Rainer Hegselmann,
University of Bayreuth

Radical Groups, Dogmatists, and Charismatic Leaders

A Simple Unifying Modeling Approach

Work in progress !

Workshop *Modeling and Control in Social Dynamics*
Rutgers University, October 6 - 10, 2014

Structure of the talk

1. The bounded confidence model: Idea and short analysis
2. Radical groups, radicalisation, charismatic leaders: A simple extension of the *BC* model
3. Without confidence dynamics: Getting an overview
4. With confidence dynamics: Getting an overview
5. Next steps and the broader perspective

§1
The bounded confidence model:
Idea and short analysis

– joint work with Ulrich Krause –

Article in JASSS

Rainer Hegselmann and Ulrich Krause (2002)

Opinion dynamics and bounded confidence: models, analysis and simulation

Journal of Artificial Societies and Social Simulation vol. 5, no. 3
<http://jasss.soc.surrey.ac.uk/5/3/2.html>

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Abstract

When does opinion formation within an interacting group lead to consensus, polarization or fragmentation? The article investigates various models for the dynamics of continuous opinions by analytical methods as well as by computer simulations. Section 2 develops within a unified framework the classical model of consensus formation, the variant of this model due to Friedkin and Johnsen, a time-dependent version and a nonlinear version with bounded confidence of the agents. Section 3 presents for all these models major analytical results. Section 4 gives an extensive exploration of the nonlinear model with bounded confidence by a series of computer simulations. An appendix supplies needed mathematical definitions, tools, and theorems.

Keywords:
Bounded Confidence; Consensus/dissent; Nonlinear Dynamical; Opinion Dynamics

Because of the complex mathematical notation in this article, it is only available in Portable Document Format (PDF) format. To read the article you will need a copy of the Adobe Acrobat reader, available free.

The article is here

For the start: Let‘s suppose ...

- a group of people, for instance a *group of experts* on something;
- each expert has an *opinion* on the topic under discussion, for instance the probability of a certain type of accident;
- *nobody is totally sure* that he is totally right;
- to some degree everybody is *willing to revise* his opinion when informed about the opinions of others, especially the opinions of '*competent*' *others*;
- the revisions produce a new opinion distribution which may lead to further revisions of opinions, and so on and so on.... .

De Vergadering (The meeting), Willy Belinfante

Basics of the bounded confidence model

Each individual takes seriously only those others whose opinions are ‚*reasonable*‘, ‚*not too strange*‘, i.e. not too far away from one's own opinion.

- There is a set of *n individuals; i, j* $\in$ *I.*
- Time is *discrete;* $t = 0, 1, 2, \ldots$.
- Each individual starts with a certain *opinion*, given by a *real number*; $x_i(t_0) \in [0,1]$.
- The *profile* of all opinions at time t is

$$X(t) = x_1(t), \ldots, x_i(t), x_j(t), \ldots, x_n(t).$$

- Each individual *i* takes into account only '*competent*' others. Competent are those individuals whose opinions are not too far away, i.e. for which $|x_i(t) - x_j(t)| \leq \varepsilon$ (*confidence interval*). The *set* of all others that *i* takes into account at time *t* is:

$$I(i, X(t)) = \{ j \mid |x_i(t) - x_j(t)| \leq \varepsilon \}.$$

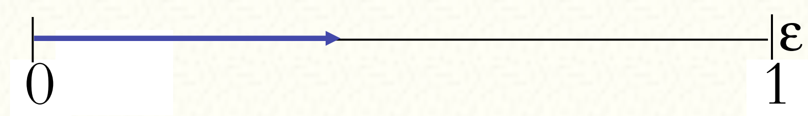
- The individuals *update* their opinions. The next period's opinion of individual *i* is the *average* opinion of all those which *i* takes seriously:

$$x_i(t+1) = \frac{1}{\#\left(I\left(i, X(t)\right)\right)} \sum_{j \in I\left(i, X(t)\right)} x_j(t)$$

How to analyse the model?

Research Questions:

- Does such a dynamics stabilize?
- Are there typical final results?
- When is consensus feasible?



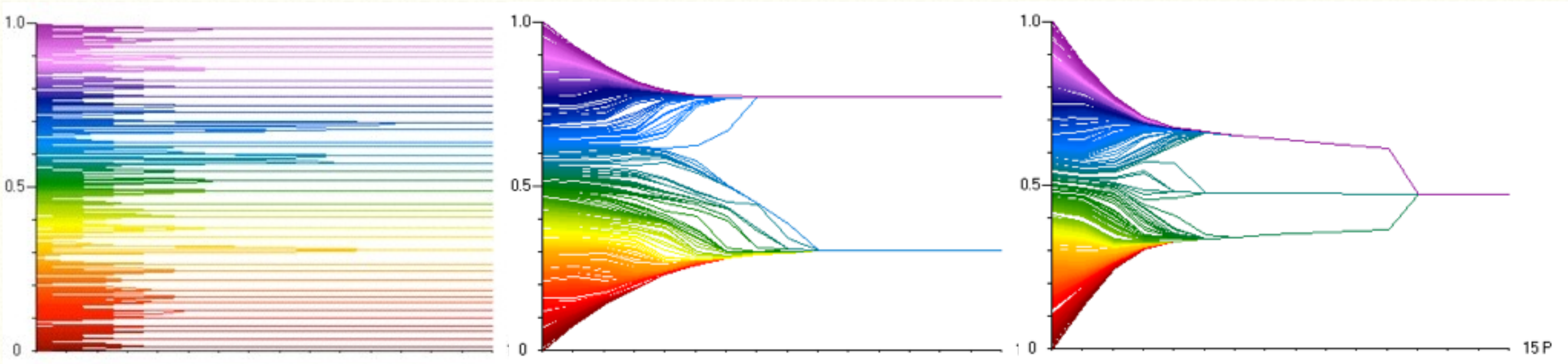
Confidence intervals: [0,1] as parameter space.

Heuristics:
‚Walking' from 0 direction 1

KISS-principle: "Keep it simple, stupid!"

- Confidence intervals: *symmetric, homogeneous,* and *constant over time.*
- Start distributions:
  random uniform distribution: $x_i(t_0) \in [0,1]$
- Updating: *simultaneous*

Effects of different confidence intervals



In principle: *phase transitions with an increasing confidence interval*

1. *Plurality*
2. *Polarization*
3. *Consensus*

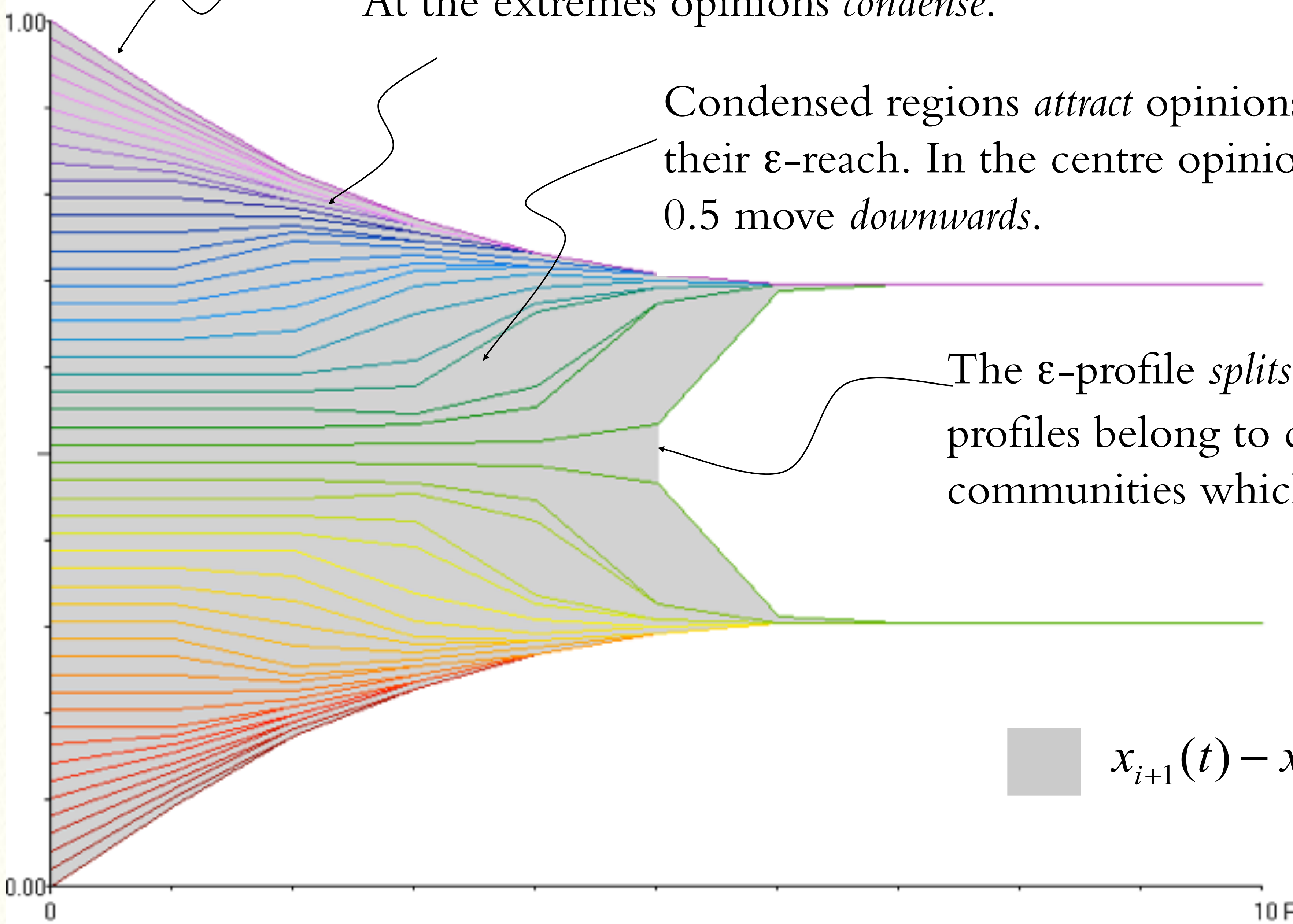
Understanding fragmentation: The ε–split

Extreme opinions are under a *one sided influence* and move direction centre. The range of the profile *shrinks*.

At the extremes opinions *condense*.

Condensed regions *attract* opinions from less populated areas within their ε–reach. In the centre opinions > 0.5 move *upwards*, opinions < 0.5 move *downwards*.

