Modeling fish migration with an interacting particle model

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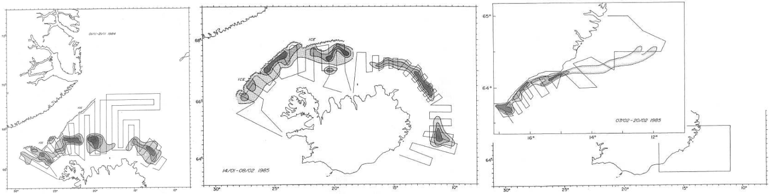
Research summary related to interacting particle models:

- Interacting particle model for fish migration
  - Behavior of the particle model with noise *(with B. Birnir, K. Taylor; simulation assistance from P. Trethewey, L. Youseff)*
  - Parallelization of the simulations *(with B. Birnir, J. Gilbert, P. Trethewey, L. Youseff)*
  - The model applied to the Icelandic Capelin *(with B. Birnir, B. Einarsson, S. Sigurdsson, The Marine Institute of Iceland)*
  - Scaling in interacting particle systems *(with B. Einarsson)* [current]
- Interacting particle models for gang dynamics
  - A coupled network model for gang rivalry formation *(with R. Hegemann, L. Smith, S. Reid, A. Bertozzi, G. Tita)*
  - A statistical mechanics approach to gang territorial development *(with L. Chayes, M. R. D'Orsogna)*
- Kinetic and hydrodynamic models for particle systems
  - Phase transition and diffusion among socially interacting self-propelled agents *(with P. Degond)*
  - Phase transition in a kinetic Cucker-Smale model with self-propulsion and friction *(with J. A. Carrillo, P. Degond)* [current]
  - A kinetic contagion model for fear in crowds *(with J. Rosado)* [current]
  - An exploration of the effect of normalization and different kinds of noise in Vicsek-type flocking models *(with M. Burger)* [current]

The Data: Icelandic stock of capelin

The Icelandic stock of capelin

An example of the acoustic data:



Our model

$$\begin{pmatrix} x_k(t+\Delta t) \\ y_k(t+\Delta t) \end{pmatrix} = \begin{pmatrix} x_k(t) \\ y_k(t) \end{pmatrix} + \Delta t \cdot v_k(t) \frac{\mathbf{D}_k(t)}{\| \mathbf{D}_k(t) \|} + \mathbf{C}(\tilde{\mathbf{p}}_k(t))$$

- Here, $\mathbf{D}_k$ is the directional heading of particle k
- Δt is the timestep
- Particle k's position in the plane is $\mathbf{p}_k$
- $\tilde{\mathbf{p}}_k$ is the nearest gridpoint to $\mathbf{p}_k$
- $\mathbf{C}(\tilde{\mathbf{p}}_k)$ is the current at $\tilde{\mathbf{p}}_k$

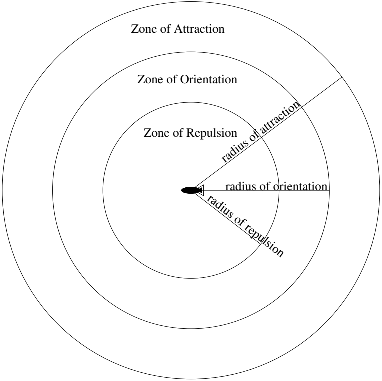
Choosing the direction

The directional heading of the particles (apart from environmental effects) is determined as follows:

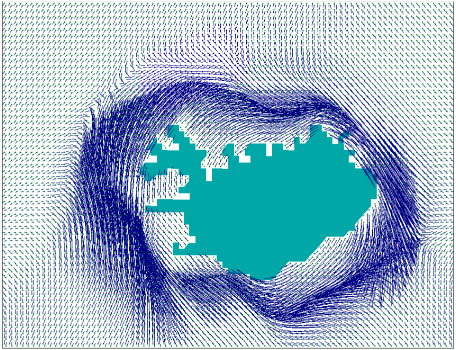
$$\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix} = \frac{\mathbf{d}_k(t+\Delta t)}{\| \mathbf{d}_k(t+\Delta t) \|}$$

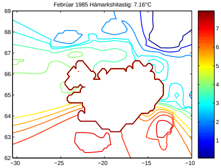
where

$$\mathbf{d}_k(t+\Delta t) := \left(\sum_{r \in R_k} \frac{\mathbf{p}_k(t) - \mathbf{p}_r(t)}{\| \mathbf{p}_k(t) - \mathbf{p}_r(t) \|} + \sum_{o \in O_k} \begin{pmatrix} \cos(\phi_o(t)) \\ \sin(\phi_o(t)) \end{pmatrix} + \sum_{a \in A_k} \frac{\mathbf{p}_a(t) - \mathbf{p}_k(t)}{\| \mathbf{p}_a(t) - \mathbf{p}_k(t) \|} \right).$$

