Fluctuation and fixation in the one-dimensional Axelrod model

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State space – F cultural features with q states

$$\eta_t : \mathbb{Z} \longrightarrow \{1, 2, \ldots, q\}^F$$

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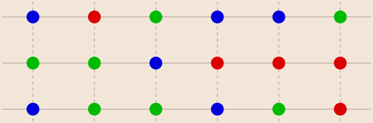

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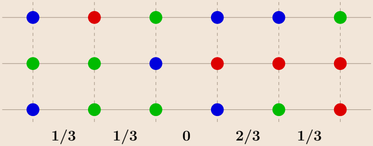

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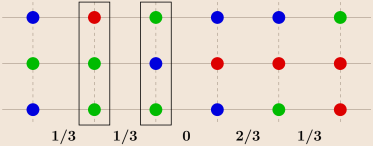

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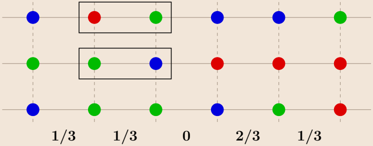

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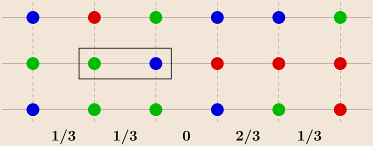

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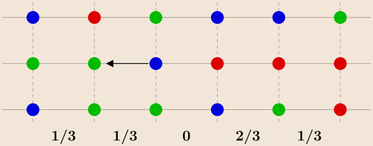

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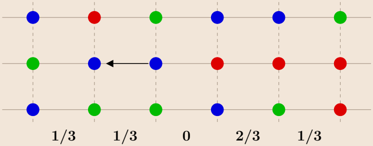

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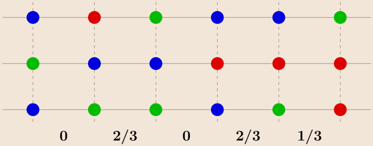

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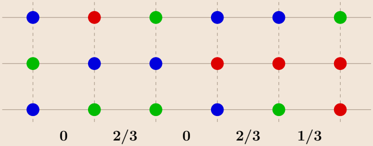

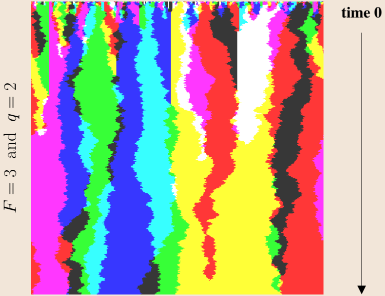

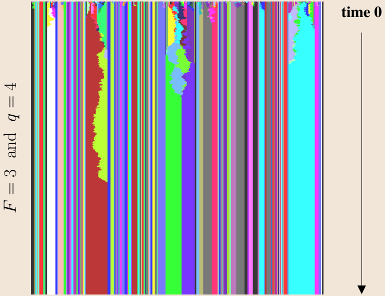

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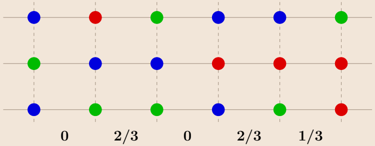
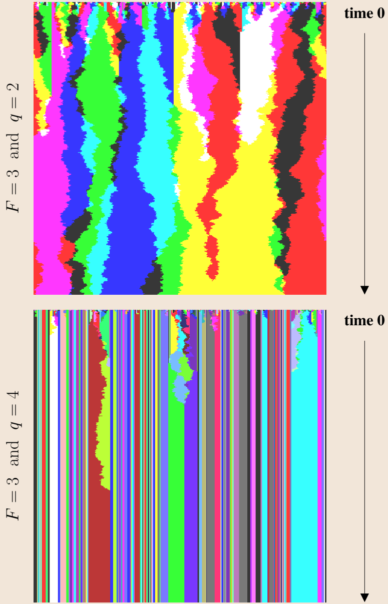


Fluctuation – **For all** $x \in \mathbb{Z}$ **and all** i

$$P\,(\eta_t(x,i) \text{ changes at arbitrarily large } t) \;=\; 1$$



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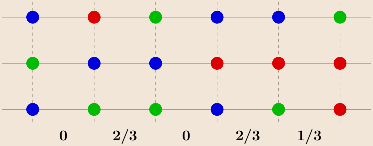

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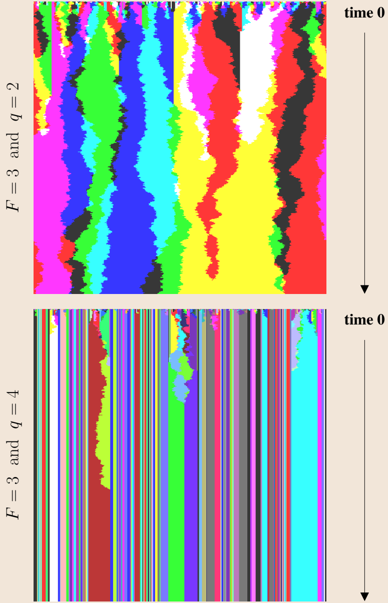

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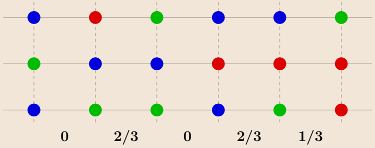

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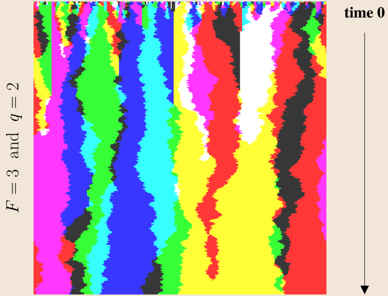
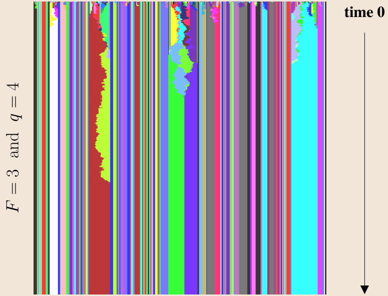
Clustering – **For all** $x, y \in \mathbb{Z}$ **and all** i

$$\lim_{t\to\infty} P\left(\eta_t(x,i) = \eta_t(y,i)\right) \ = \ 1$$

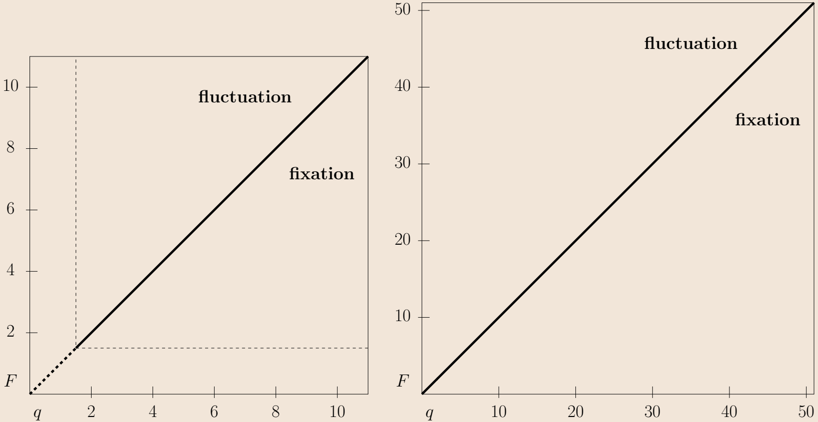




Fluctuation versus fixation

Main conjecture

- **Fluctuation (clustering) when** $F > q$
- **Fixation (no clustering) when** $F < q$