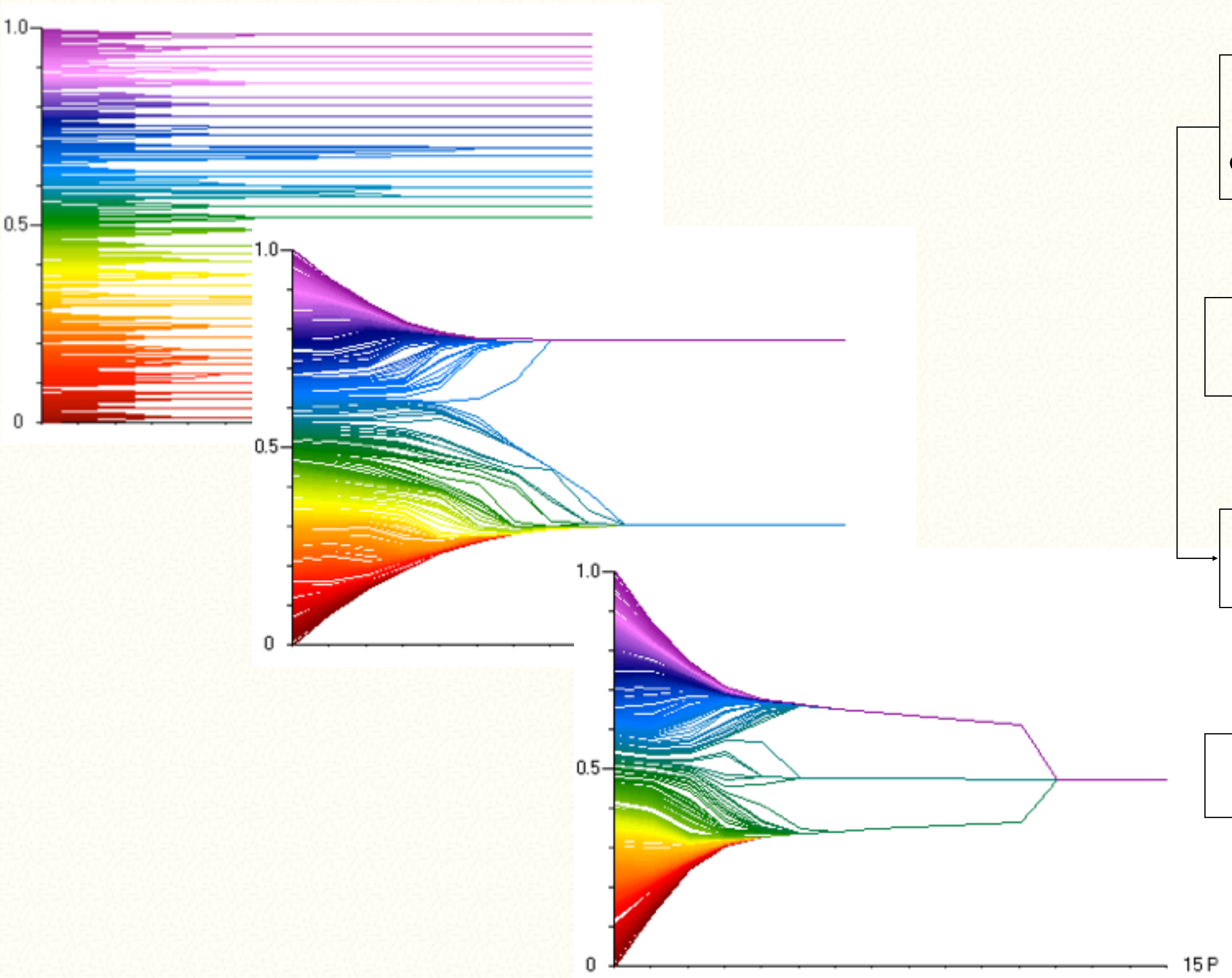
The ε-profile *splits* in t_6. From now on the split sub-profiles belong to different 'opinion worlds' or communities which *do no longer interact*.

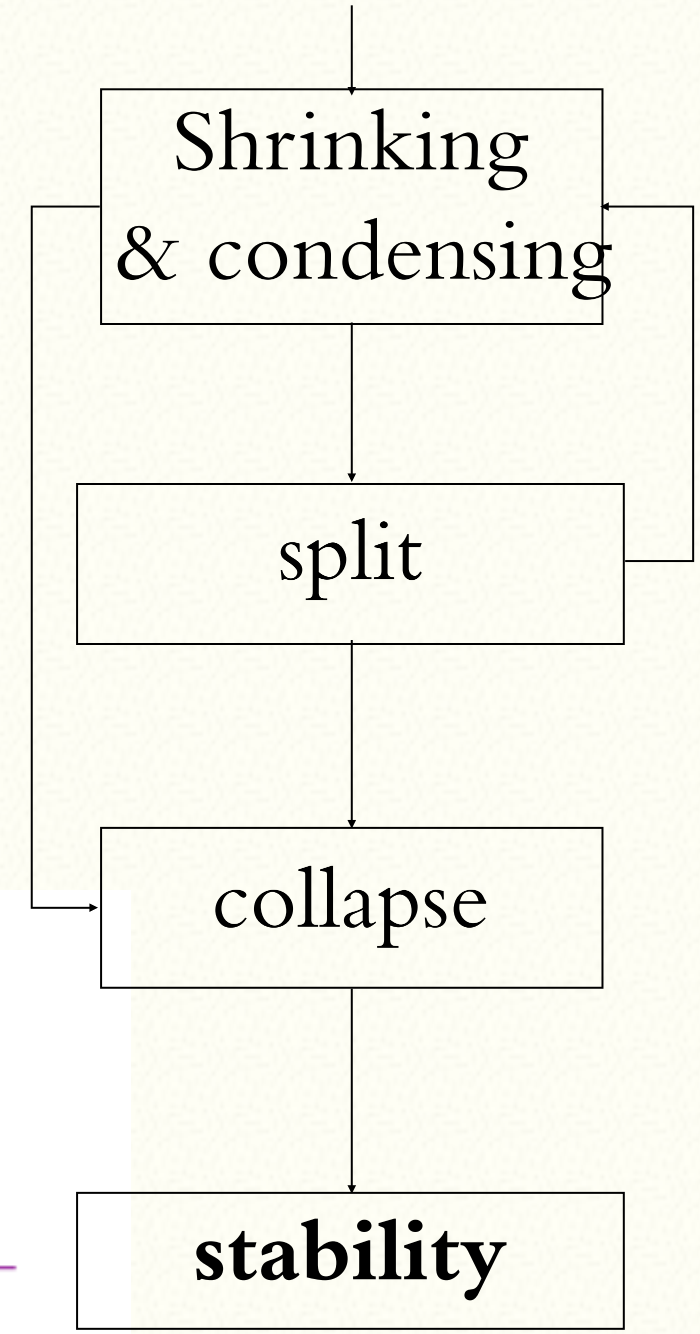


$x_{i+1}(t) - x_i(t) \le \varepsilon$

Dynamics with 50 opinions, simultaneous updating, regular start profile, ε = 0.2.

Understanding fragmentation: summary





§2

Radical groups, radicalisation, charismatic leaders, dogmatists: A simple extension of the BC-model

Some starting points (‚stylized facts‘)

A radical group

- has – compared to ‚normal‘ agents – a comparatively stable in-group consensus on an extreme opinion. No other opinion is taken seriously.

A group of dogmatists

- is like a radical group, but with an in-group consensus which is not necessarily an extreme opinion.

A charismatic leader

- counts for ‚normal‘ agents that are under his/her influence much more than other ‚normal‘ agents.

In a process of radicalisation or dogmatisation

- ‚normal agents‘ tend to get less and less open-minded.

Formal description by heroic abstractions

The set of agents is partitioned into two sets:
a set of radicals (with $\#_{radicals}$ elements) and a set of normal agents (with $\#_{normals}$ elements).

radicals

$x_i(t_0)$ for all radical agents i is an extreme opinion R, e.g. 0.9

The confidence interval ε of all radical agents is 0.

normals

The opinions of normals are distributed over the whole opinion space.

The confidence interval ε of normals is strictly greater than 0.

set of agents within ε:

$$I(i, X(t)) = \left\{ j \,\middle\|\, x_i(t) - x_j(t) \right| \leq \varepsilon \}$$

Only radical opinions count

$$x_i^{radical}(t+1) = x_i^{radical}(t) = R$$

All opinions within ε count, whether radical or not.

$$x_i^{normal}(t+1) = \frac{1}{\#\left(I\left(i, X(t)\right)\right)} \sum_{j \in I(i, X(t))} x_j(t)$$

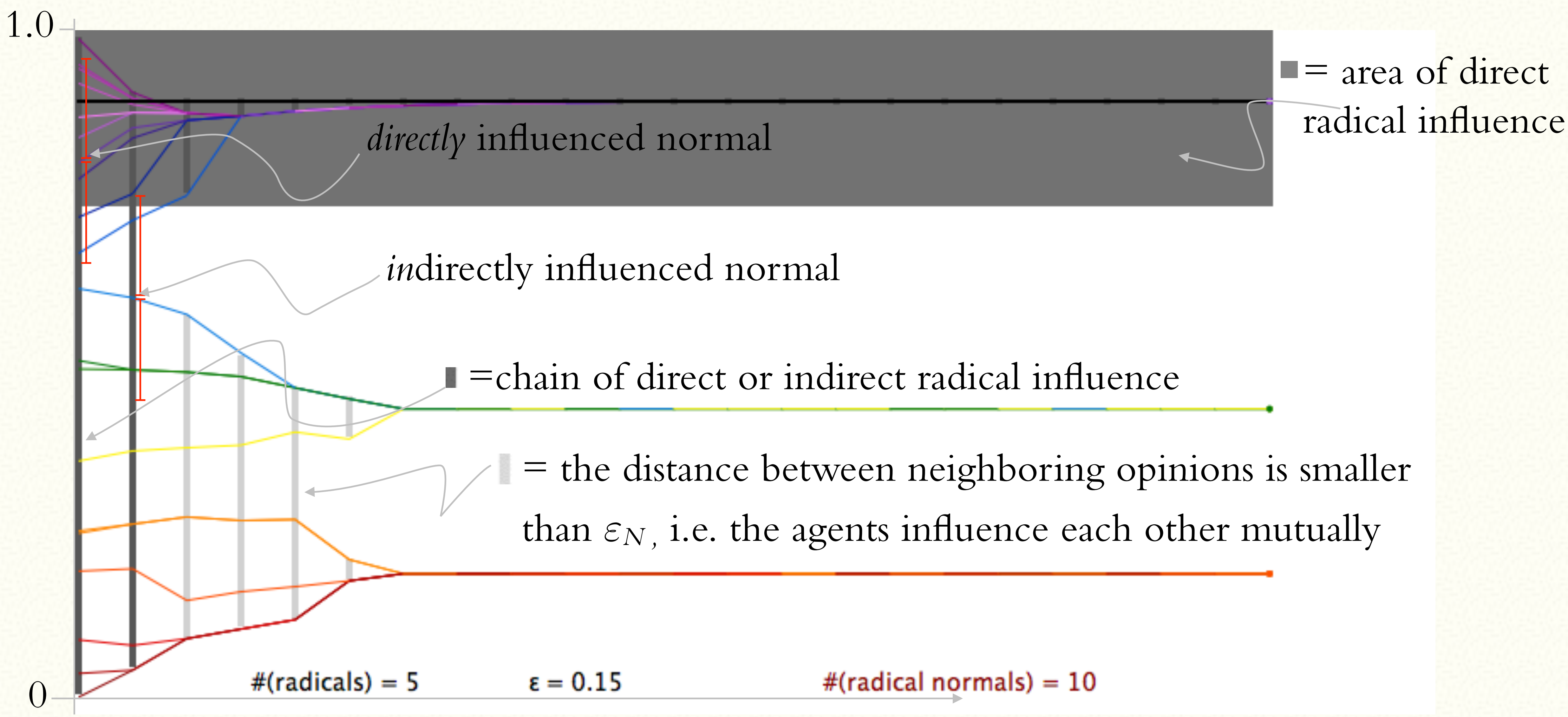
Direct and indirect radical influence
Some visualization

colored trajectories:

Normals (#$_N$= *20,* $\varepsilon_N = 0.15$)

black trajectory:

Radicals (#$_R$ = 5, $\varepsilon_R = 0$), $R = 0.9$



NOTE: The chain of direct or indirect radical influence is drawn *second*. Consequence: It overdraws mutual influence of normals, which is drawn *first*!

Our starting points and their formal description

A radical group

- has – compared to ‚normal' agents – a comparatively stable in-group consensus on an extreme opinion. No other opinion is taken seriously.

✓

A group of dogmatists

- is like a radical group, but with an in-group consensus which is not necessarily an extreme opinion.

✓

A charismatic leader

- counts for ‚normal' agents that are under his/her influence much more than other ‚normal' agents.

We take the group of $\#_R$ radicals as *one* charismatic leader that counts $\#_R$-times more than a normal agent. $\#_R$ is a sort of ‚degree of charismaticity'.

In a process of radicalisation or dogmatisation

- ‚normal agents' tend to get less and less open-minded.

?

... less and less open-minded.