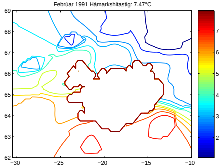


Environmental information

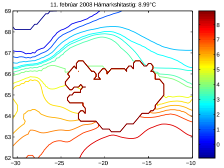




February, 1985



February 1991



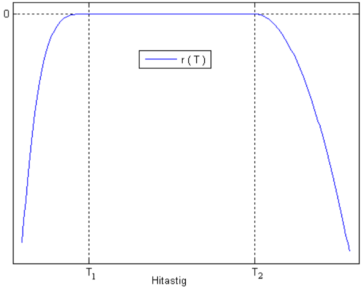
February 2008 [1]

[1] http://www.wetterzentrale.de/topkarten/fsfaxsem.html

Temperature function

Function $r(T)$ determines the reaction of the particles to the temperature field.

$$r(T) := \begin{cases} -(T - T_1)^4 & \text{if} \quad T \leq T_1 \\ 0 & \text{if} \quad T_1 \leq T \leq T_2 \\ -(T - T_2)^2 & \text{if} \quad T_2 \leq T \end{cases}$$



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where

$$\mathbf{D}_k(t+\Delta t) := \left(\alpha \underbrace{\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix}}_{\text{interaction term}} + \beta \underbrace{\frac{\nabla r(T(\mathbf{p}_k(t)))}{\| \nabla r(T(\mathbf{p}_k(t))) \|}}_{\text{temperature term}} \right)$$

for $\alpha + \beta = 1$.

Acoustic data from 1984-1985

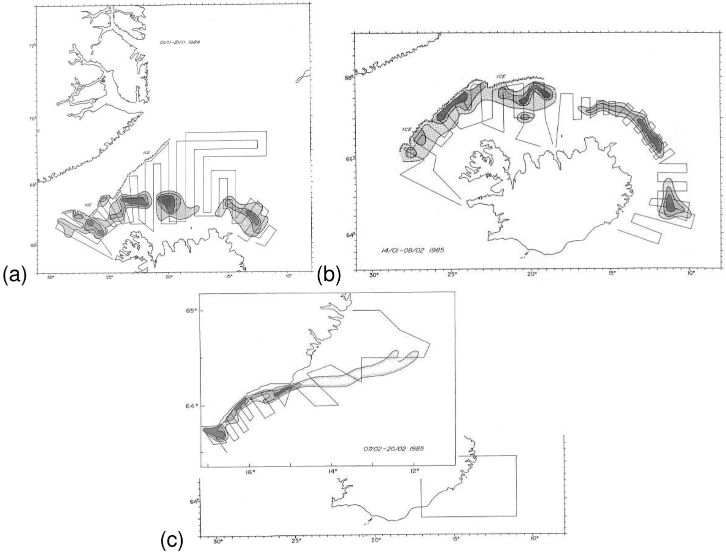


Figure : The distribution of capelin during the spawning migration of 1984-1985.
(a) Acoustic data from November 1 to November 21 (b) Acoustic data from January 14 to February 8
(c) Close up of the distribution of capelin from February 7 to February 20 of 1985.