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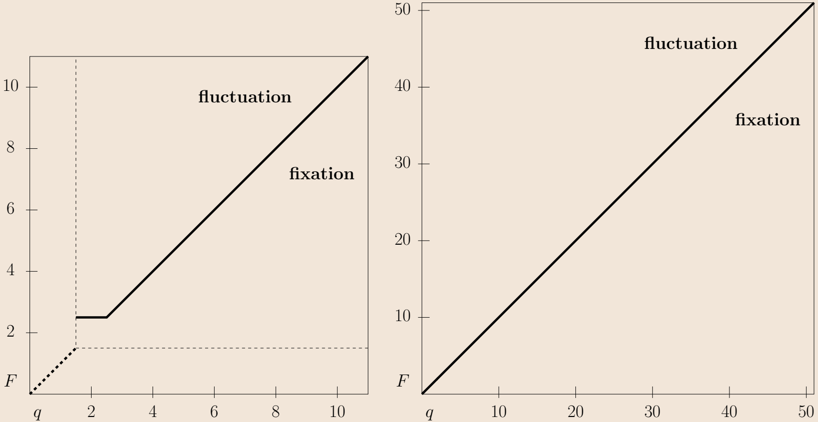

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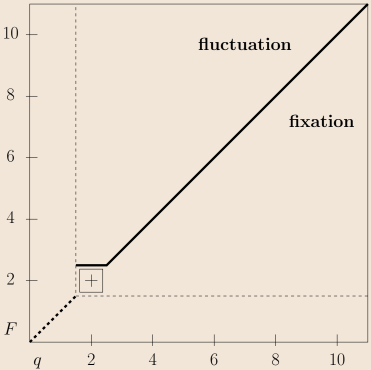

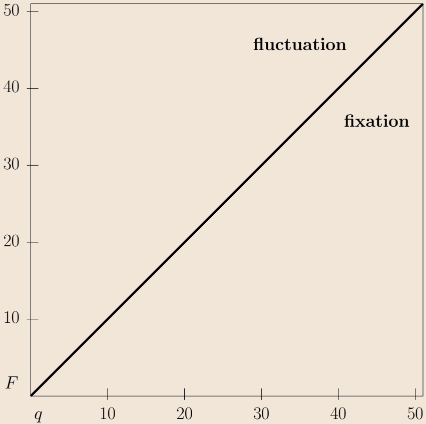

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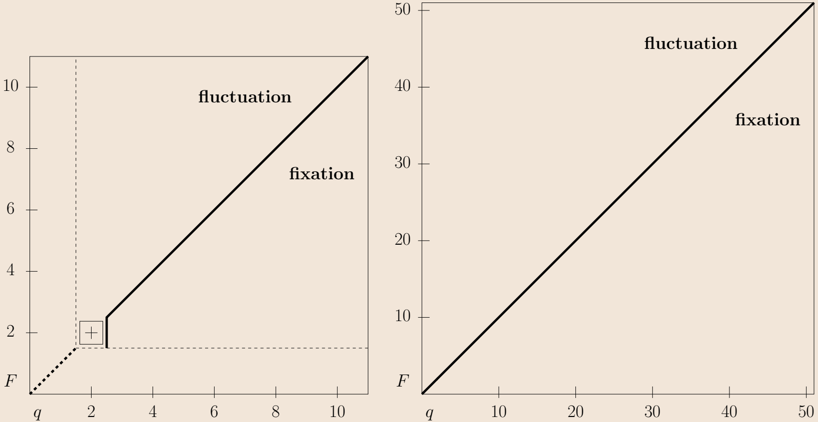

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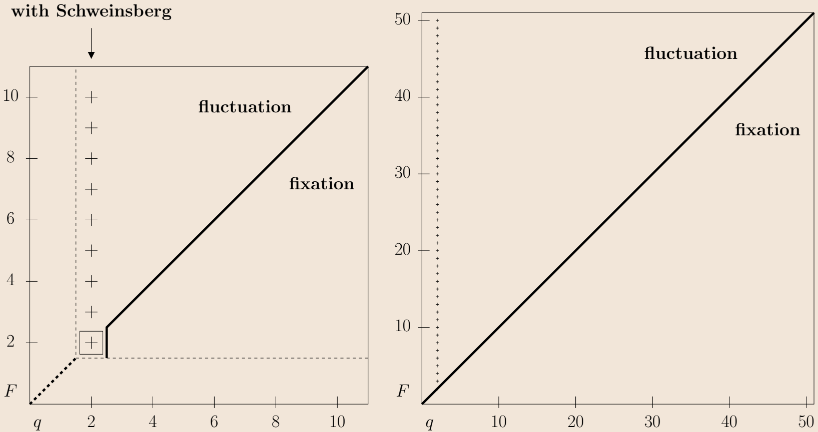

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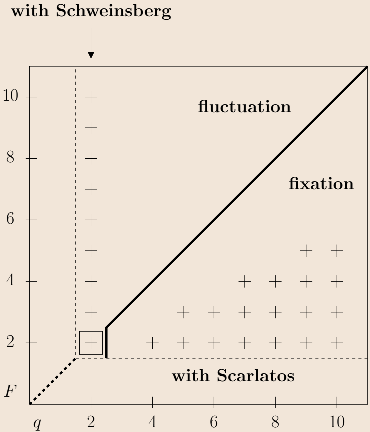

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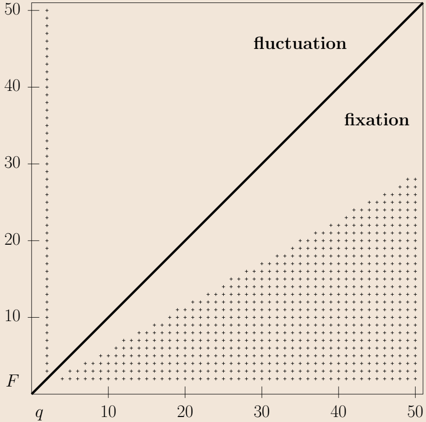

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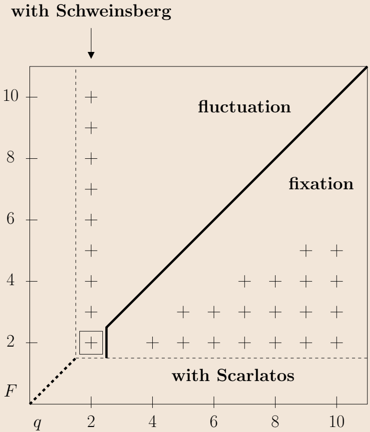

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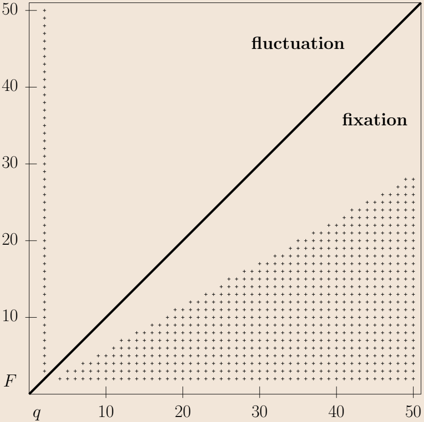

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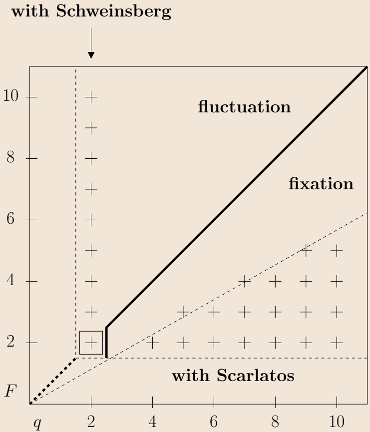