Idea:

Normal agents do not only average over the opinions of others that are within their confidence interval. *They average as well over the confidence intervals of all others that are within their confidence interval.*

Consequence: Normals become affected by the 0-confidence interval of radicals, charismatic leaders, or dogmatists.

more formally: set of agents *j* that are in agent's *i time dependent* confidence interval ε_i

$$I\left(i, X(t), \varepsilon_i(t)\right) = \left\{ j \,\middle\|\, x_i(t) - x_j(t) \right| \le \varepsilon_i(t) \right\}$$

$$x_i^{normal}(t+1) = \frac{1}{\#\left(I\left(i, X(t), \varepsilon_i(t)\right)\right)} \sum_{j \in I\left(i, X(t), \varepsilon_i(t)\right)} x_j(t)$$

opinion dynamics

$$\varepsilon_i^{normal}(t+1) = \frac{1}{\#\left(I\left(i, X(t), \varepsilon_i(t)\right)\right)} \sum_{j \in I\left(i, X(t), \varepsilon_i(t)\right)} \varepsilon_j(t)$$

confidence dynamics (CD)

§ 3

Without confidence dynamics: Getting an overview

How to get an overview? The idea

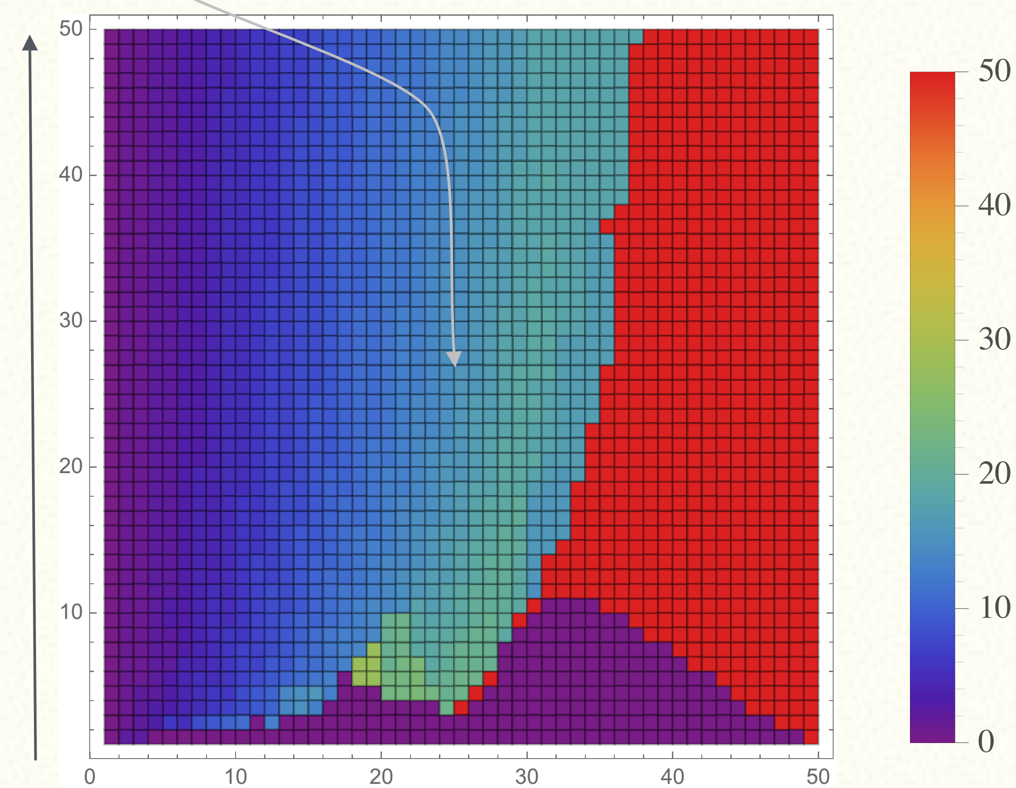
Questions:

- How many normals end up at the radicals positions *R*?
- How far into the opinion profile can radicals successfully penetrate?
- Mean and median of the stabilized opinions of normals (compared to a situation with no radicals at all)?
- Typical dynamical patterns of radical influence?
- How are the stabilized opinions clustered (consensus, polarization, fragmentation)

simulation runs for each $< \varepsilon, \#_{radicals} >$ value combination until the dynamics is almost stable.

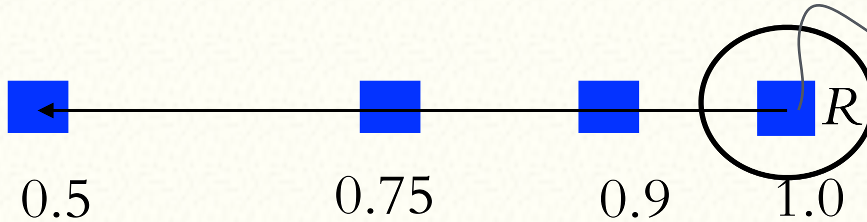
Colors: number of normals that end up at the radicals' position R.

stepwise increase of the number of radicals ($\#_{radicals}$)



ε

stepwise increase of the confidence interval (step size 0.01).



stepwise *decrease* of the *radical Position R*

(*step size 0.01*)

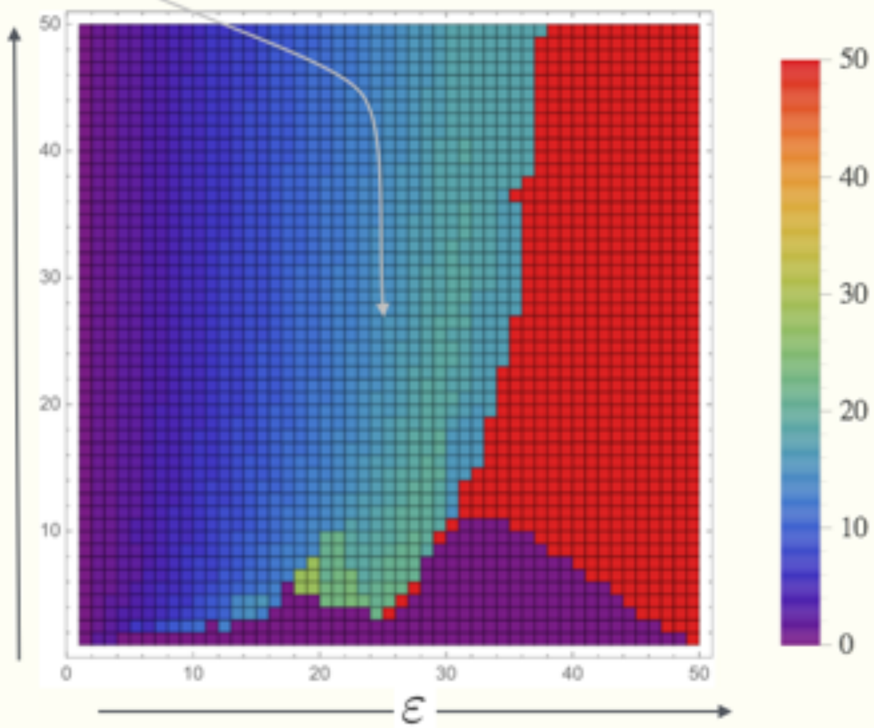
normal agents: $\#_{normals} = 50$

How to get an overview? The idea

simlation runs for each < ε, $\#_{radicals}$> value combination until the dynamics is almost stable.

Colors: number of normals that end up at the radicals' position R.

stepwise increase of the number of radicals ($\#_{radicals}$)



stepwise increase of the confidence interval (step size 0.01).

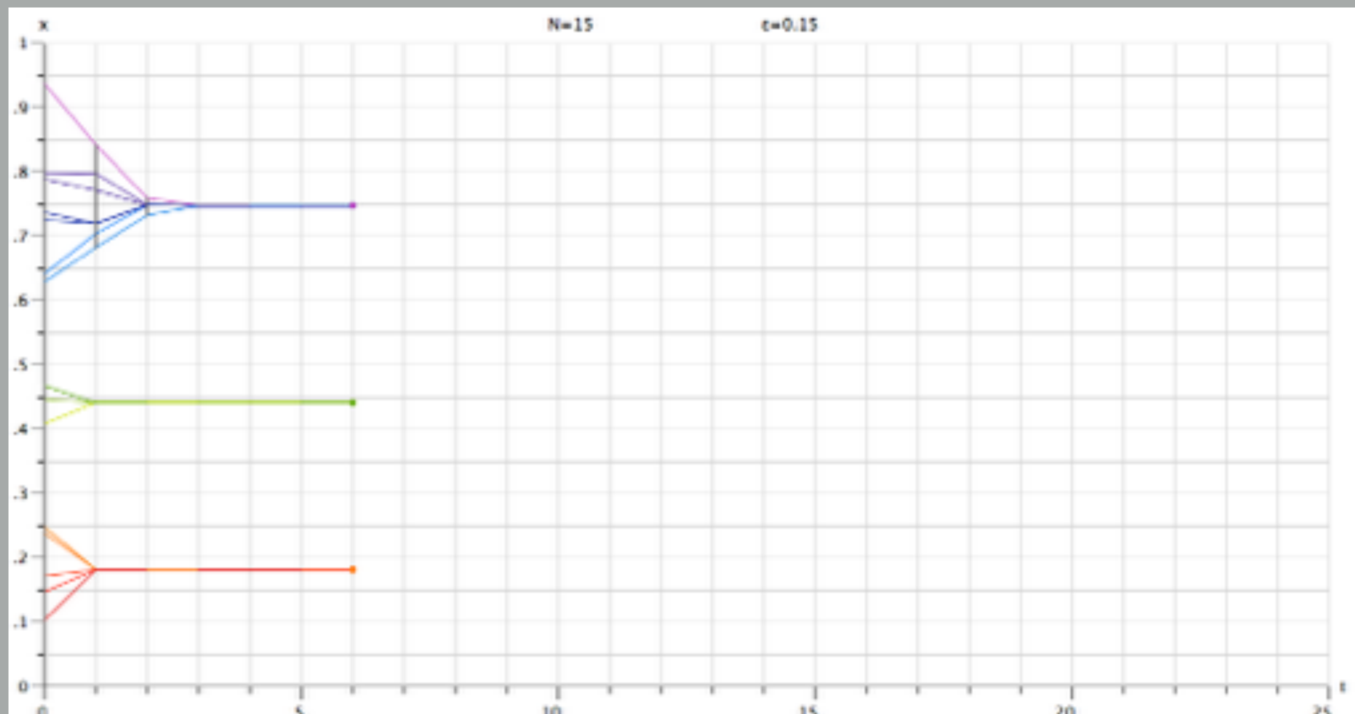
Definitions:

A simulation run < ε, $\#_{radicals}$ > is ***considered stabilized*** at time t iff it holds: For all i ($|x_i(t+1) - x_i(t)| \leq 10^{-5}$)

A normal i with an opinion x_i ***ends up at the radical position*** R *iff* after stabilization at time t it holds $|x_i(t) - R| \leq 10^{-3}$.

Even random and expected value distribution

Even random distribution



Confession & warning:

I always use that type of distribution!