Acoustic data from 1990-1991

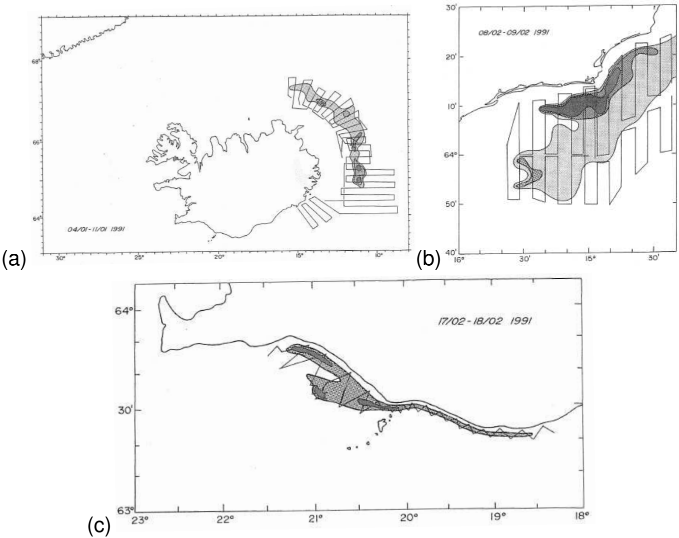
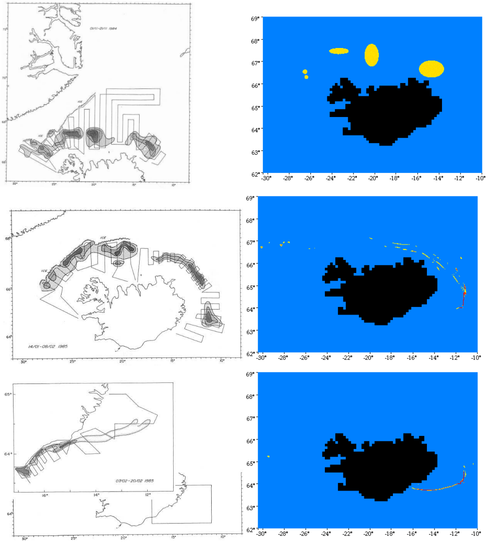


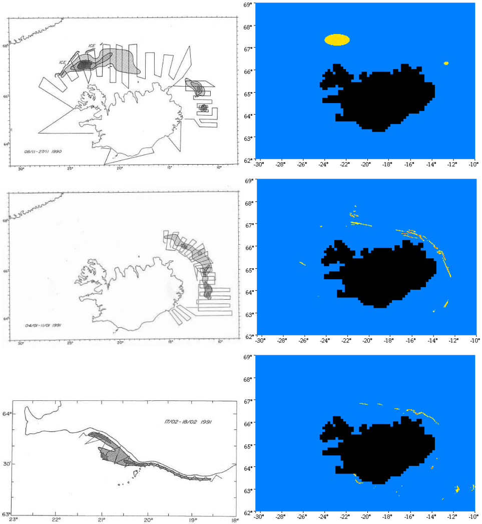
Figure : The distribution of capelin during the spawning migration of 1991.
(a) Acoustic data from January 4 to January 11.
(b) Close up of the distribution of capelin southeast of Iceland from February 8 to February 9 of 1991.
(c) Acoustic data from February 17 to February 18.

1984-1985



14/01-08/02 1985
07/02-20/02 1985
69° 68° 67° 66° 65° 64° 63° 62°
-30° -28° -26° -24° -22° -20° -18° -16° -14° -12° -10°

1990-1991



ICE
08/11-27/11 1990
04/01-11/01 1991
17/02-18/02 1991
69°
68°
67°
66°
65°
64°
63°
62°
-30°
-28°
-26°
-24°
-22°
-20°
-18°
-16°
-14°
-12°
-10°
23°
22°
21°
20°
19°
18°
30'

2008

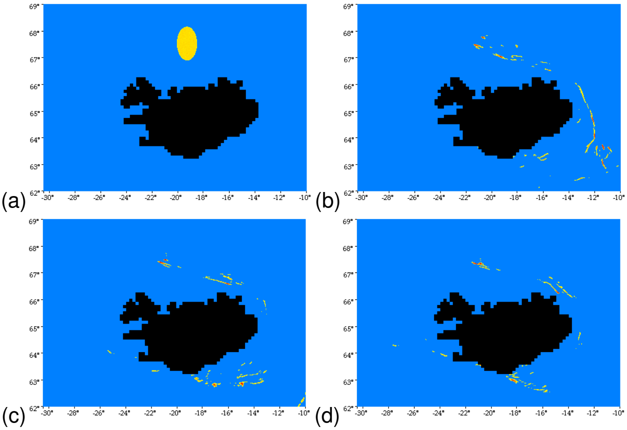


Figure : Simulation of the 2007-2008 spawning migration.
(a) Early January, day 0
(b) Mid-February, day 47
(c) Late February, day 59
(d) Early March, day 65.