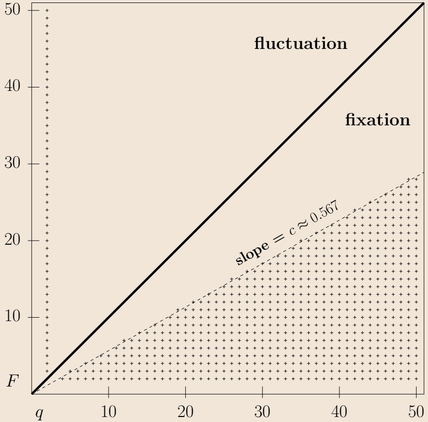

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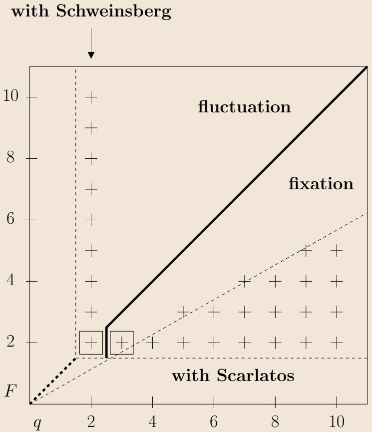
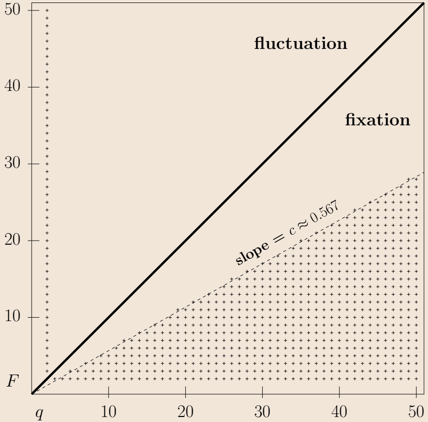
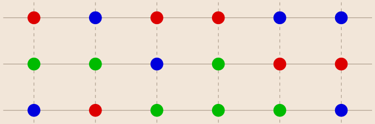


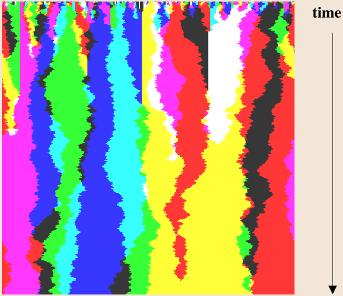


System of random walks



$F = 3$ and $q = 2$

time 0



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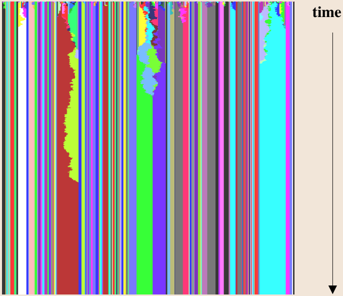

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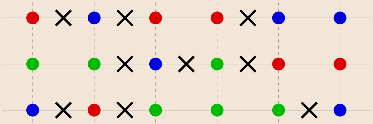

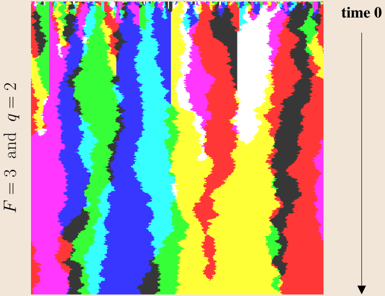

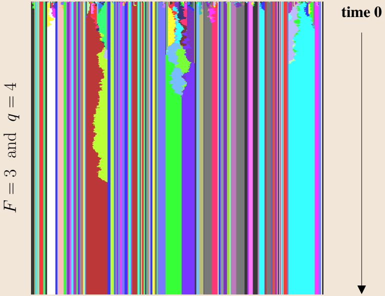

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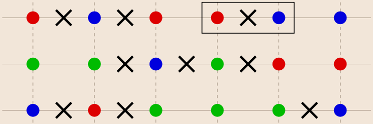

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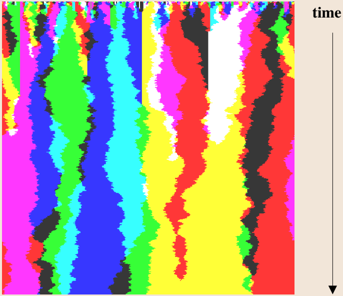

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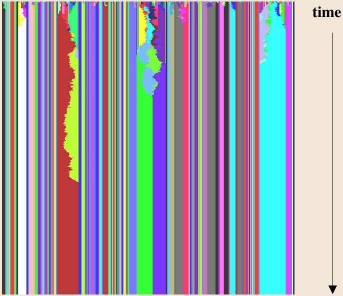

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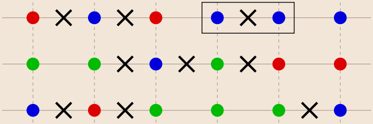

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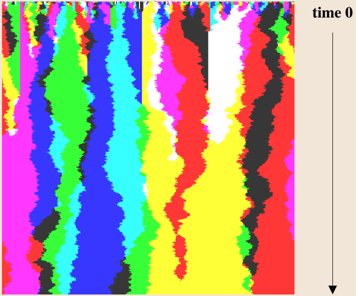

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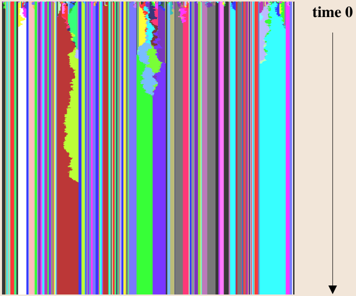

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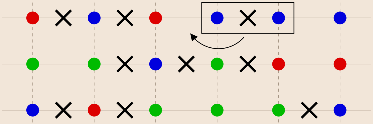

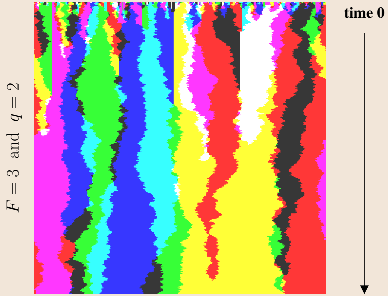

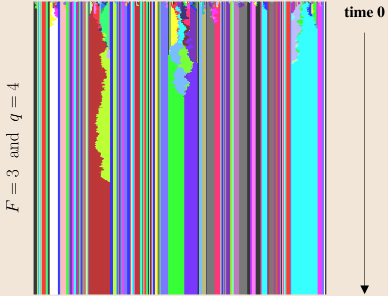

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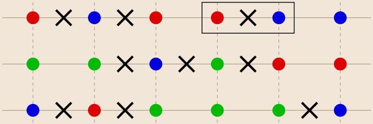

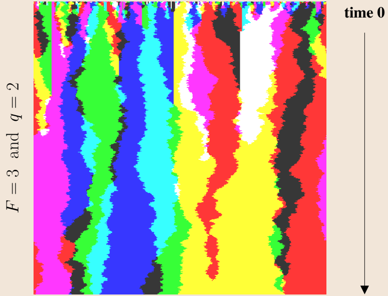

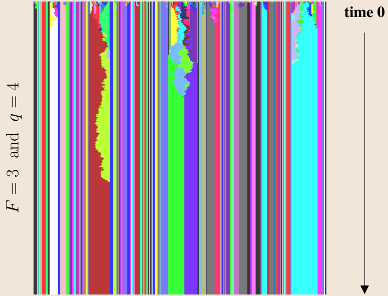

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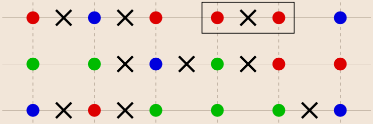

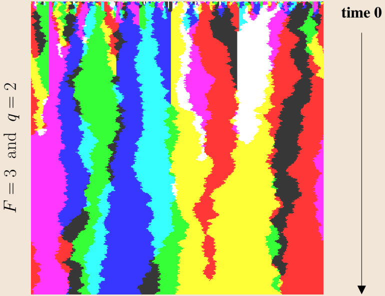

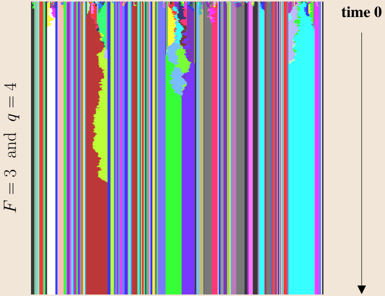

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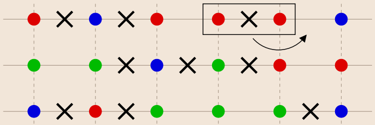