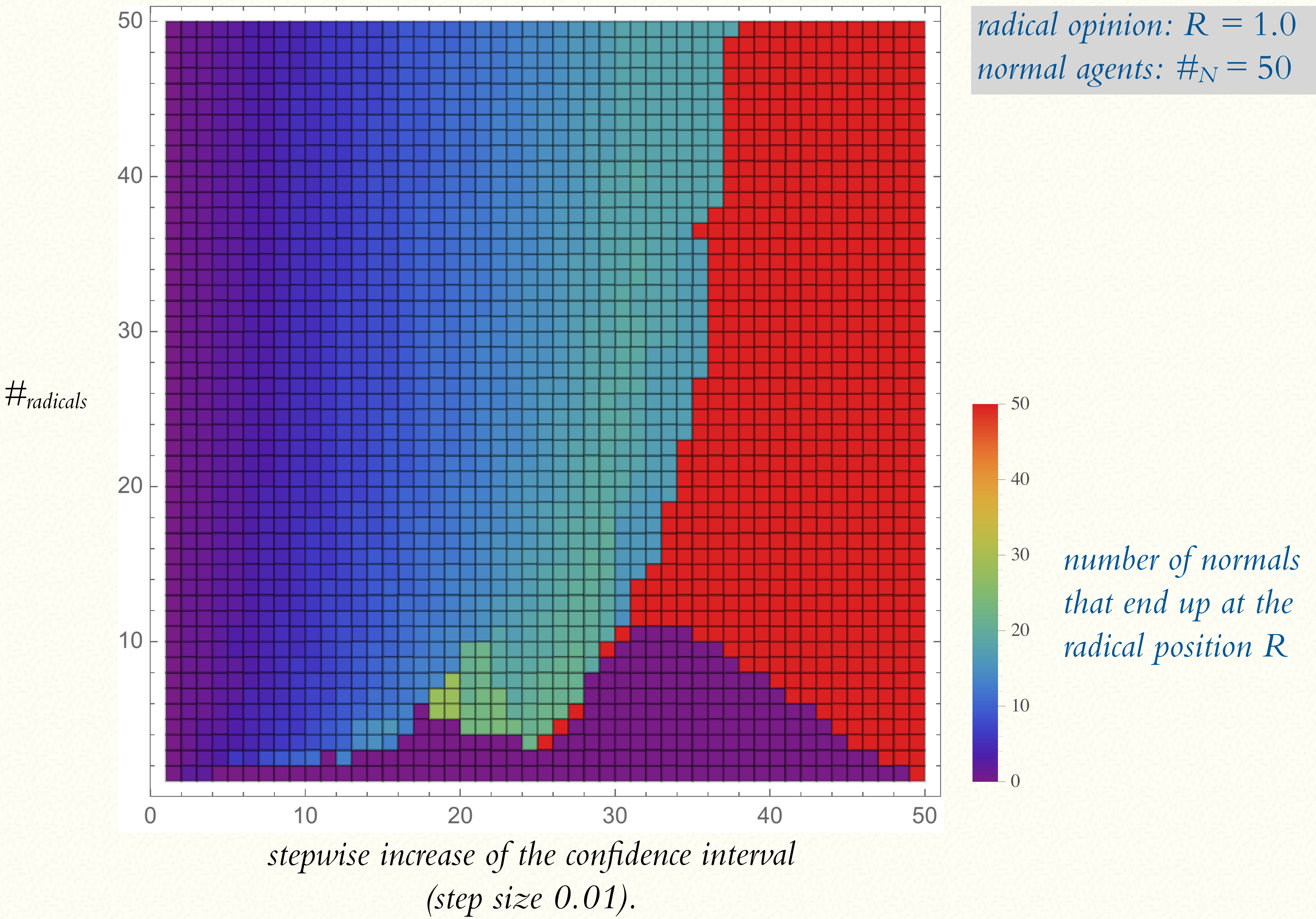
Expected value distribution

The r^{th} opinion (r = 1, ..., n) is $r/(n+1)$.

This distribution directly realizes the *expected* value for the value of the r^{th} position in an ordered profile that was generated as an even random distribution.

It also realizes the *expected* distances between neighboring opinions that are randomly distributed.

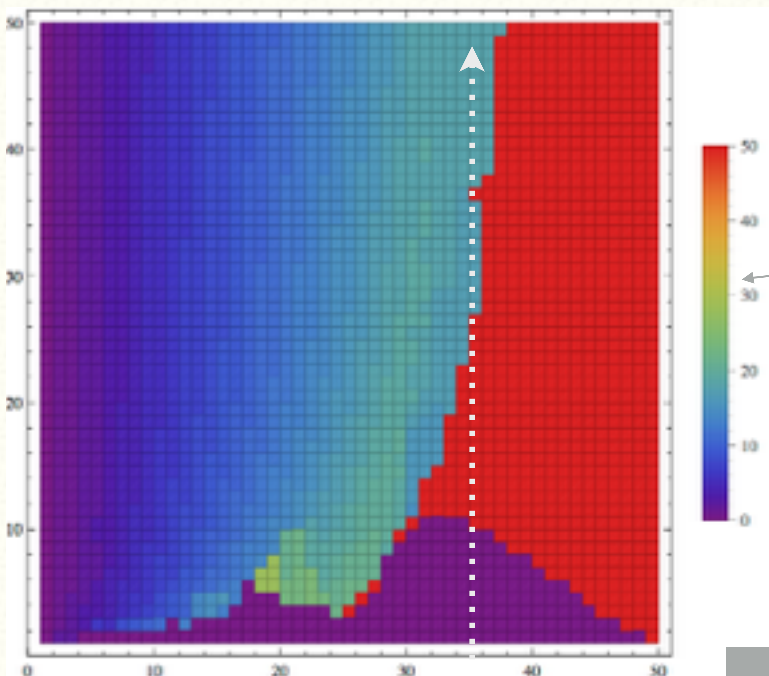
The number of normals that get radical



Normals that end up radical

[R = 1.0 $\#_{normals}$ = 50]

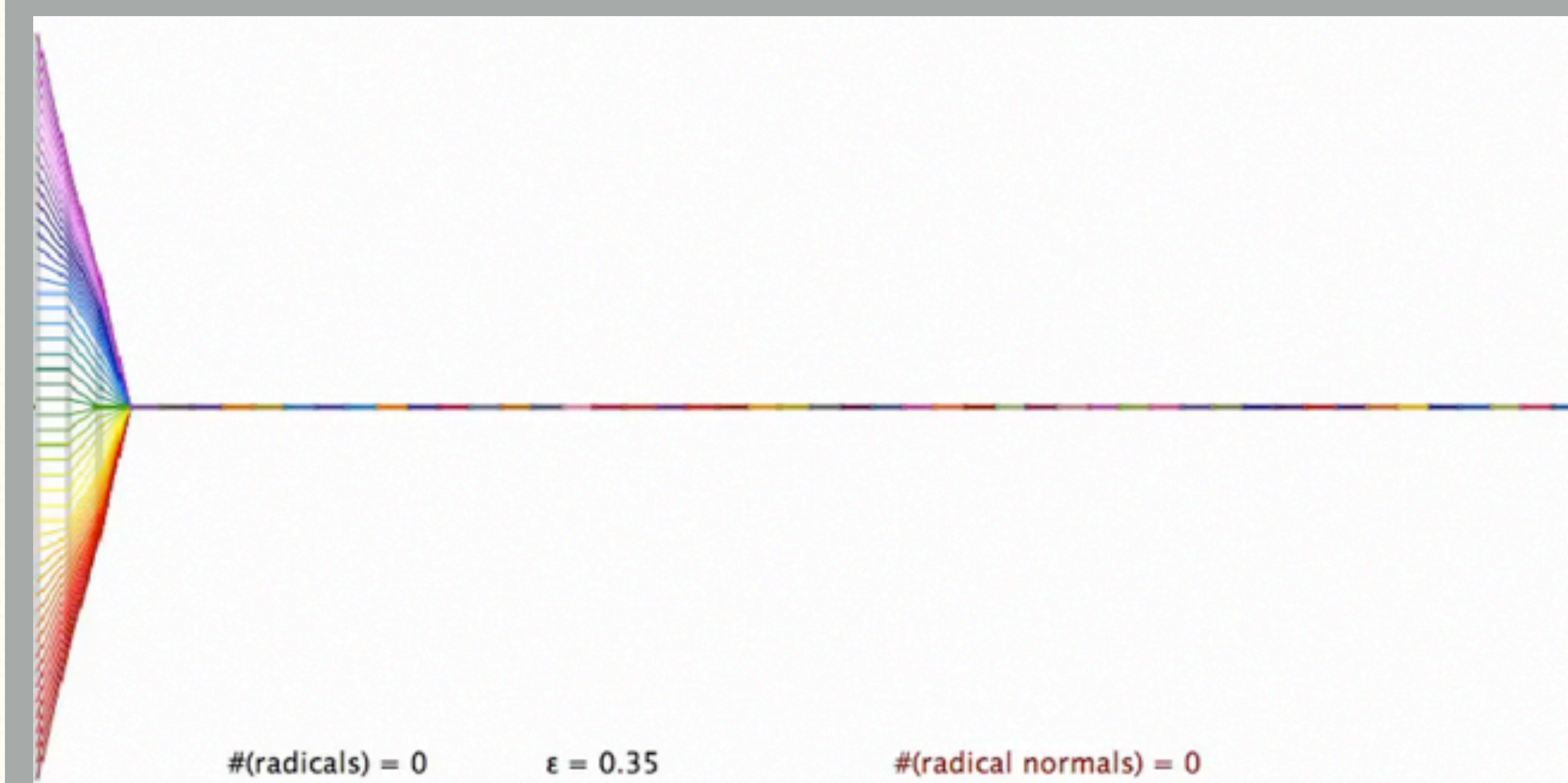
$\#_{radicals}$



$\varepsilon = 0.35$

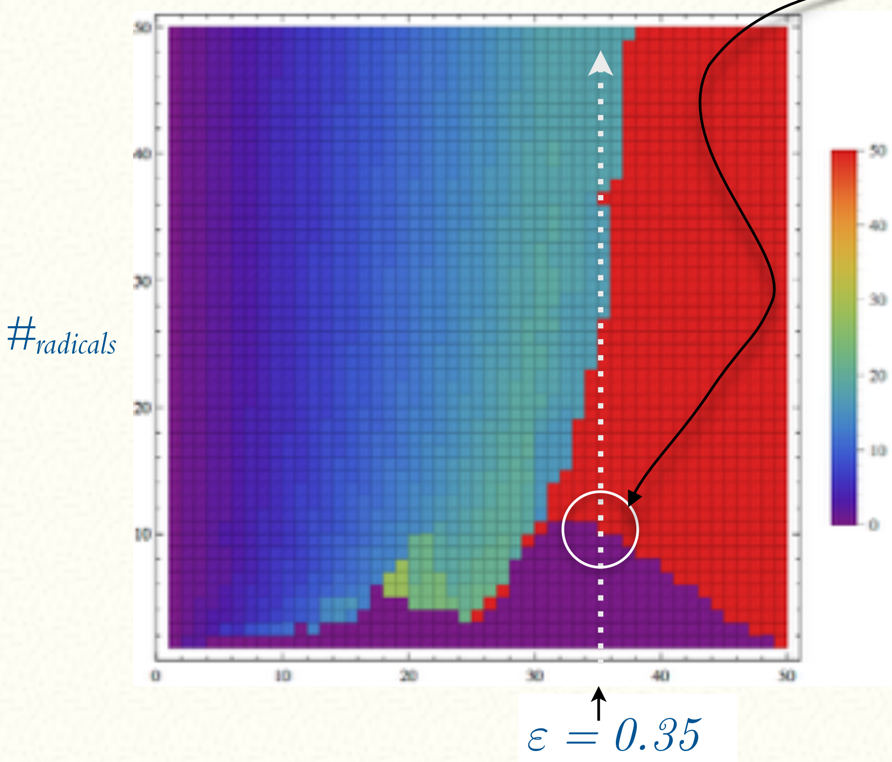
Colors:
number of normals that end
up at the radicals' position R= 1.0