Acoustic data from 2008

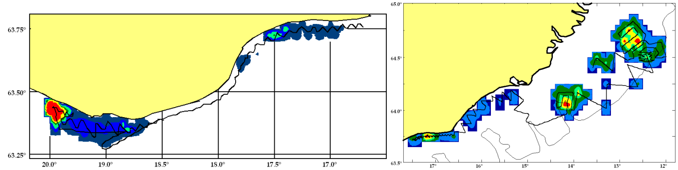
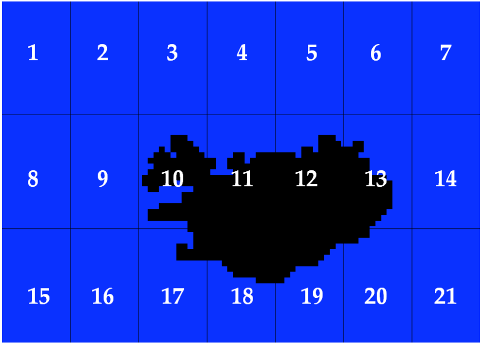


Figure : Collected migration data:
(a) Measured distribution of capelin near south coast of Iceland from February 26 to February 27 of 2008.
(b) Measured distribution of capelin near the southeast coast of Iceland from February 29 to March 3 of 2008.

Sensitivity to perturbed parameters



We measure the sensitivity of the system by seeing how the migration route and timing change. For details, see to [2].

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- Each particle must therefore be thought of as a superindividual
- With these superindividuals, we captured the migration

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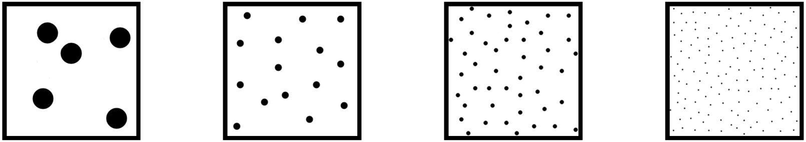
So this leads to a question: how does the system change as we change the number of particles? We need to make some assumptions:

- We assume uniform density of particles and fish in the schools
- The interaction length of the particles should be much less than the size of the school
- We further assume the velocities of the particles are equal

Propagation of information through the school

- If sufficiently dense, local interactions between particles allows information to propagate through a school
  - Temperature information
  - Information about predators
  - Information about food
- We want to preserve the speed at which this information propagates through the school

Varying Numbers of Particles



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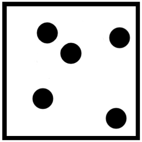
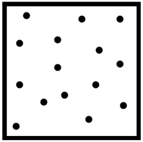
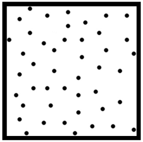
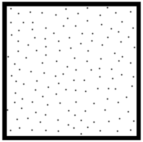

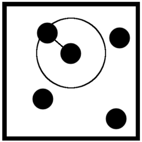

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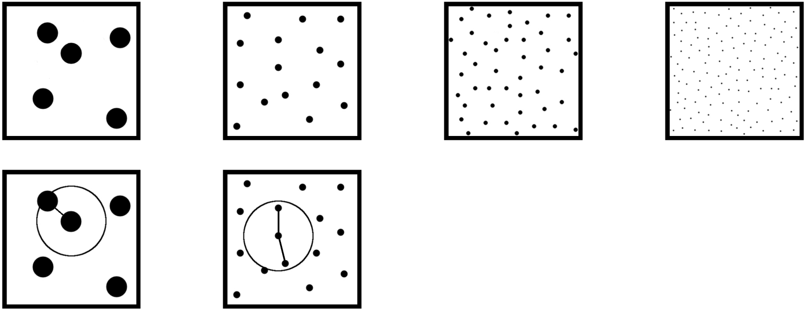

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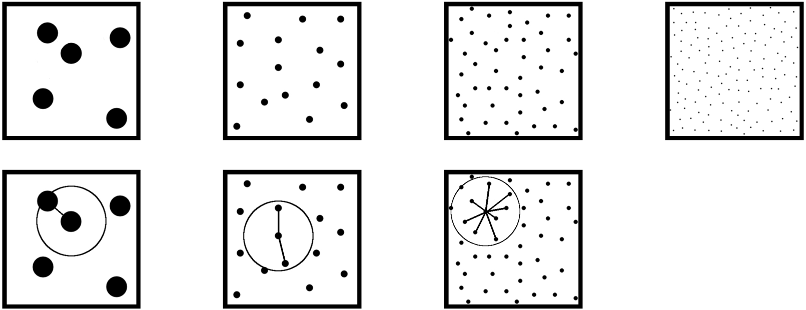