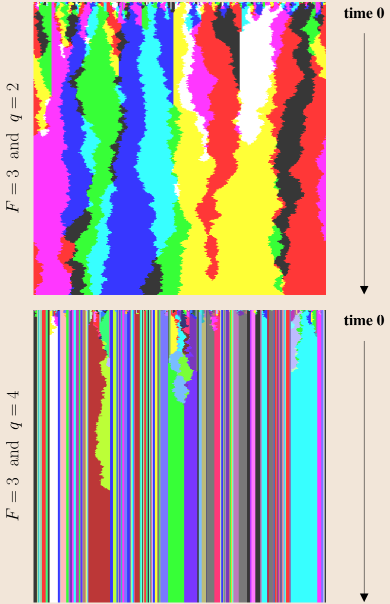

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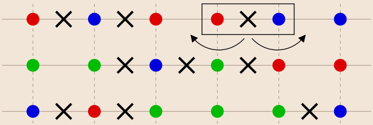

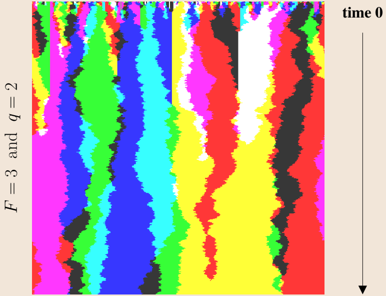

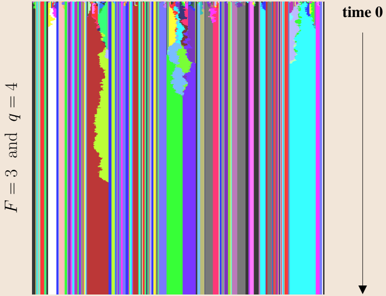

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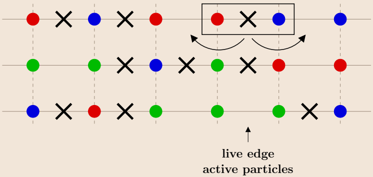

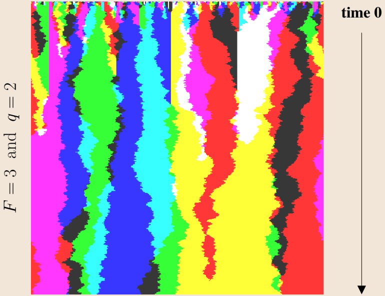

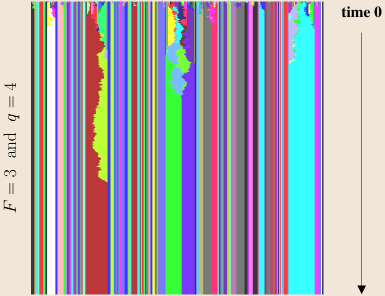

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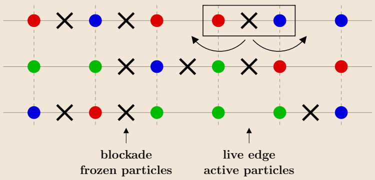

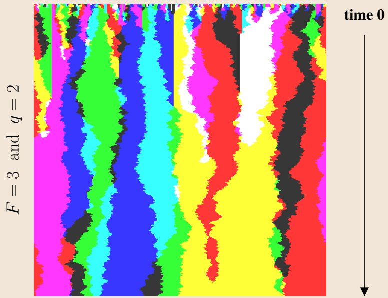

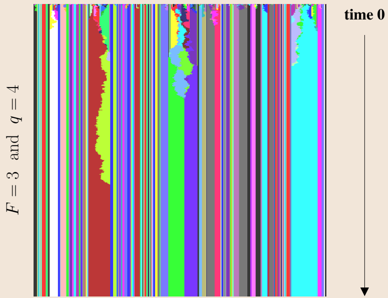

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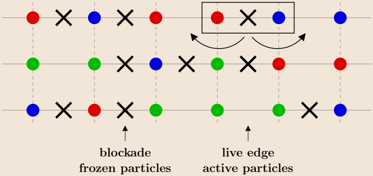
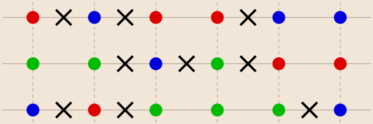
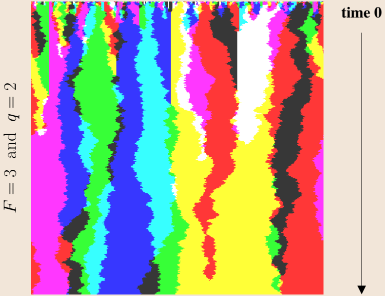
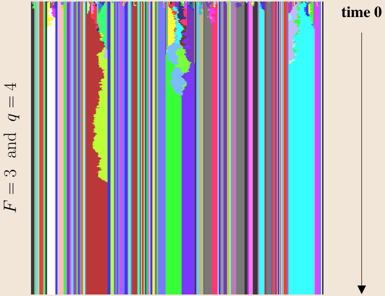


Annihilating events







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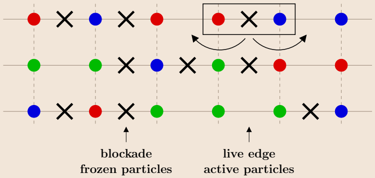

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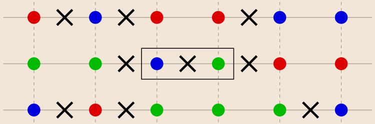

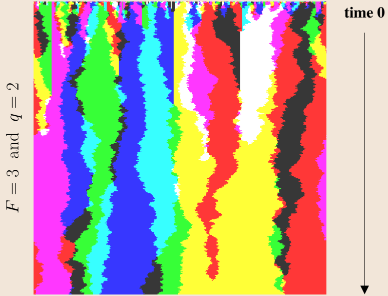

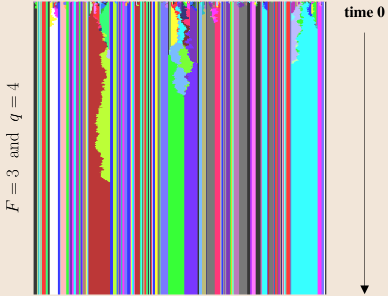

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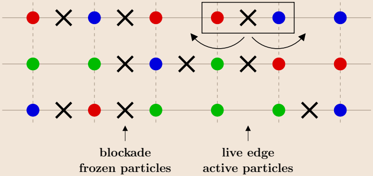

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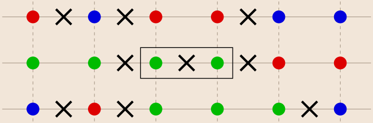
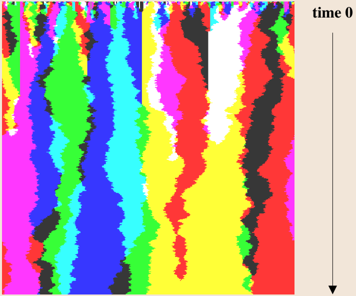


$F = 3$ and $q = 2$



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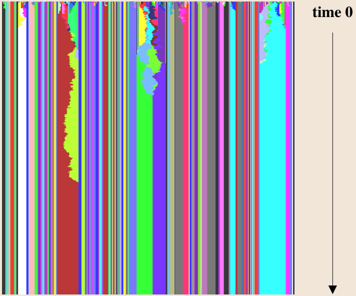

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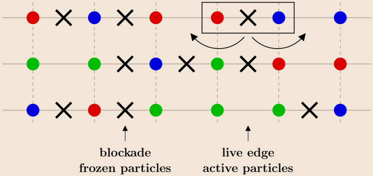

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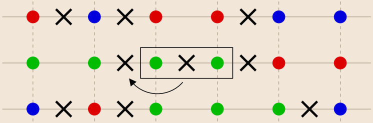
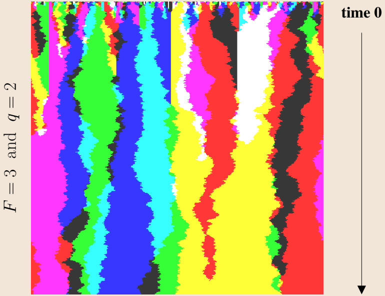
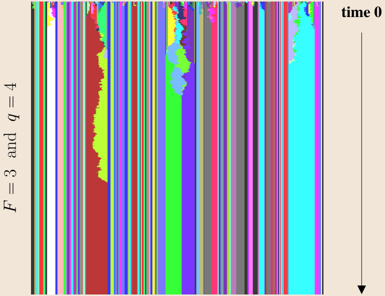
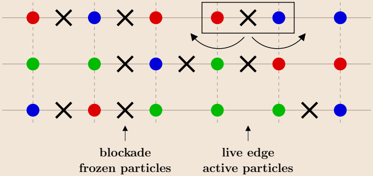