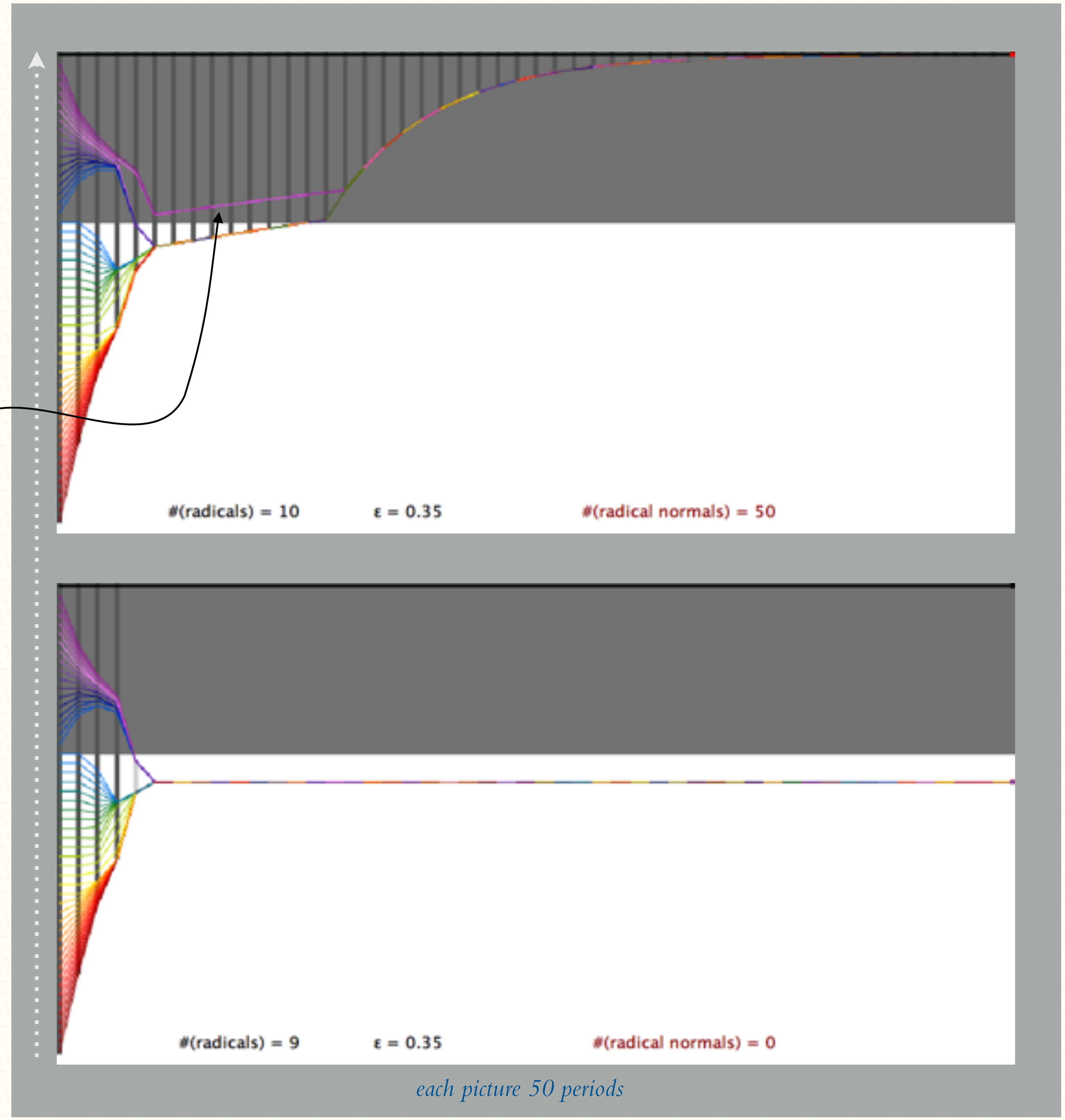
Increasing number of radicals: $\#_{radicals}$ = 0, 1, …, 50



each picture 50 periods

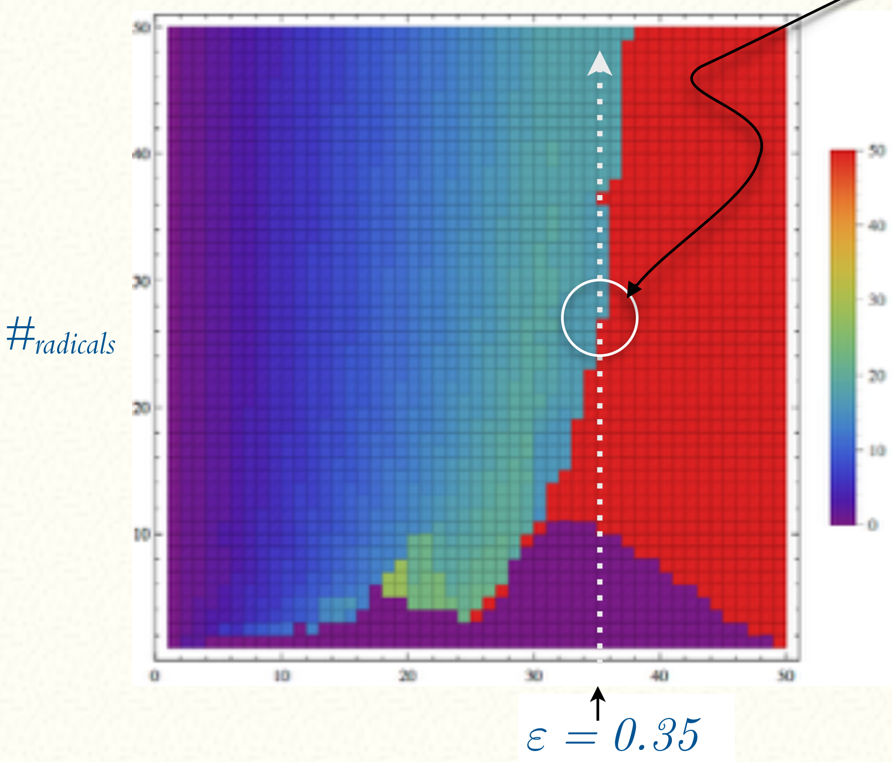
Explanandum 1

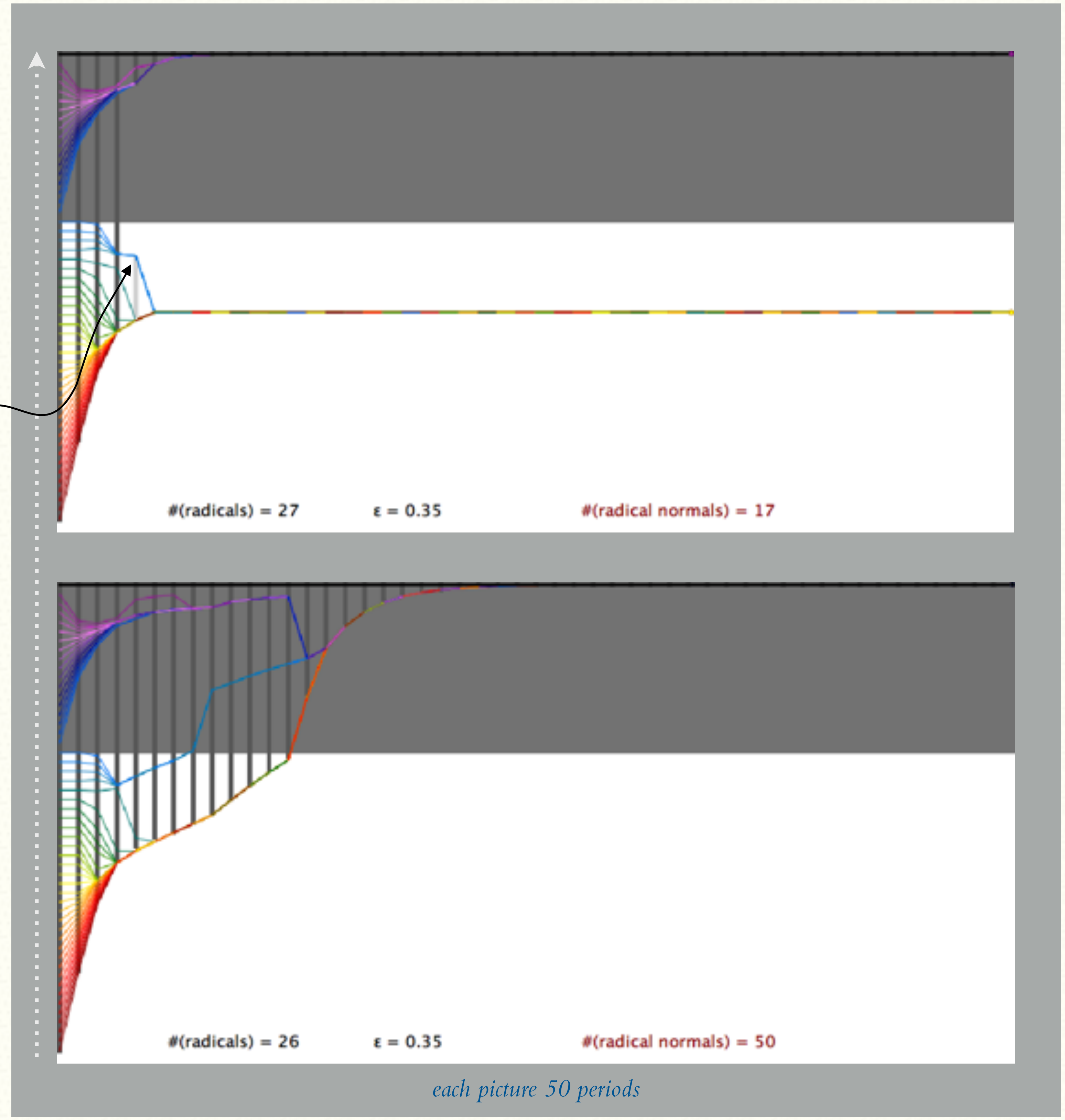




Idea for an explanation:
A bridging group appears.

