}_k = (z_k, \eta, z_1, \ldots, z_{k-1}, z_{k+1}, \ldots, z_{N-1})$
- Assume $W = W(t, \mathbb{Z})$ solves equation

$$\partial_t W(t,\mathbb{Z}) + \sum_{k=1}^{N-1} \partial_{z_k} W(t,\mathbb{Z}) \left(f(\mathbb{Z}_k) - \frac{1}{\nu}\partial_\eta W(t,\mathbb{Z}) \right)$$
$$+ f(\mathbb{Z})\partial_\eta W(t,\mathbb{Z}) = -h(\mathbb{Z}) + \frac{1}{2\nu}(\partial_\eta W(t,\mathbb{Z}))^2$$

- Then, $V_i(t, X) = W(t, x_i, X_{-i})$ is solution to ith HJB equation
- Meanfield limit in the equation for $N \to \infty$ for W leading to an equation for $\mathbf{W}(t, x, m)$

Meanfield limits for $W(t,\mathbb{Z})$ $T \to B$

$$\partial_t W(t,\mathbb{Z}) + \sum_{k=1}^{N-1} \partial_{z_k} W(t,\mathbb{Z}) \left(f(\mathbb{Z}_k) - \frac{1}{\nu} \partial_\eta W(t,\mathbb{Z}) \right)$$

$$+ f(\mathbb{Z}) \partial_\eta W(t,\mathbb{Z}) = -h(\mathbb{Z}) + \frac{1}{2\nu} (\partial_\eta W(t,\mathbb{Z}))^2$$

Function $W(t,\mathbb{Z}) = W(t,\eta,z_1,\ldots,z_{N-1})$ is symmetric in $(z_1,\ldots,z_{N-1})$ and therefore we may expect a limit $\mathbf{W}(t,\eta,m)$

$$W(t,\mathbb{Z}) = W_N(t,\eta,m_{Z_{-N}}^{N-1}) \sim \mathbf{W}(t,\eta,m_Z^N),$$

$$\partial_t W(t,\mathbb{Z}) \sim \partial_t \mathbf{W}(t,\eta,m_Z^N), \quad \partial_\eta W(t,\mathbb{Z}) \sim \partial_\eta \mathbf{W}(t,\eta,m_Z^N),$$

Meanfield limit for the sum $\sum_{k=1}^{N-1} \partial_{z_k} W(t,\mathbb{Z}) f(\mathbb{Z}_k)$ $T \to B$

- As before for a symmetric function g and

$$\sum_{j=1}^{N} c(x_j) g(X) = \frac{d}{dt} g(\dots, C_i(t), \dots) = \frac{d}{dt} \mathbf{G}(m_X^N(t)) \sim < \partial_m \mathbf{G}(m), c > .$$

- Apply to f if symmetric in all arguments $f(\mathbb{Z}_k) \sim \mathbf{f}(m)$ to obtain

$$\sum_{k=1}^{N-1} \partial_{z_k} W(t,\mathbb{Z}) f(\mathbb{Z}_k) = \frac{d}{dt} \mathbf{W}(t, \eta, m_Z^N) - \partial_t \mathbf{W}(t, \eta, m_Z^N),$$

$$\partial_t \left(m_Z^N \right) + \partial_x \left(f^N(m_Z^N) m_Z^N \right) = 0.$$

- Hence: $\sum_{k=1}^{N-1} \partial_{z_k} W(t,\mathbb{Z}) (f(\mathbb{Z}_k) - \frac{1}{\nu} \partial_\eta W(t,\mathbb{Z}))$
$\sim < \partial_m \mathbf{W}(t,\eta,m), \mathbf{f}(m) - \frac{1}{\nu} \partial_\eta \mathbf{W}(t,\eta,m) > .$