System of random walks



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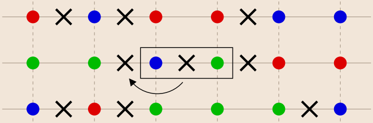
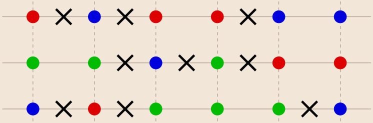
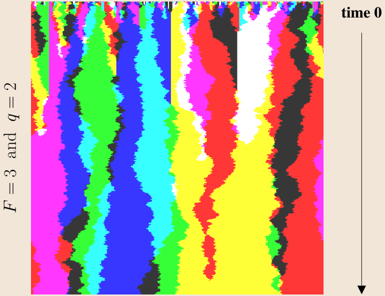
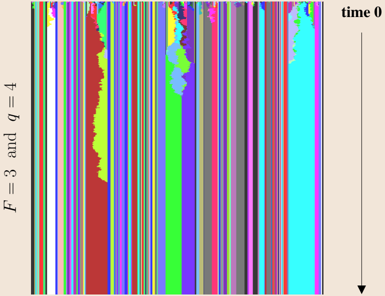


Coalescing events







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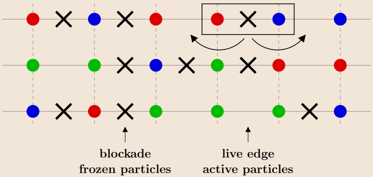

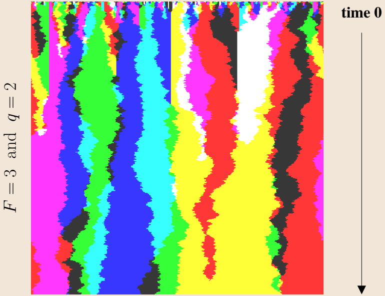

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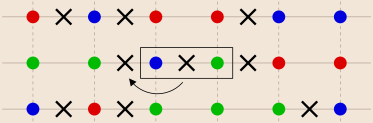

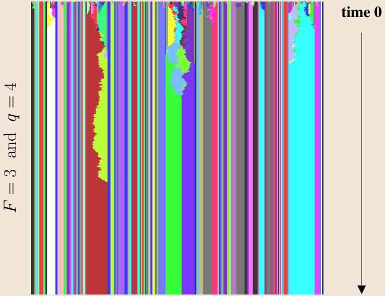

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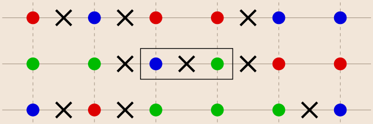

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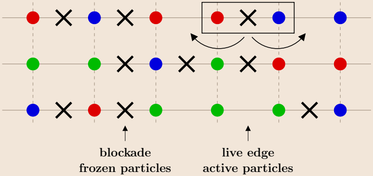

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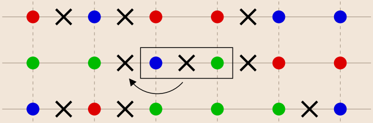

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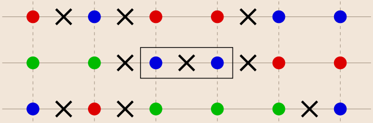

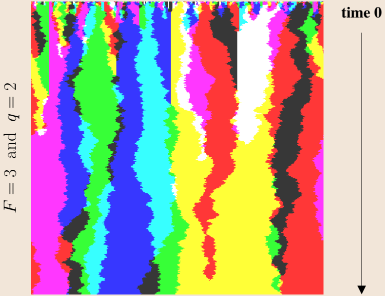

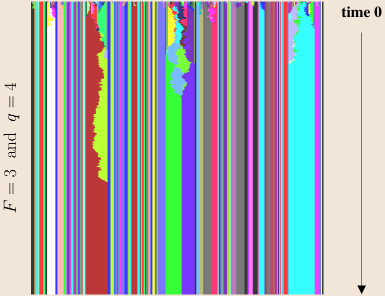

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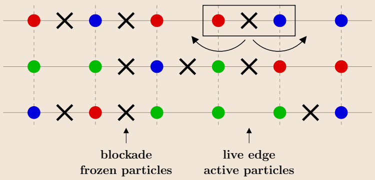

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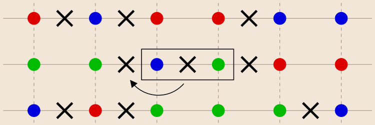

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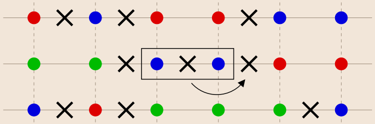

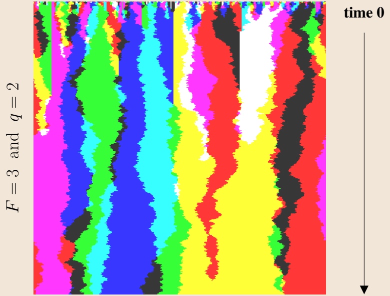

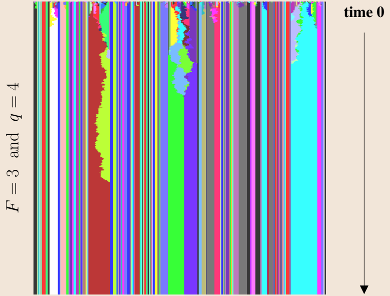

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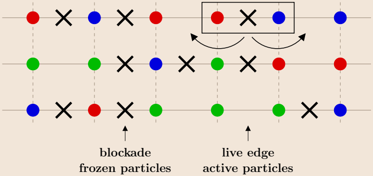

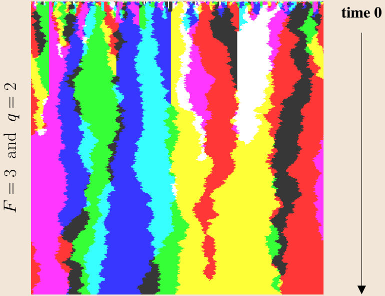

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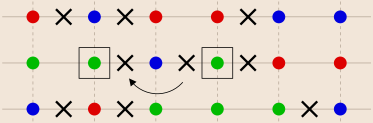

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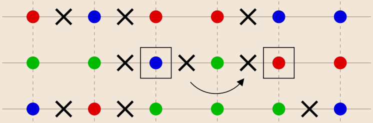

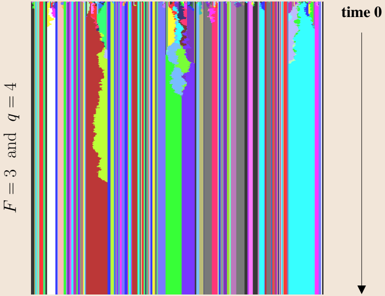

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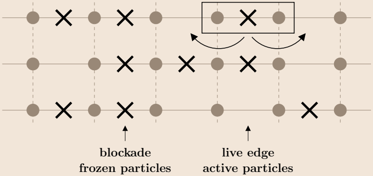

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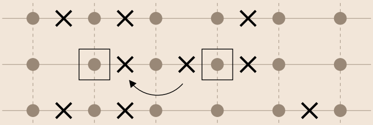

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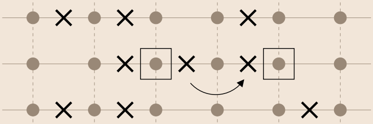