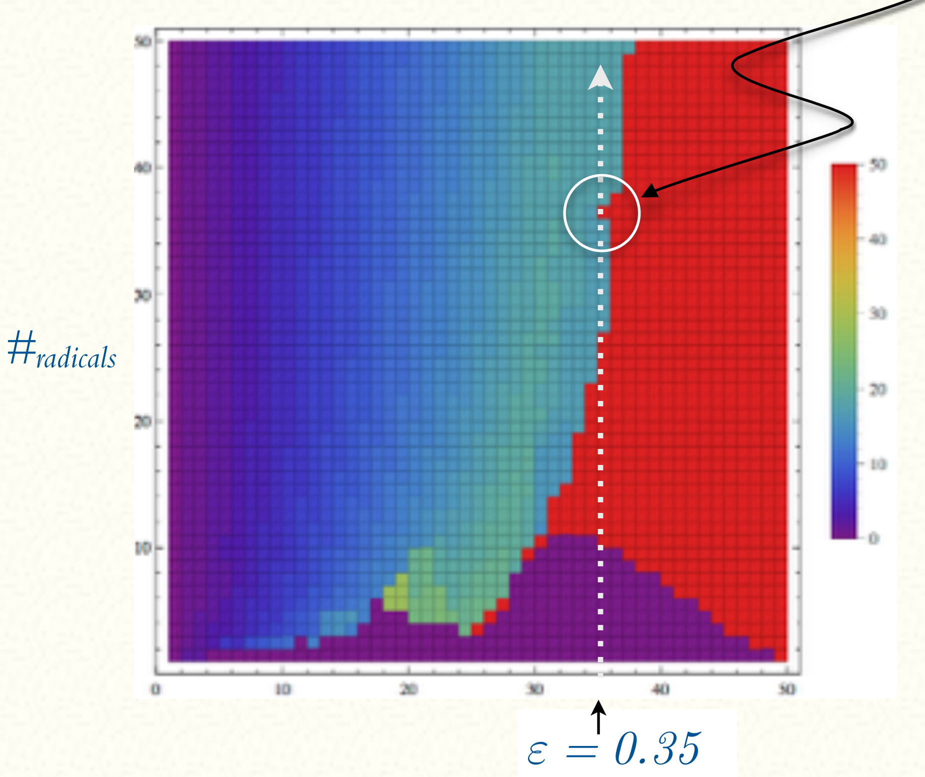
Explanandum 2

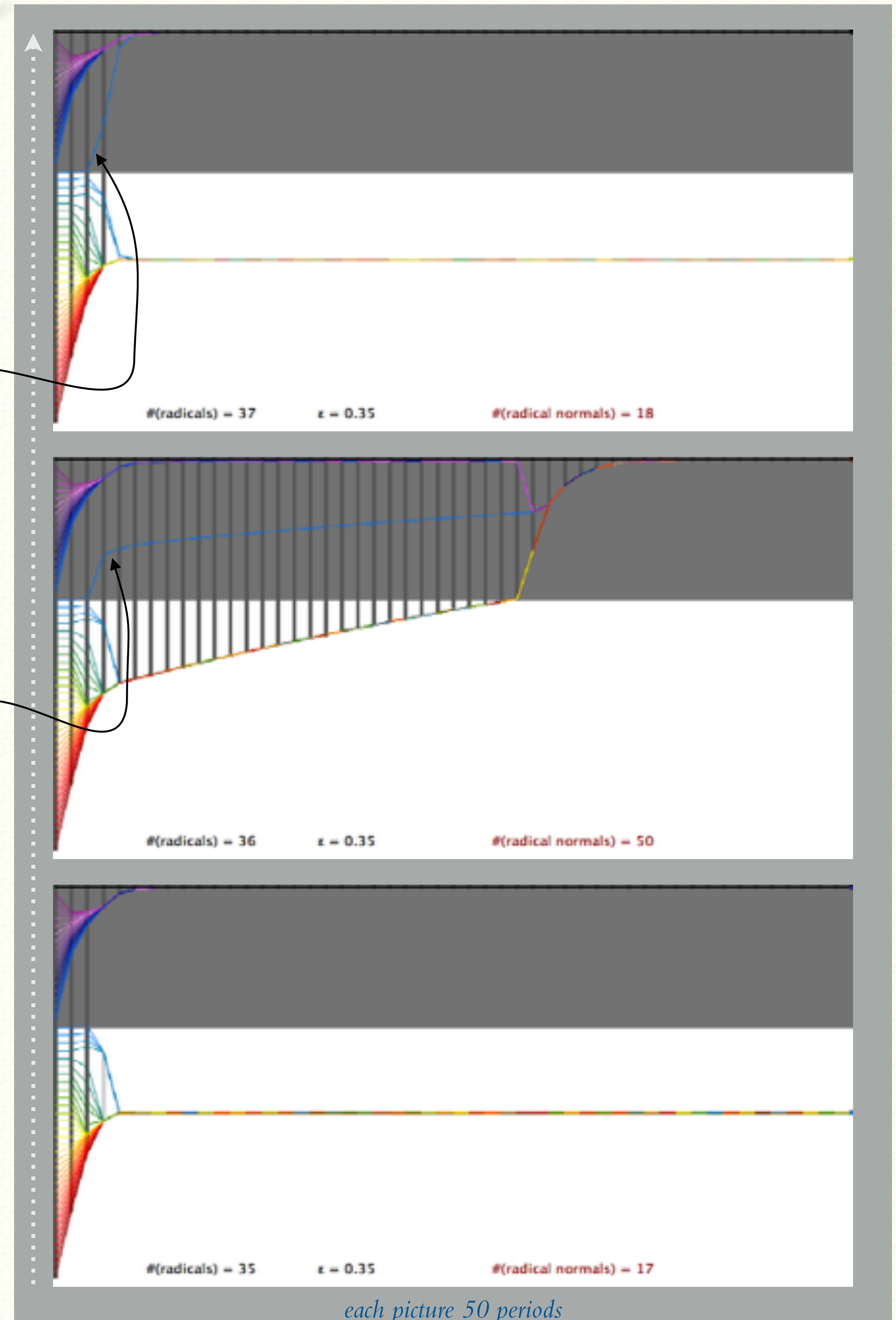




<u>Idea for an explanation:</u>
A bridging group disappears.

Explanandum 3





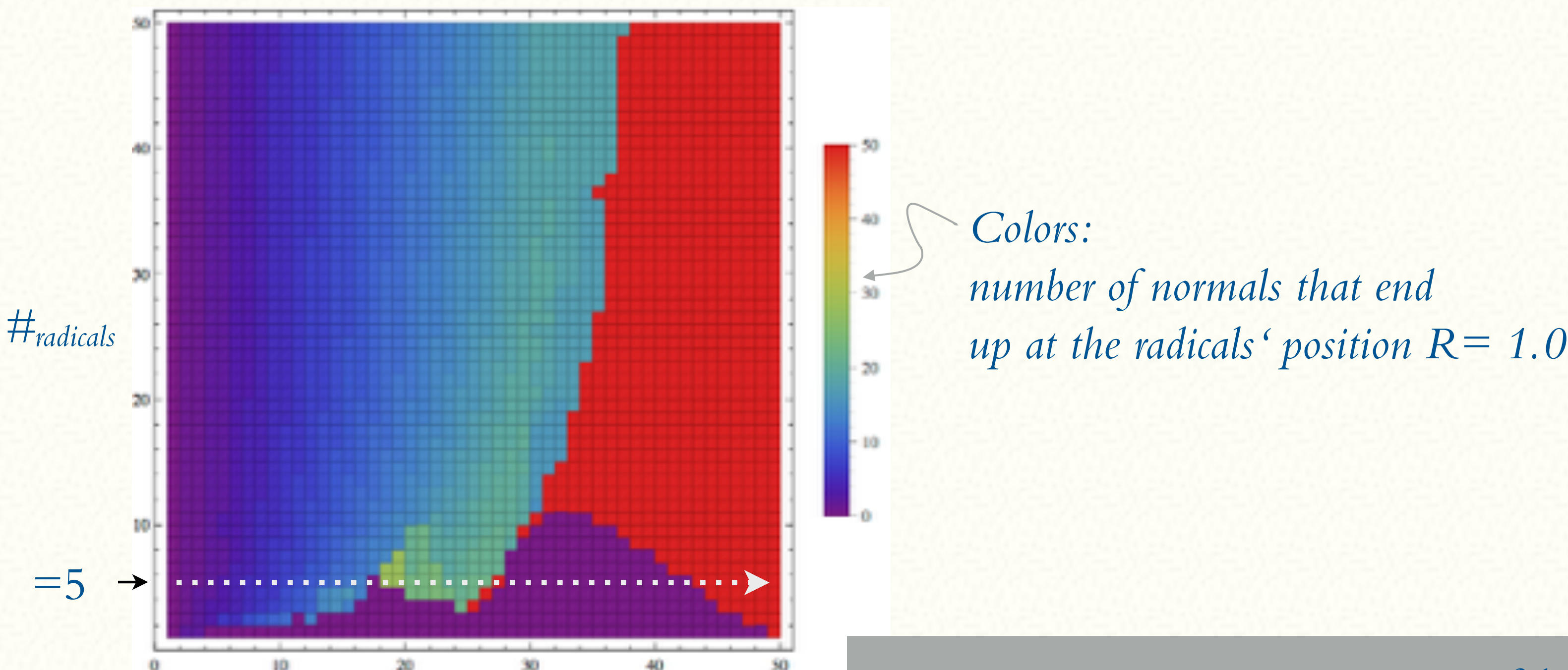
<u>Idea for an explanation:</u>

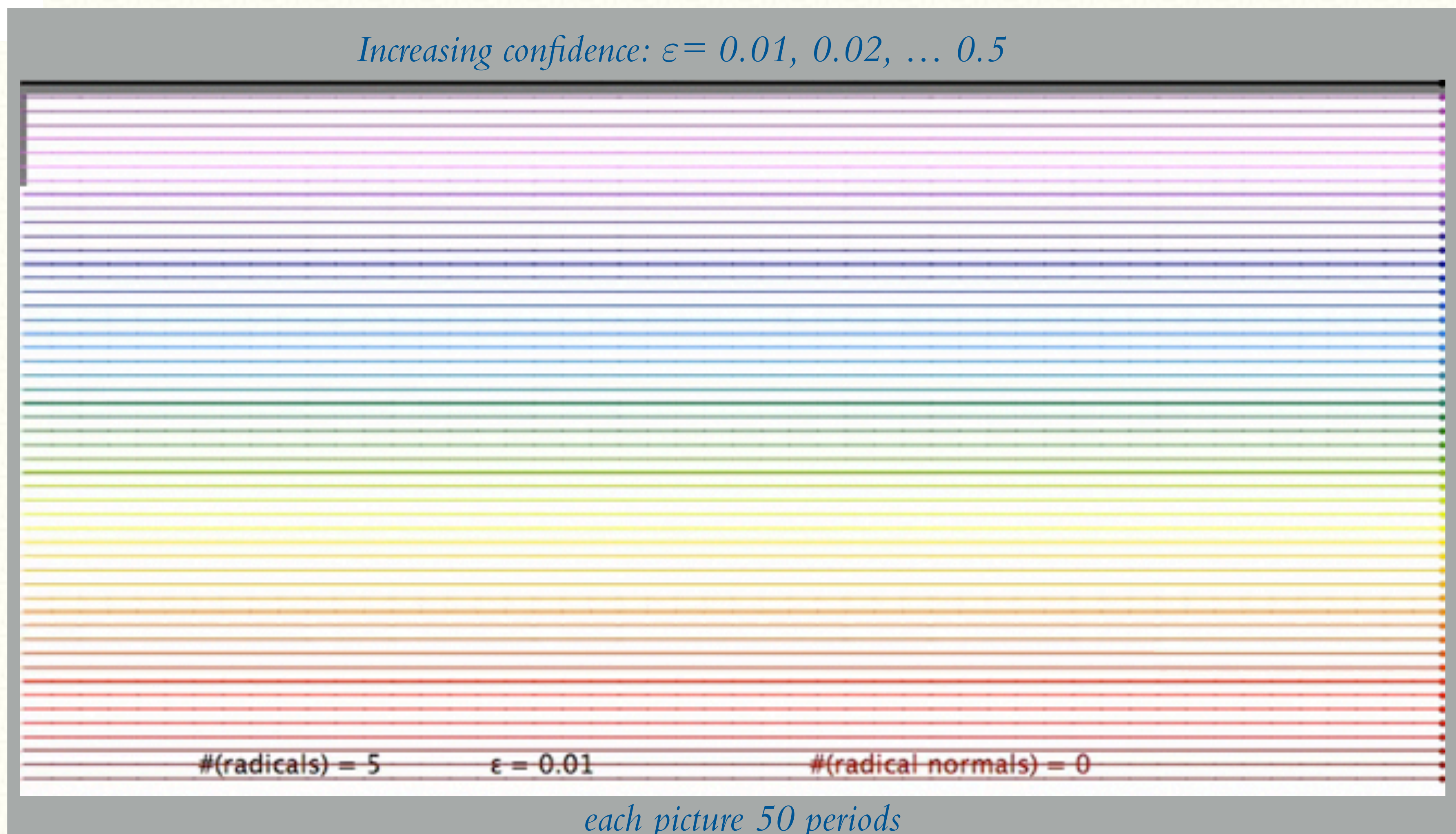
A bridging agent disappears.

A bridging agent appears.