Summary of meanfield limit for HJB $T \to B$

$$\partial_t \mathbf{W}(t,\eta,m) + < \partial_m \mathbf{W}(t,\eta,m), \mathbf{f}(m) - \frac{1}{\nu}\mathbf{W}(t,\eta,m) >$$
$$+ \mathbf{f}(m)\partial_\eta \mathbf{W}(t,\eta,m) = -\mathbf{h}(\eta,m) + \frac{1}{2\nu}(\partial_\eta \mathbf{W}(t,\eta,m))^2$$

Change η to x and introduce $\mathbf{w}(t,x) = \mathbf{W}(t,x,m(t))$ where $m(t)(\cdot)$ is solution to conservation leads to

$$\partial_t \mathbf{w}(t,x) + \mathbf{f}(m)\partial_x \mathbf{w}(t,x) = -\mathbf{h}(x,m) + \frac{1}{2\nu}(\partial_x \mathbf{w}(t,x))^2$$
$$\partial_t m + \partial_x \left(m \left(\mathbf{f}(m) - \frac{1}{\nu}\partial_x \mathbf{w}(t,x) \right) \right) = 0$$

MPC for meanfield equation

$$\partial_t \mathbf{w}(t,x) + \mathbf{f}(m)\partial_x \mathbf{w}(t,x) = -\mathbf{h}(x,m) + \frac{1}{2\nu}(\partial_x \mathbf{w}(t,x))^2$$

$$\partial_t m + \partial_x \left(m \left(\mathbf{f}(m) - \frac{1}{\nu}\mathbf{w}(t,x) \right) \right) = 0$$

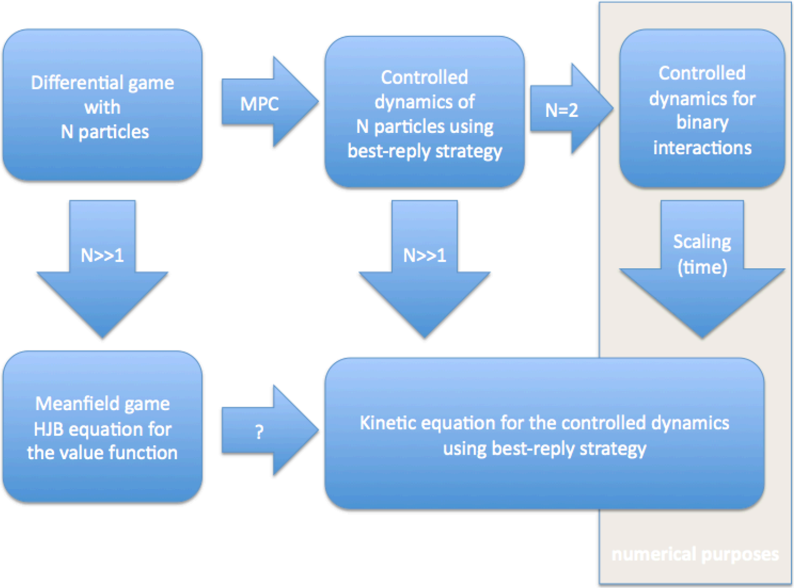
- Terminal time was arbitrary; set $T = t + \Delta t$
- Terminal condition on $\mathbf{w}(T,x) = 0$ and explicit backwards Euler discretization leads to

$$\mathbf{w}(T - \Delta t, x) = \mathbf{h}(x,m)$$

- Taylor expansion yields kinetic equation equivalent to the MPC approach applied to particle system

$$\partial_t m + \partial_x \left(m \left(\mathbf{f}(m) - \frac{1}{\nu}\partial_x \mathbf{h}(x,m) \right) \right) = 0$$

Meanfield games, MPC and numerics



Efficient computation of controlled particle systems

$$(P) \quad \frac{d}{ds} x_i = f_i(X) + u_i, \; u_i = \text{ argmin }_{\tilde{u}} \int_0^T \left(\frac{\nu}{2} \tilde{u}^2 + h_i(X)) \right) ds$$