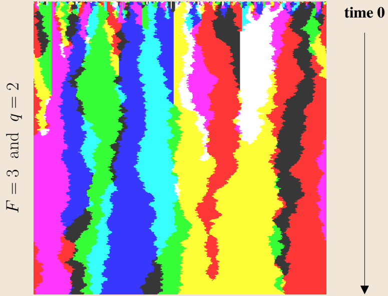

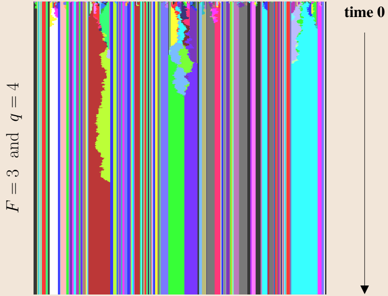

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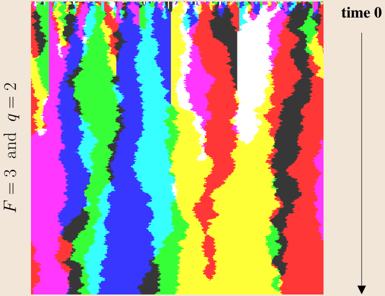

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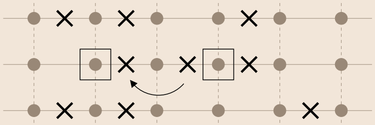

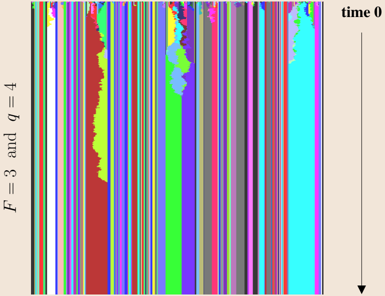

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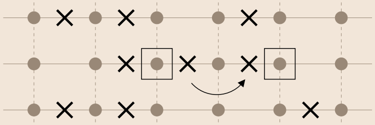

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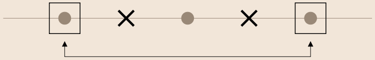

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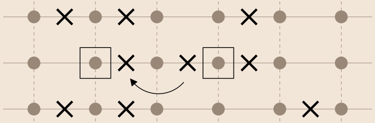

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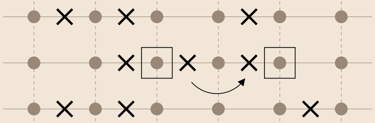
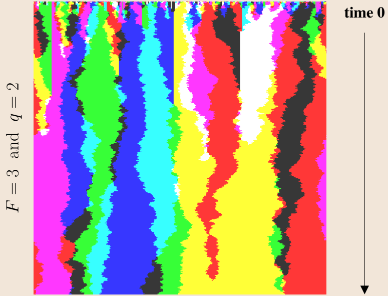
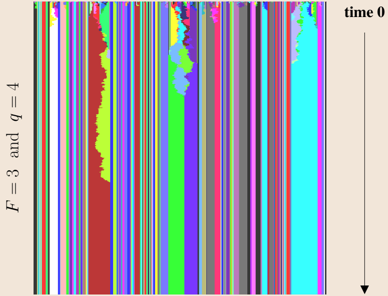
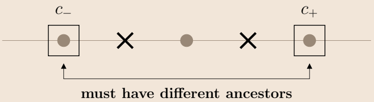




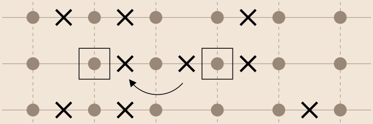


System of random walks

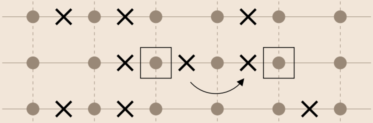


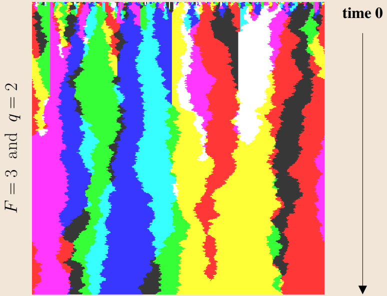
Colors c_- and c_+ are independent uniform random variables on the set $\{1, 2, \dots, q\}$ . . .

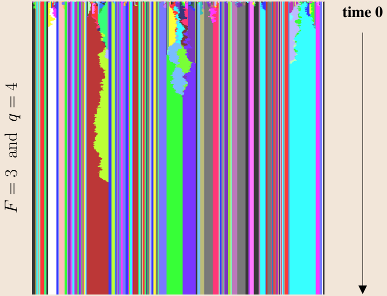
Annihilating events



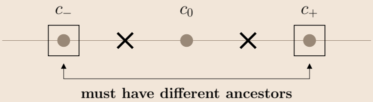
Coalescing events





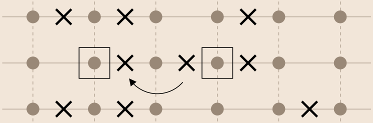


System of random walks

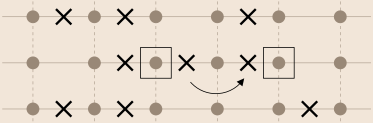


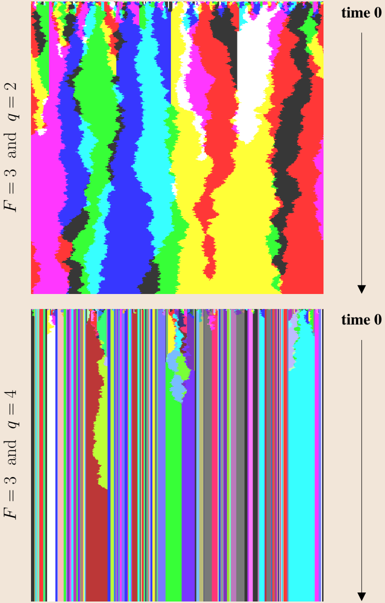
Colors c_- and c_+ are independent uniform random variables on the set $\{1, 2, \ldots, q\}$ ... conditioned to be different from c_0

Annihilating events

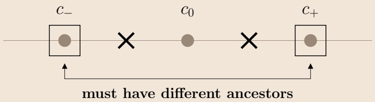


Coalescing events



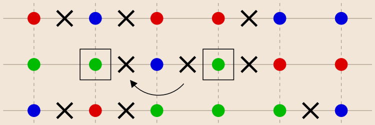


System of random walks

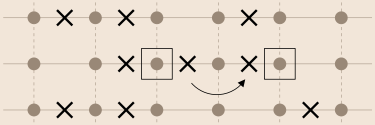


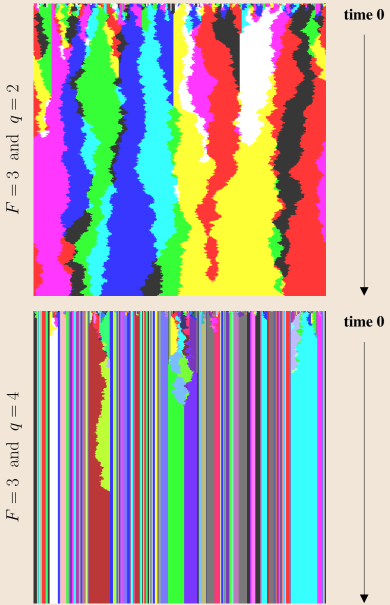
Colors c_- and c_+ are independent uniform random variables on the set $\{1, 2, \dots, q\}$ ... conditioned to be different from c_0

Annihilating events: probability $(q-1)^{-1}$

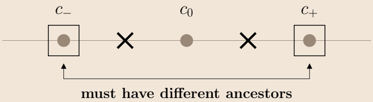


Coalescing events





System of random walks



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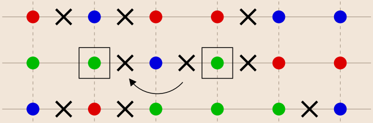
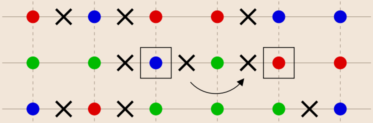
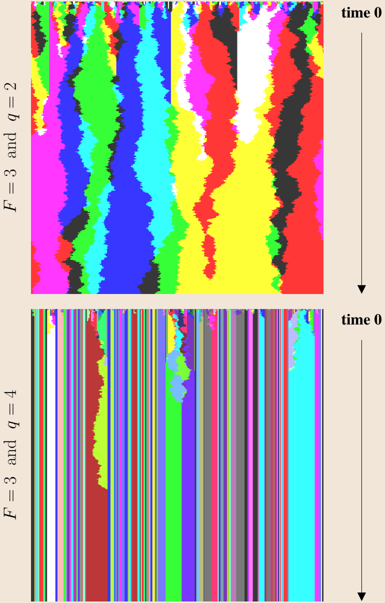
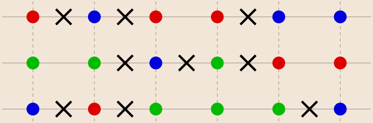


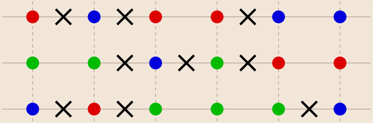
Coalescing events: probability $(q-2)\,(q-1)^{-1}$





Fixation when $F \leq cq$ – survival of the blockades





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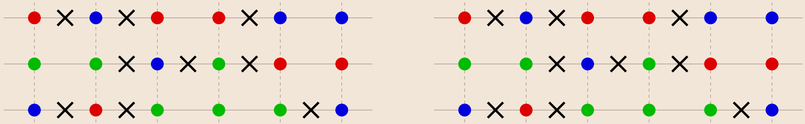

How many blockades required to absorb all the active particles?