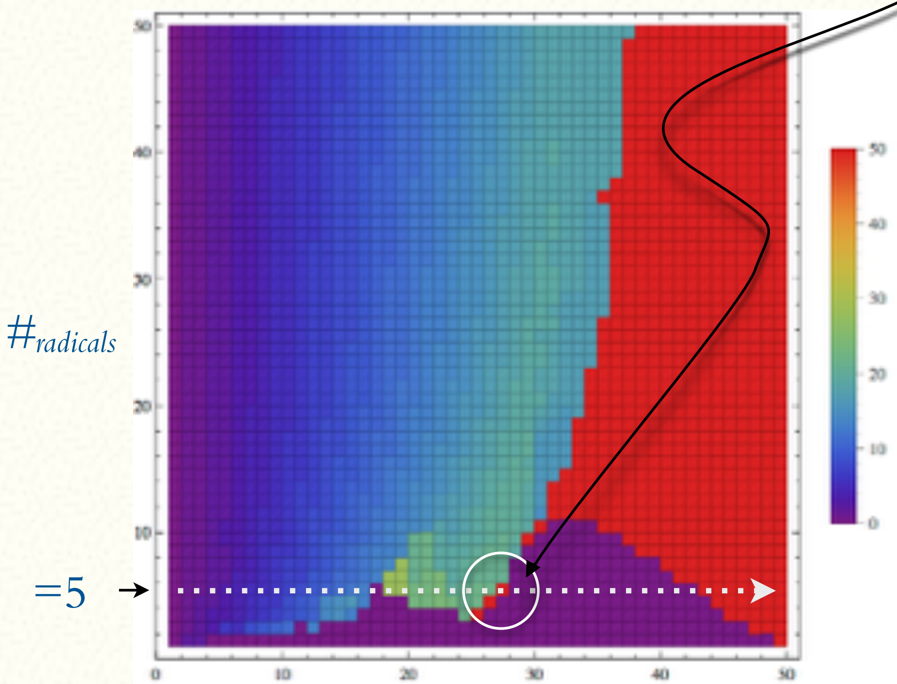
Normals that end up radical

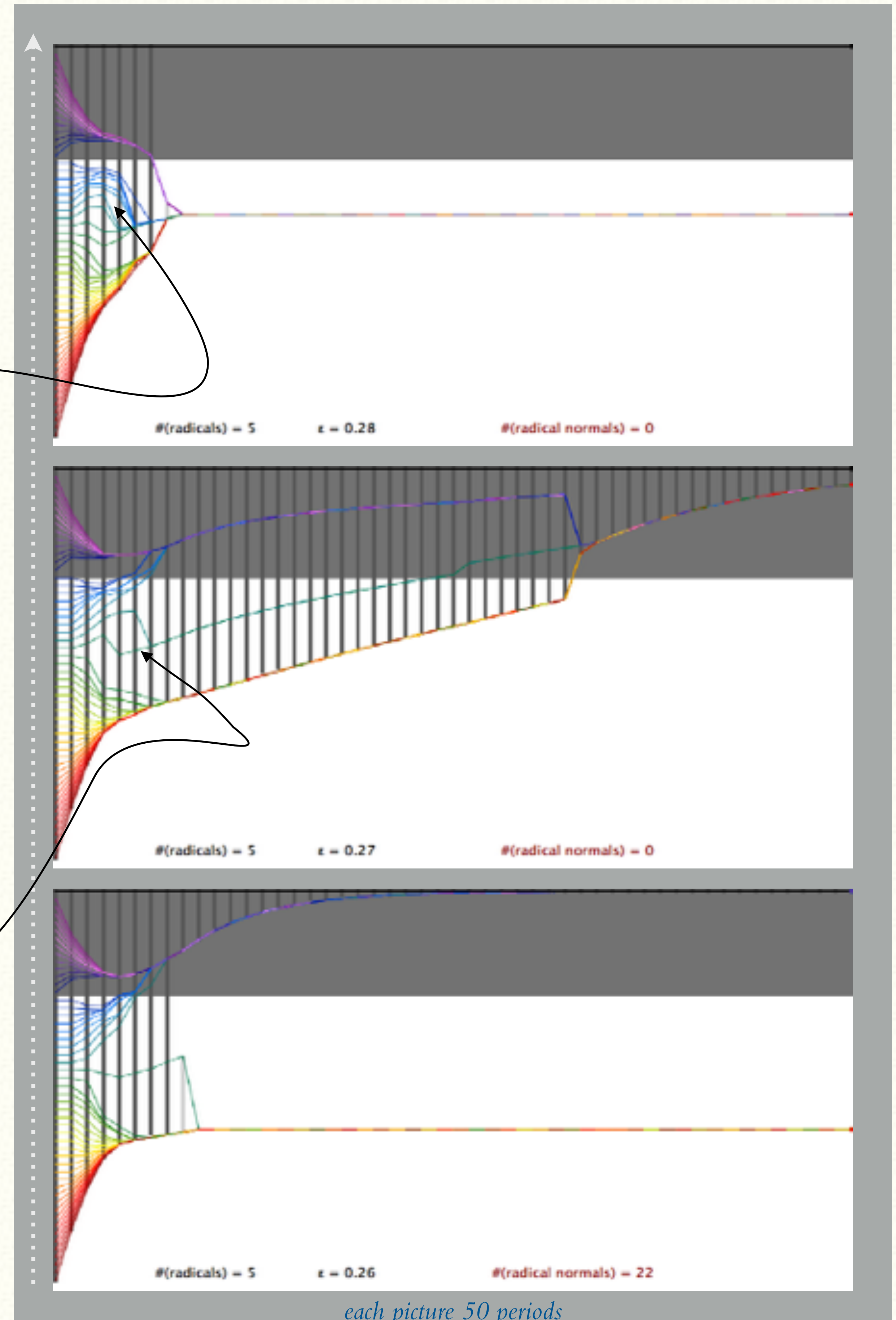
[R = 1.0 $\#_{normals}$ = 50]





Explanandum 4



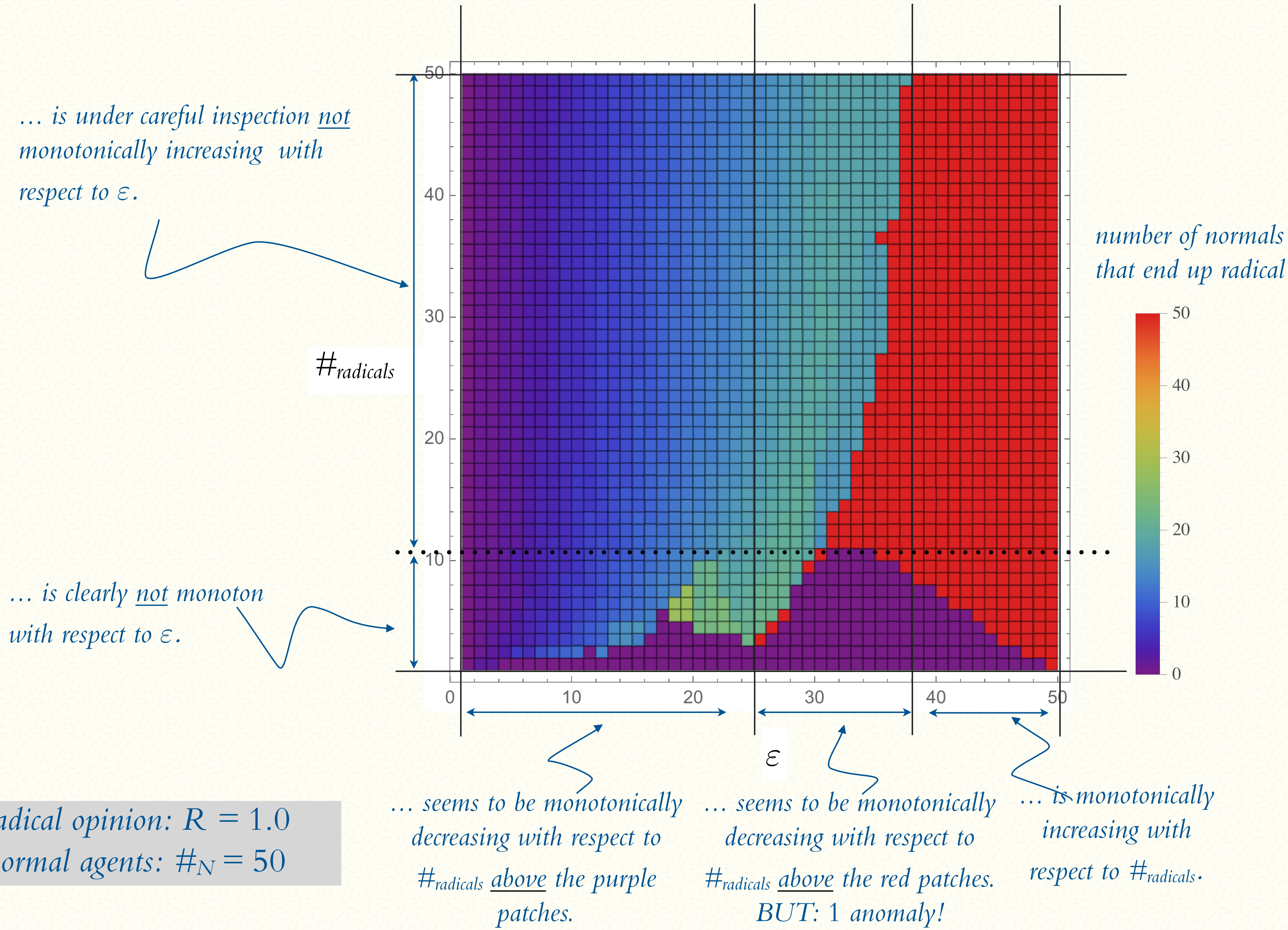


Idea for an explanation:

A bridging group appears that pulls all normals above out of the direct influence of the radicals.

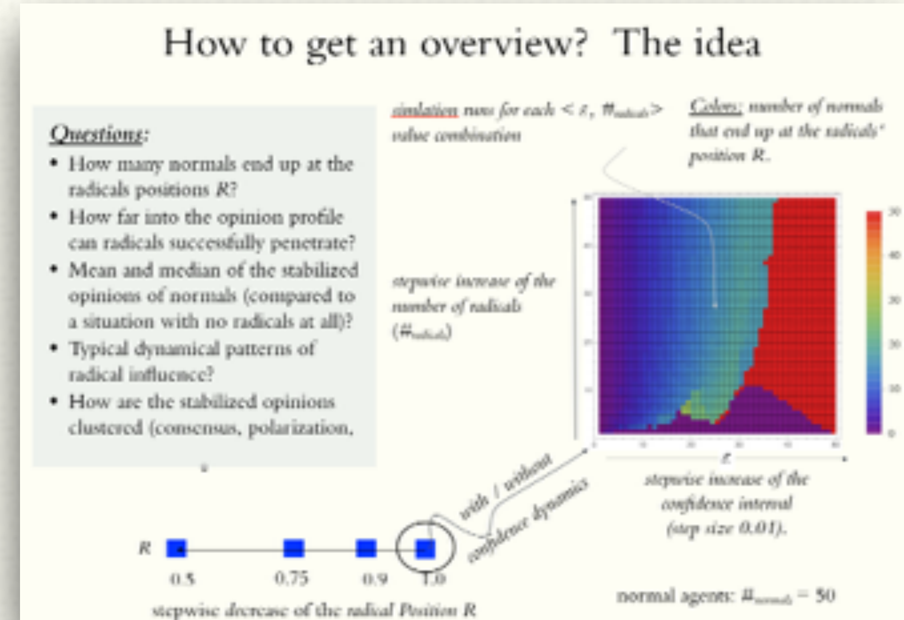
A bridging group appears that finally pulls all normals below them into the area of the direct influence of the radicals.