- Apply MPC approach at every time t with time horizon Δt to obtain closed formula for $u_i = -\frac{1}{\nu} \partial_{x_i} h_i(X)$
- Straight–forward discretization $x_i^n = x_i(t_n)$ requires to evaluate N collisions per time step (similar to explicit spatial discretization of kinetic equation)
- Consider *binary* discretized interaction model where $f^{bin} = f(X)$ and $N = 2$

$$x_i^{n+1} = x_i^n + \Delta t \; f^{bin}(x_j^n, x_i^n) - \frac{\Delta t}{\nu} \partial_{x_i} h^{bin}(x_i^n, x_j^n),$$
$$x_j^{n+1} = x_j^n + \Delta t \; f^{bin}(x_i^n, x_j^n) - \frac{\Delta t}{\nu} \partial_{x_j} h^{bin}(x_i^n, x_j^n),$$

Remarks on controlled binary interaction dynamics

$$x_i^* = x_i + \tau\, f^{bin}(x_j, x_i) - \frac{\tau}{\nu}\partial_{x_i} h^{bin}(x_i, x_j),$$
$$x_j^* = x_j + \tau\, f^{bin}(x_i, x_j) - \frac{\tau}{\nu}\partial_{x_j} h^{bin}(x_i, x_j),$$

- Pre-collision states (x_i, x_j), post–collision states (x_i^*, x_j^*) out of $i, j = 1, \ldots, N$, interaction strength $\tau = \Delta t$
- Write kinetic equation for the single particle distribution with γ interactions per Δt
- Choose a scaling of the rate γ such that binary interaction model yields up to $O(\tau^2)$ the MPC *meanfield* kinetic equation

$$\partial_t m + \partial_x \left(m \left(\mathbf{f}(m) - \frac{1}{\nu}\partial_x \mathbf{h}(x, m) \right) \right) = 0$$

- Approach possible for alignment models as for example opinion or wealth formation or Cucker–Smale model
- Numerical examples computed as presented

Sznadj's model with $h_i(X) = \frac{1}{2}(x_i - w_d)^2$

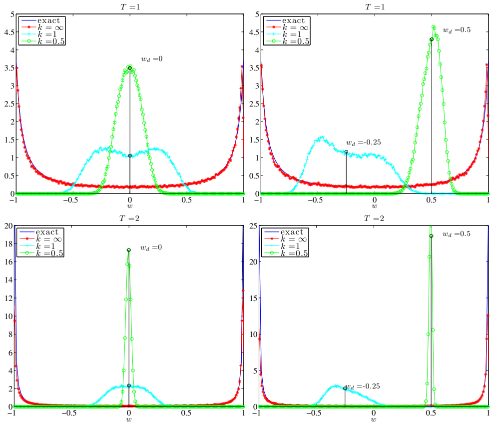


Figure: Solution profiles at time $T = 1$, first row, and $T = 2$, second row, for uncontrolled, mildly controlled case, strong controlled case. On the left: desired state is set to $w_d = 0$, on the right $w_d = 0.5$ for the strongly controlled case, and $w_d = -0.25$ for the mildly controlled case.