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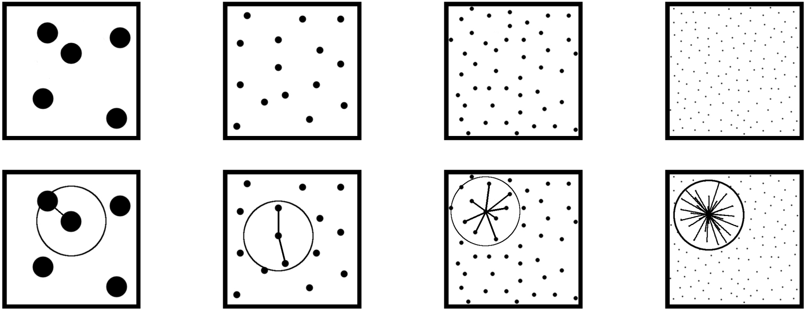

Relationship between Parameters in the Simulation

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&= \left(\text{Fish per particle}\right)\left(\frac{\text{\# Particles in simulation}}{\text{\# Zones of interaction}}\right)\left(\frac{\text{\# Zones of interaction}}{\text{Size of domain}}\right) \\
&= \left(\frac{F}{N_i}\right)\left(\frac{N_i}{D/\pi {R_i}^2}\right)\left(\frac{D/\pi {R_i}^2}{D}\right)
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$$\frac{\text{\# Real fish}}{\text{Size of domain}} = \left(\frac{F}{N_i}\right)(M_i)\left(\frac{1}{\pi {R_i}^2}\right)$$

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- So, if we want to maintain the number of interaction partners, the radii and the number of particles should relate as follows:
$$R \propto \frac{1}{\sqrt{N}}.$$

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- In this way, we see:

$$\Delta t \propto R \propto \frac{1}{\sqrt{N}}.$$

Constant density of actual fish

- In the actual migration, there are a given number of fish within a given area
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- Schematic:
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- Let N denote the total number of particles in a simulation, F denote the number of fish in the migration, and A_w denote the total area of the region
- Let M denote the number of particles per interaction zone
  - Constant across interaction zones due to constant density assumption
  - For computational intensity, need M is constant across different simulations (so the number of neighbors for each particle remains constant)

Relating time and space to the number of particles

- Then for a given simulation indexed by i, $\frac{F}{A_w} = (M)(\frac{F}{N_i})(\frac{A_w}{\pi r_i^2}) \Rightarrow \frac{1}{A_w^2} = \frac{M}{(\pi r_i^2)N_i}$

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- For two different simulations:
  - $\frac{M}{(\pi r_0^2)N_0} = \frac{M}{(\pi r_1^2)N_1} \Rightarrow (\frac{r_1}{r_0})^2 = \frac{N_0}{N_1}$

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- $r_1 = r_0\sqrt{\frac{N_0}{N_1}}$
- Considering r_0 and N_0 to have come from a reference simulation:
  - $\Delta t \propto r \propto \sqrt{\frac{1}{N}}$

Our parameters for the migrations

- $\Delta t = 0.05$ days
- Initial speed $v_k \simeq 4 - 8$ km/day
- $r_r = 0.01$ or about ~ 120 m
- $r_o = r_a = 0.1$ or about ~ 1.2 km
- Number of particles is roughly $5 \cdot 10^4$

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  These are all biologically reasonable!

Where to go from here

Toward Data

- Einarsson, Birnir, and Sigurdsson have created a dynamic energy budget (DEB) model for the physiology of the capelin [9]
- Next step: Incorporate this DEB model into the simulations of the spawning migration

Toward Mathematics

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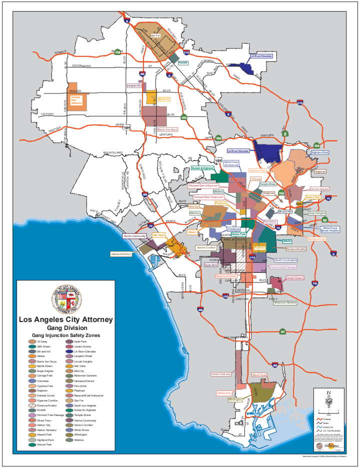
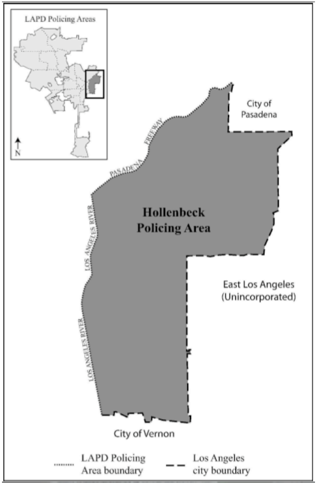
Toward Mathematics