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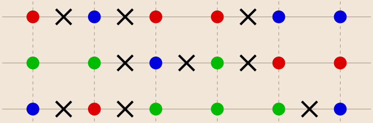

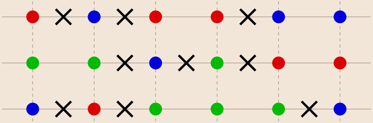

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Each active particle is assigned a weight of -1

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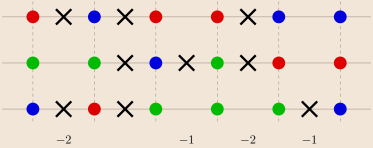

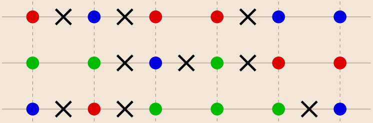

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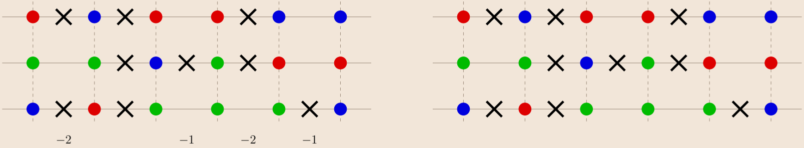

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Number of collisions to break a blockade

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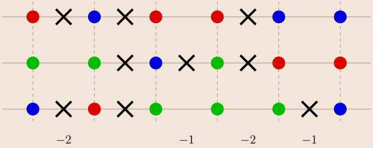

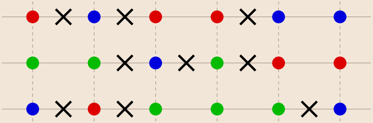

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Number of collisions to break a blockade: $Z =$ **Geometric with mean** $q - 1$

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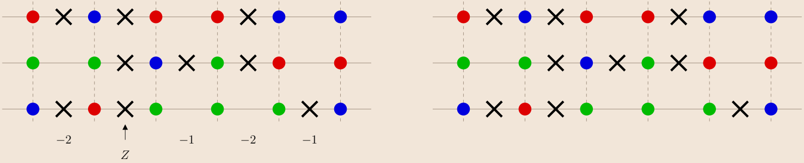

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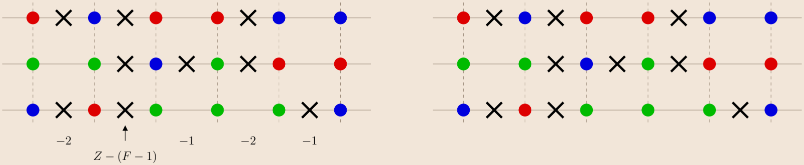

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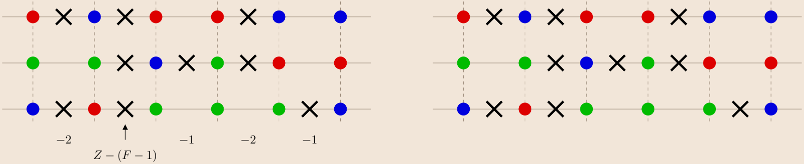

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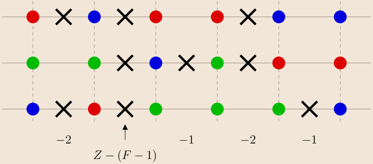

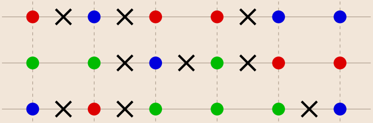

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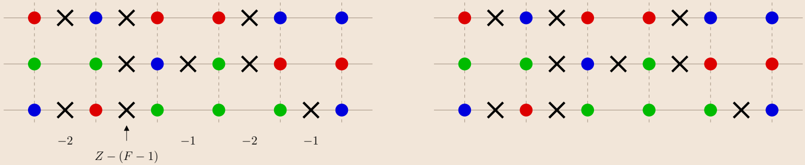

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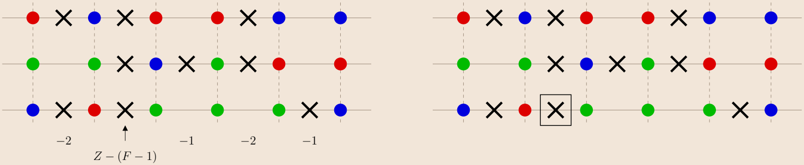

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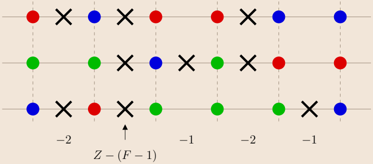

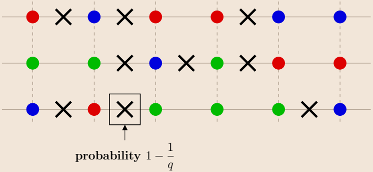

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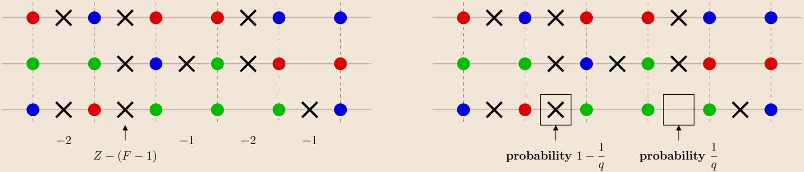

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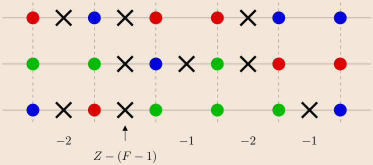

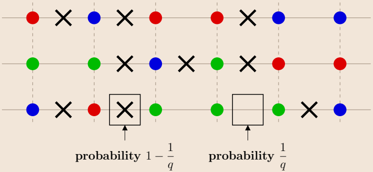

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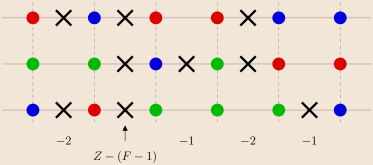

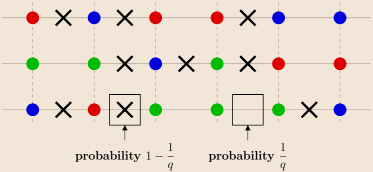

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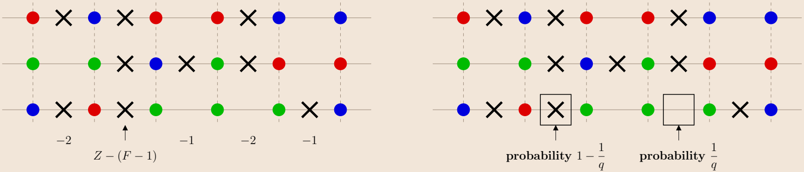