Cucker–Smale model with control on velocity

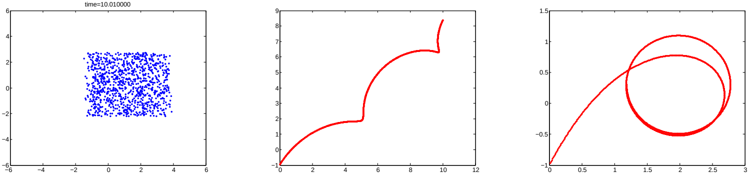
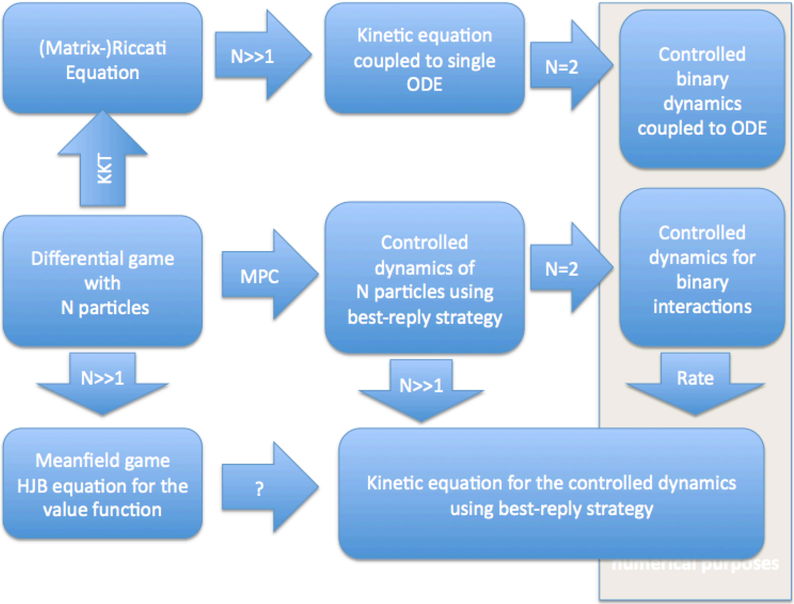


Figure: Trajectory of the center of mass in the controlled and uncontrolled case. Terminal particle distribution in the controlled case at time $T = 10$.

Meanfield games, MPC and Riccati



Optimality of the MPC approach

$$(P) \quad \frac{d}{ds}x_i = f_i(X) + u_i, \; u_i = \text{ argmin }_{\tilde{u}} \int_0^T \left(\frac{\nu}{2}\tilde{u}^2 + h_i(X)) \right) ds$$

- Corresponds to solve HJB equations on a moving horizon Δt
- u_i is optimal solution for the value function $V_i^{\Delta t}(t,Y) = \int_t^{t+\Delta t} \left(\frac{\nu}{2}u_i^2 + h_i(X) \right) ds$ (leads to approximation and cumulative errors)
- *Simplified setting*
  $f_i(X)$ linear, $h_i(X)$ quadratic independent of i, single control $u_i \equiv u$
- Problem (P) has an explicit solution with $u = -\frac{1}{\nu}\sum_{j=1}^{N}(K(t)X)_j$ where $K(t)$ solves a backwards in time Riccati equation
- Function $\frac{1}{2}X^T K(t)X = V(t,X)$ fulfills HJB equation (independent of i)

Riccati equation

$$(P) \quad \frac{d}{ds} x_i = f_i(X) + u, \;\; u = \text{ argmin }_{\tilde{u}} \int_0^T \left(\frac{\nu}{2} \tilde{u}^2 + h(X) \right) ds,$$

$$f_i(X) = (AX)_i, \;\; h(X) = \frac{1}{2} X^T M X, \;\; u = -\frac{1}{\nu} \sum_{j=1}^N (K\, X)_j$$

- Riccati equation for $K(t) \in \mathbb{R}^{N \times N}$ with $K(T) = 0$ given by

$$-\frac{d}{dt} K(t) = KA + A^T K - \frac{1}{\nu} K\, \mathbf{1}\, K^T + M.$$

- Toy model and explicit Euler discretization leads to particular structure of

$$K(t) = \mathcal{K}(t) \mathbf{1}$$

  and $\mathcal{K}(t) \in \mathbb{R}$ fulfills an ordinary differential equation
- Meanfield limit for $N \to \infty$ leads to kinetic equation coupled to a single ODE for $\mathcal{K}$

Explicit computation for toy model

$$(P) \quad \frac{d}{ds}x_i = \frac{1}{N}\sum_{i=1}^{N} P(x_j - x_i) + u, \; u = \text{ argmin }_{\tilde{u}} \int_0^T \left(\frac{\nu}{2}\tilde{u}^2 + \frac{1}{2}X^T X) \right) ds,$$