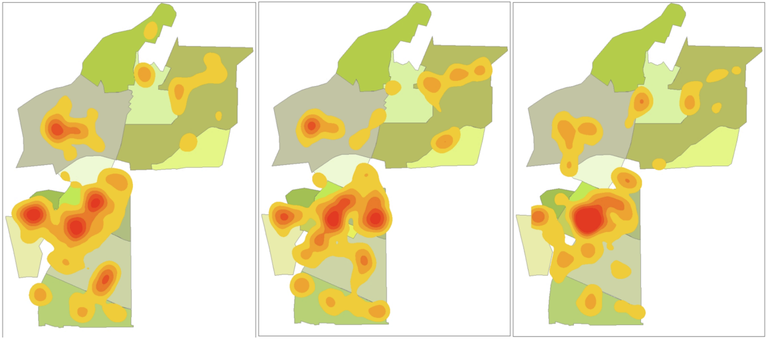
- Numerical validation of the proposed scaling laws
- Kinetic and hydrodynamic versions of similar models have been and are being studied
- Models taking into consideration the number of interaction neighbors have also been proposed and studied
- Including emotional influences into the model

Gangs are a problem in Los Angeles!

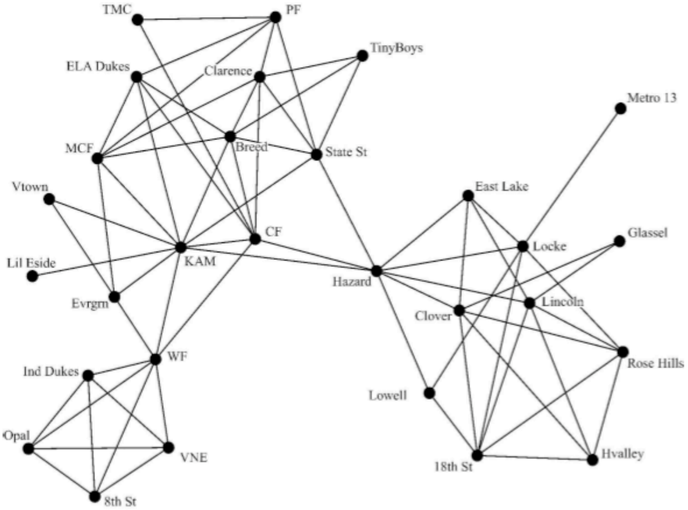




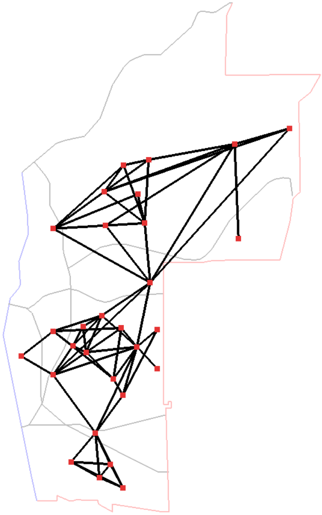
Violence data: 1998, 1999, and 2000



The Rivalry Network



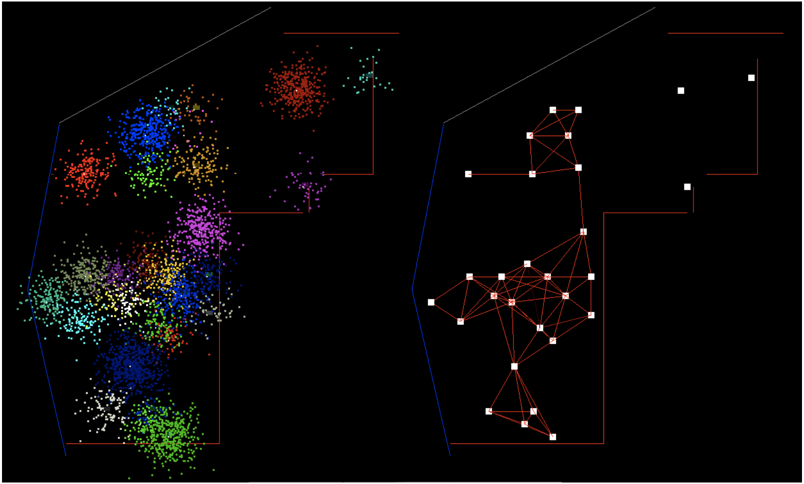
Observed Rivalry Network Among Hollenbeck Gangs [2]