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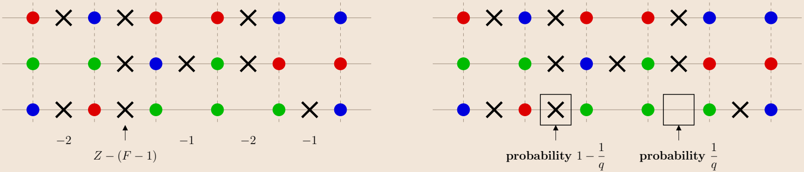

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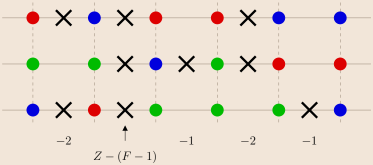

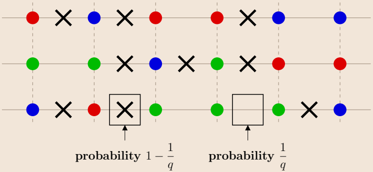

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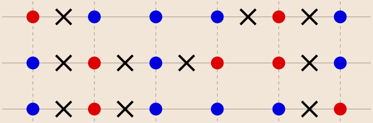
Flux when $q = 2$ – extinction of the blockades



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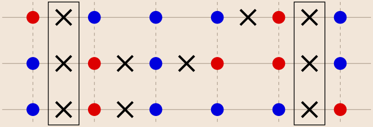

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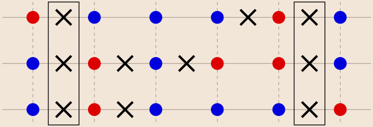

When $q = 2$ only annihilating events so the parity is preserved

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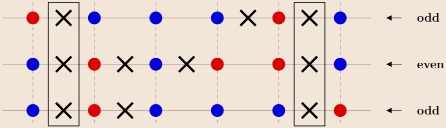

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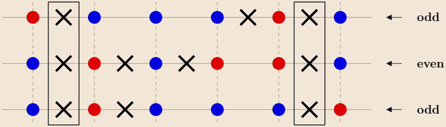
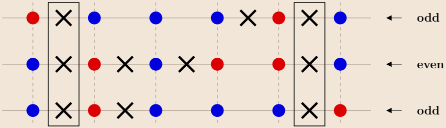


When $q = 2$ only annihilating events so the parity is preserved

Different parities imply that one of the two blockades is destroyed eventually

Flux when $q = 2$ – extinction of the blockades



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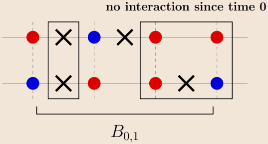

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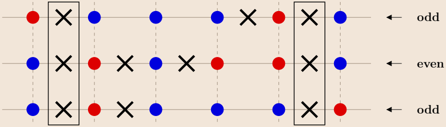

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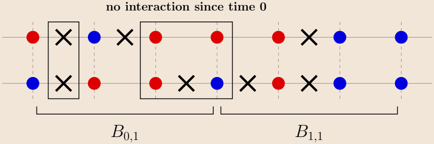

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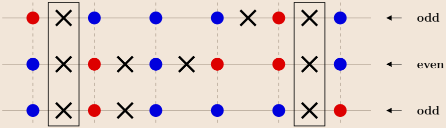

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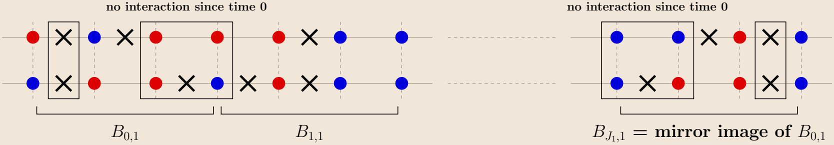

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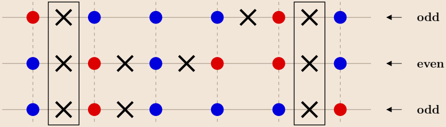

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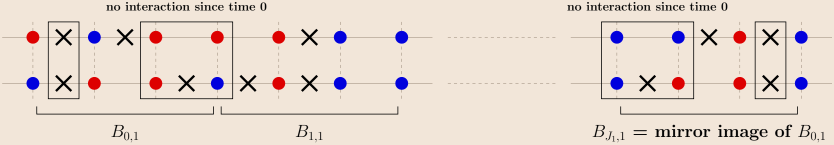
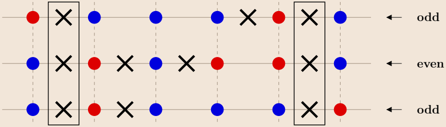


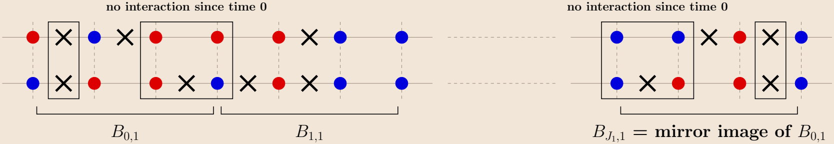
If different parities, left or right blockade destroyed eventually

Flux when $q = 2$ – extinction of the blockades



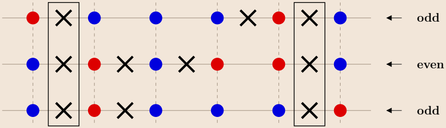
When $q = 2$ only annihilating events so the parity is preserved

Different parities imply that one of the two blockades is destroyed eventually



If different parities, left or right blockade destroyed eventually
By symmetry, left blockade destroyed before right blockade with probability $1/2$

Flux when $q = 2$ – extinction of the blockades



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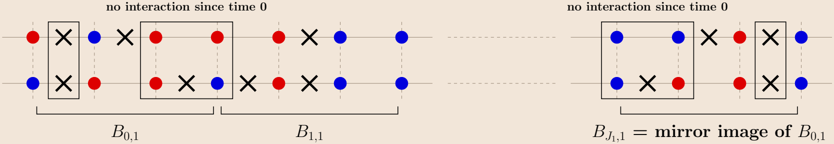

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If same parity or right blockade destroyed first then . . .

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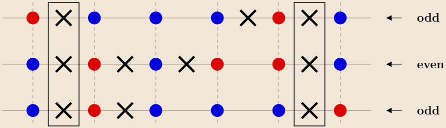

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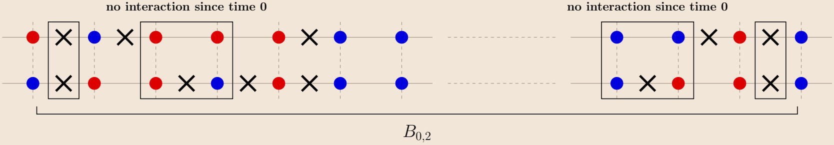

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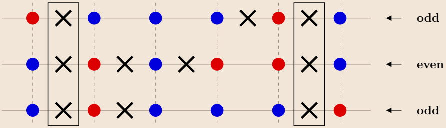

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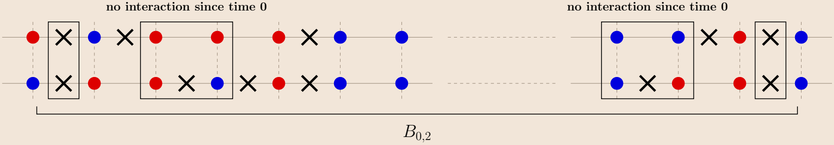

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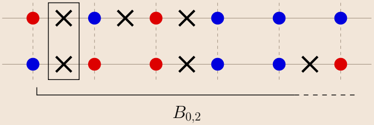

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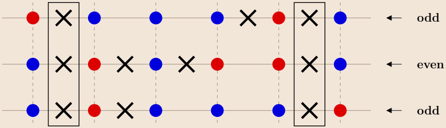

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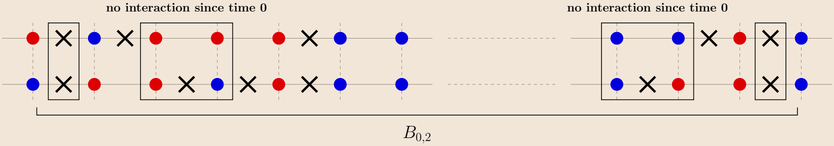

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