Acting control for particle i with the *binary* interaction with $\tau = \Delta t$

$$\text{Riccati} \quad u = -\frac{\tau}{\nu}\mathcal{K}(t)x_i, \; \mathcal{K}(t) = \frac{1}{\sqrt{\nu}}\tanh(\frac{T-t}{\sqrt{\nu}})$$

$$\text{MPC} \quad u = -\frac{\tau^2}{2(\nu+\tau^2)}(x_i + x_j)$$

MPC vs optimal (Riccati) control

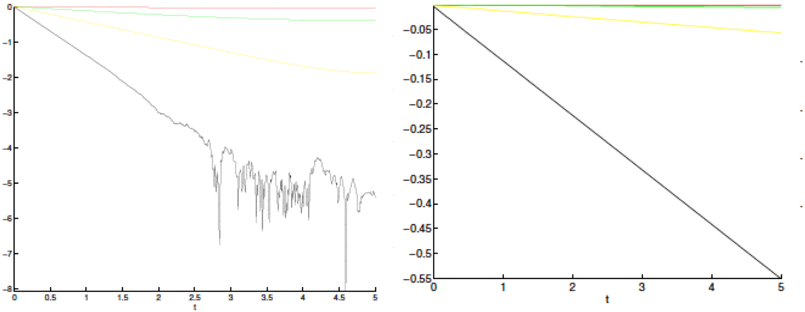
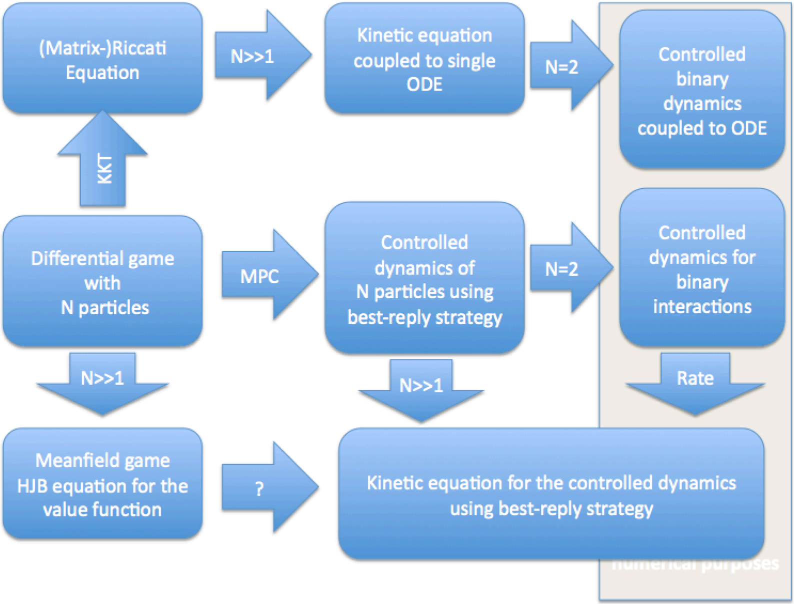


Figure: Evolution of the mean $\int xf(x,t)dx$ for in the Riccati control case (left) and the MPC case (right). Plots are in log–scale and for different penalization of the control ν. Left plot scales to 10^{-8}, right to $10^{-0.55}$.

Meanfield games, MPC and Riccati



Performance bounds on the MPC

Is there a quantitative estimate on the performance of a general MPC?

- Comparison in terms of the value function (optimal)

$$V^*(\tau, y) = \text{argmin}_u \int_\tau^T h(X) + \frac{\nu}{2} u^2 ds$$

where $x_i'(t) = f(X(t)) + u,\ t \in (\tau, T), x_i(0) = y_i$.