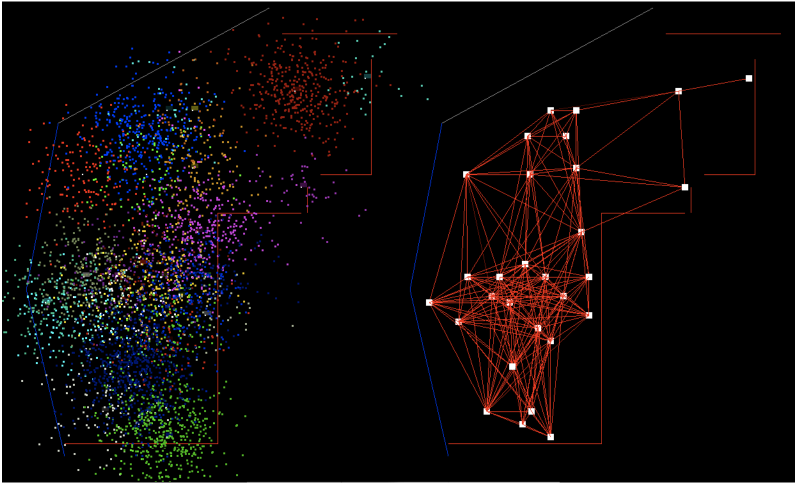
- 29 Active Gangs in Hollenbeck
- 69 Rivalries Among the Gangs
- A Set Space is a gang's center of activity where gang members spend a large quantity of their time
- Gang activity in Hollenbeck is generally isolated from gang activity outside of Hollenbeck
- Freeways and other geographic features influence the rivalry network

[2] S. Radil, C. Flint, and G. Tita,"Spatializing Social Networks: Using Social Network Analysis to Investigate Geographies of Gang Rivalry, Territoriality, and Violence in Los Angeles." 2010.

A control: just diffusion



What's going wrong?



Potential models

- Graph Generating Methods
  - Geographical Threshold Graphs

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  - Biased Lévy Flights with Semi-Permeable Boundaries

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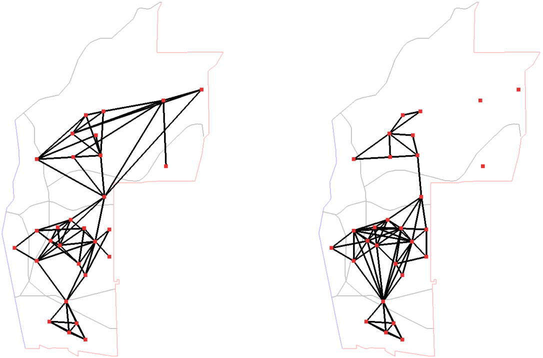
- Graph Generating Methods
  - Geographical Threshold Graphs
- Agent-Based Methods
  - Brownian Motion with Semi-Permeable Boundaries
  - Biased Lévy Flights with Semi-Permeable Boundaries
    - Coupling the rivalry network and avoidance strength
    - Decay on the edges of graph
    - Heading home
    - Avoiding rivals' set spaces
    - Semi-permeable freeways

Geographical Threshold Graphs [3]

- Geographical Threshold Graphs (GTGs) randomly assign weights η_i to the N nodes
- The edge between nodes n_i and n_j exists only if $\frac{F(\eta_i,\eta_j)}{d(n_i,n_j)^\beta} \geq$ Threshold

We construct a specific realization of GTGs:

- η_i = size of gang i
- $F(\eta_i, \eta_j) = \eta_i \cdot \eta_j$, and $\beta = 2$
- Threshold to have the same number of rivalries as observed network



[3] M. Bradonjic, A. Hagberg, A. Percus. *Giant Component and Connectivity in Geographical Threshold Graphs (2007).*

Biased Lévy walk with semi-permeable boundaries

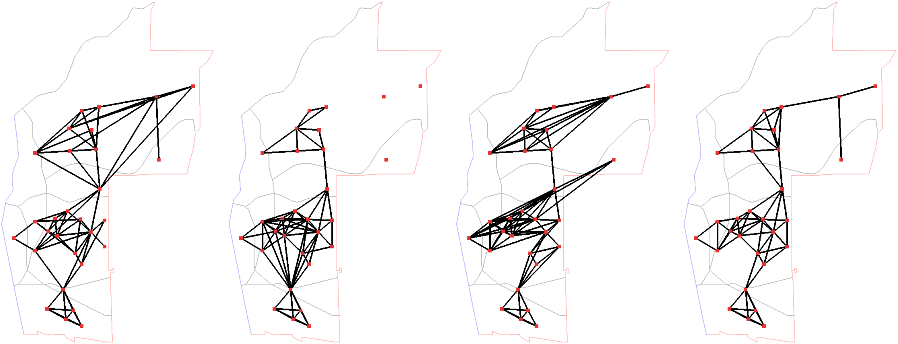
Movement dynamics:

- Agents move in free space according to a biased Lévy walk
- Choose direction of bias according to location of other gangs' set spaces and location of the agent's own set space
- Agents have some probability of crossing a boundary

Interactions:

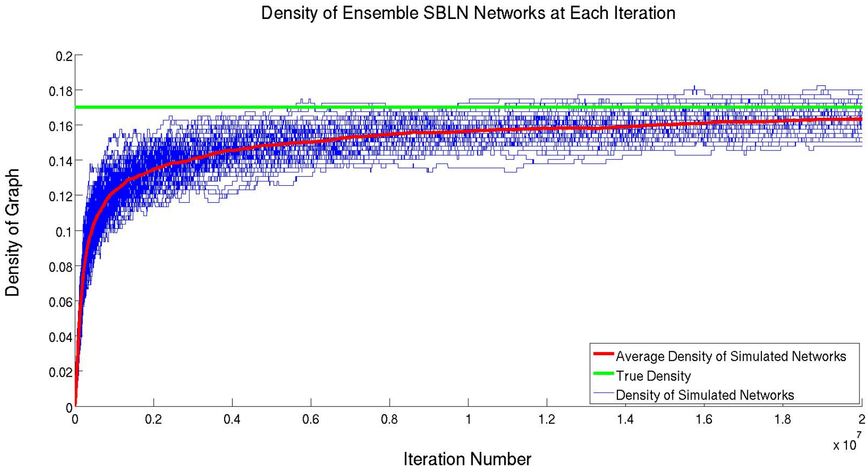
- If two gang members from different gangs cross paths, then an interaction has occurred and the rivalry between the gangs is excited
- At the end of the simulation, we exclude rivalries where the number of interactions is mutually insignificant to both gangs

Comparison of Networks: Observed, Geogrpahical Threshold Graph (GTG), Brownian Motion Network (BMN), Simulated Biased Lévy walk Network (SBLN)



Observed GTG BMN SBLN

Limiting Behavior of ensemble SBLN: graph density

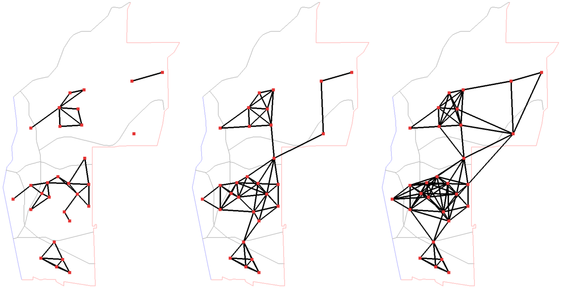


	Density	Variance	Centrality
Observed	0.169951	4.32105	0.201058
GTG	0.169951	9.976219	0.277778
Ensemble BSN	0.163547 ± 0.005593	3.6642331 ± 0.483954	0.1503968 ± 0.018831

Table : The table provides the shape measures for the observed network, GTG, BMN, and ensemble BSN.

Performance of the models

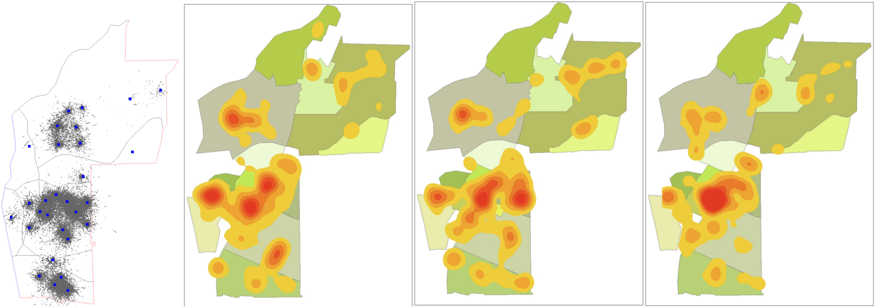
- SBLN and GTG both performed quite well in metric comparisons (accuracy, shape, community structure metrics)
- SBLN allows us to explore *evolution* of the rivalries
- SBLN produces dynamic stochastic networks:



Comparison (left to right) of ensemble SBLN 100% edge agreement, ensemble SBLN 50% edge agreement, and ensemble SBLN 1% edge agreement

Static network model vs. SBLN model

- SBLN allows us to see where interactions take place



Simulated 1998 1999 2000

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