- MPC controlled dynamics are $(x_i^{MPC})'(t) = f(X^{MPC}(t)) + u^{MPC}$ and future costs are

$$V^{MPC}(\tau, y) = \int_\tau^T h(X^{MPC}) + \frac{\nu}{2}(u^{MPC})^2 ds$$

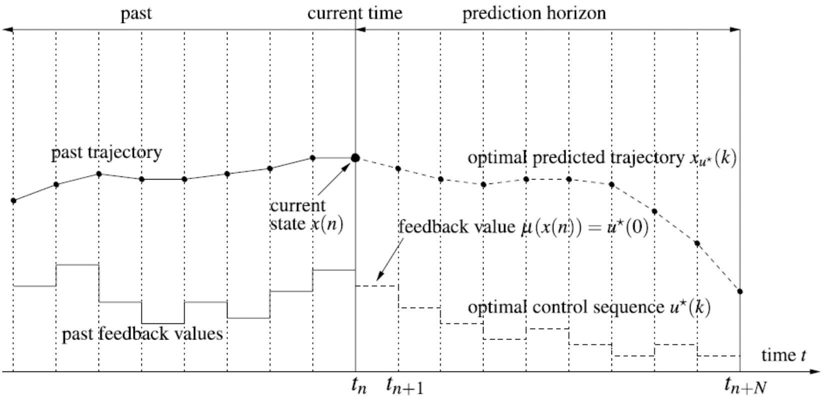
- Finite dimensional result [1]

$$V^{MPC}(\tau, y) \leq \alpha V^*(\tau, y)$$

for some $0 < \alpha < 1$ provided that V^* fulfills a growth condition. α depends on the growth of the running cost h **and** the MPC horizon M

[1] Grüne, 2009

MPC = Receding horizon control on short time horizon M



Performance bounds on the MPC (cont'd)

Is there a quantitative estimate on the performance of a general MPC?

$$V^{MPC}(\tau, y) \leq \alpha(M, h) V^*(\tau, y)$$

- Result extends to the meanfield limit under same assumptions (plus symmetry of running cost and dynamics)
- Observed numerically for an opinion formation model

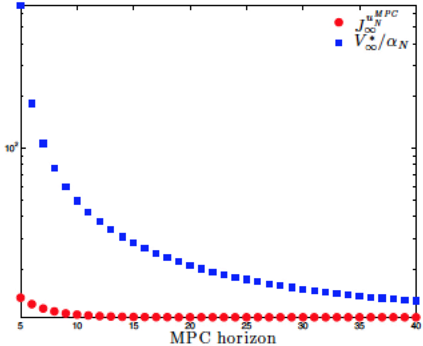
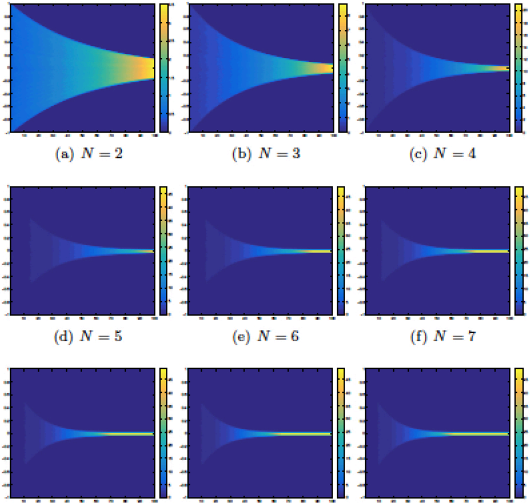


Illustration of the effect of longer MPC horizon (N) on opinion formation dynamics



(a) $N = 2$ (b) $N = 3$ (c) $N = 4$

(d) $N = 5$ (e) $N = 6$ (f) $N = 7$

(g) $N = 8$ (h) $N = 9$ (i) $N = 10$

FIGURE 3. Experimental results for the optimization problem with

Thank you for your attention.

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