Monotonicity analysis:
The number of normals that get radical …



radical opinion: $R = 1.0$
normal agents: $\#_N = 50$

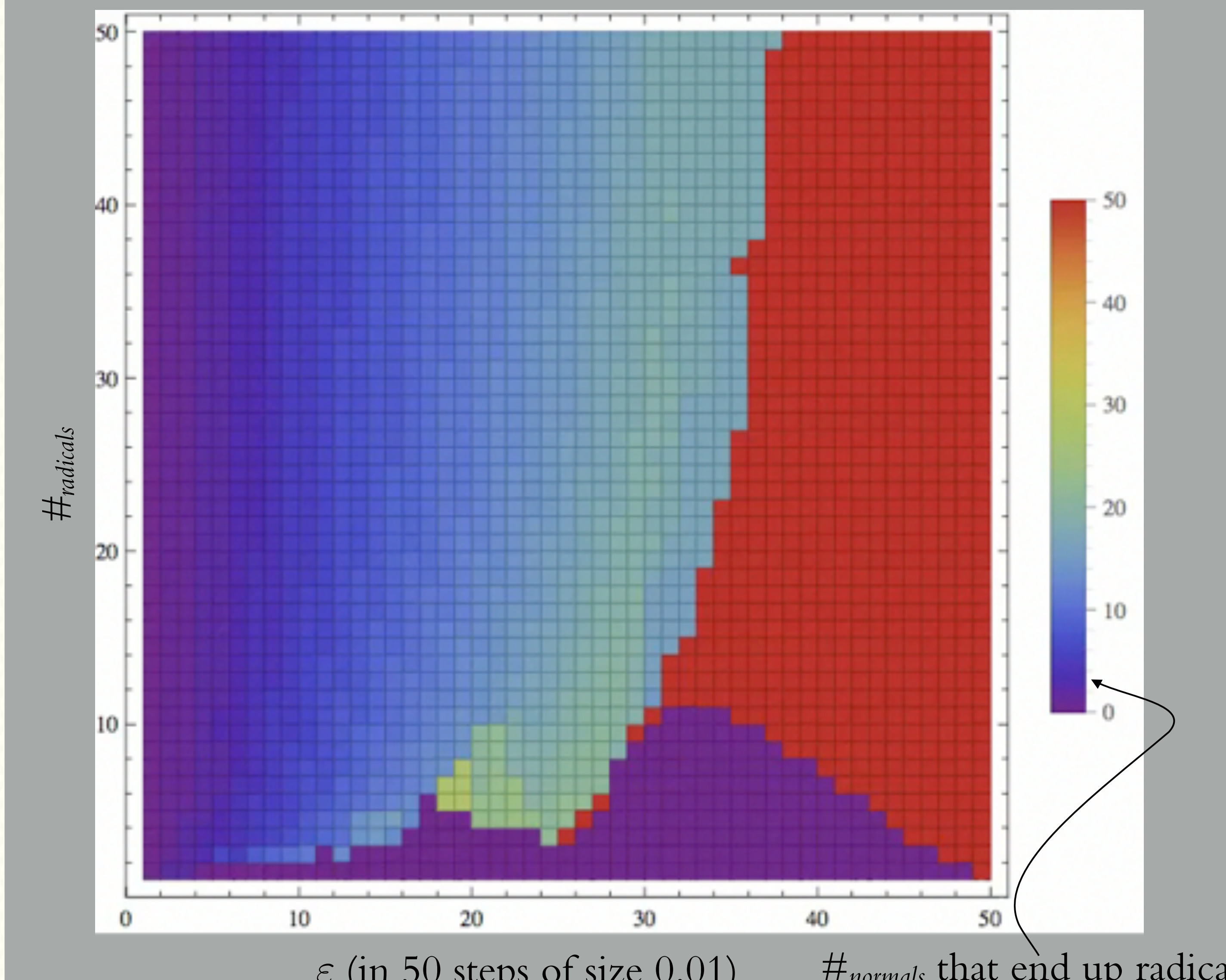
If the radical position R moves direction center



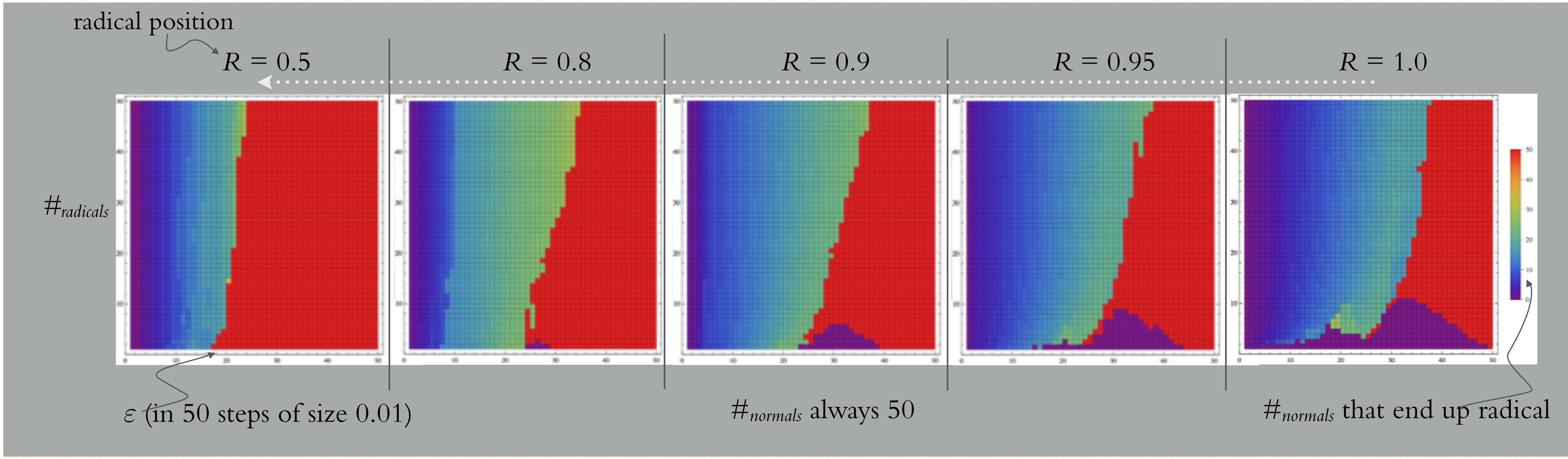
MOVIE:

R = 1.0, 0.99, 0.98 …, 0.51, 0.5 (51 pictures)

– without CD –



Normals that end up at the radical position



If the radical position R is at the upper bound of the opinion space,

- … given the confidence level, *more* radicals may lead to *less* radicalisation of normals.
- … given a small number of radicals, an increasing confidence level results in *up and down jumps* in the number of radicalised normals.

As the radicals' position R moves direction centre

- … the dynamics becomes *less and less ‚wild'*.
- … the number of normals that end up at the radical position becomes more and more independent of the number of radicals.
- … and is more and more monotonically increasing - though with a sudden jump – with regard to the confidence level.

§ 4

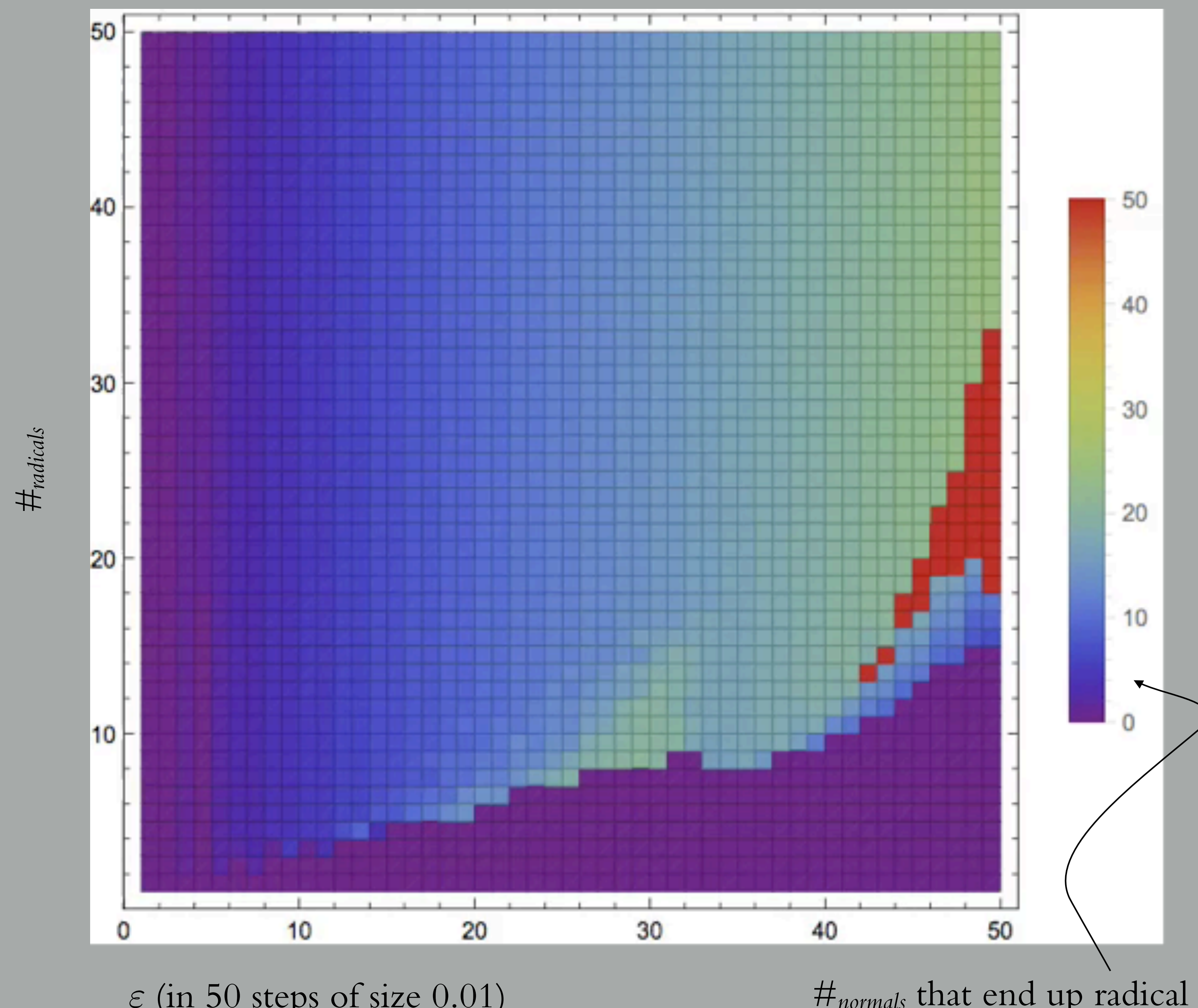
With confidence dynamics: Getting an overview

With CD: The complete overview

MOVIE:

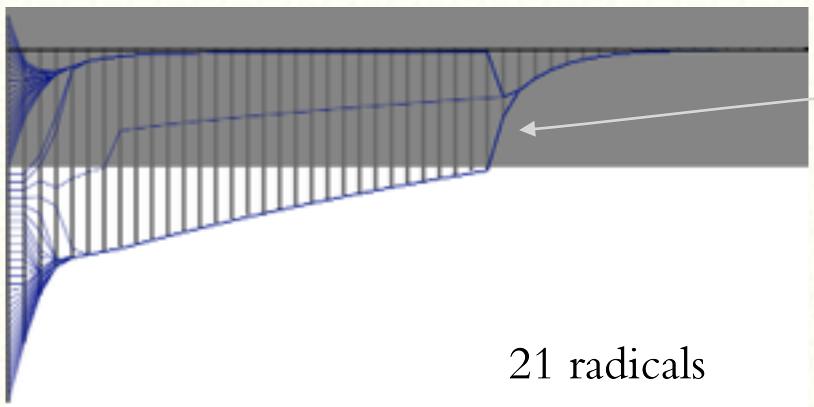
R = 1.0, 0.99, … 0.51, 0.5 (51 pictures)

– with CD –



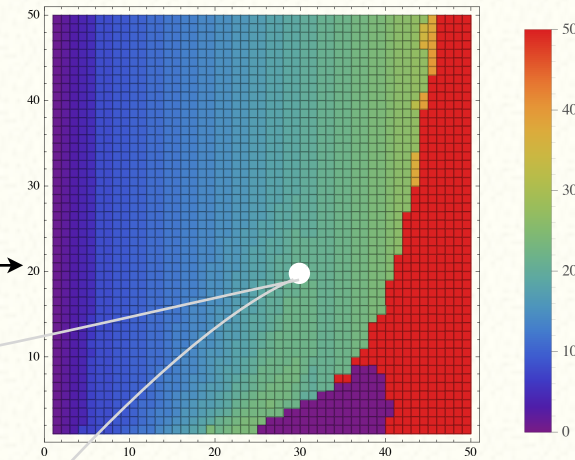
To see more: Coloring the size of $\varepsilon_i(t)$

without CD



with CD

$\#_{Radicals} = 21$



$\varepsilon = 0.3$

$R = 0.9$

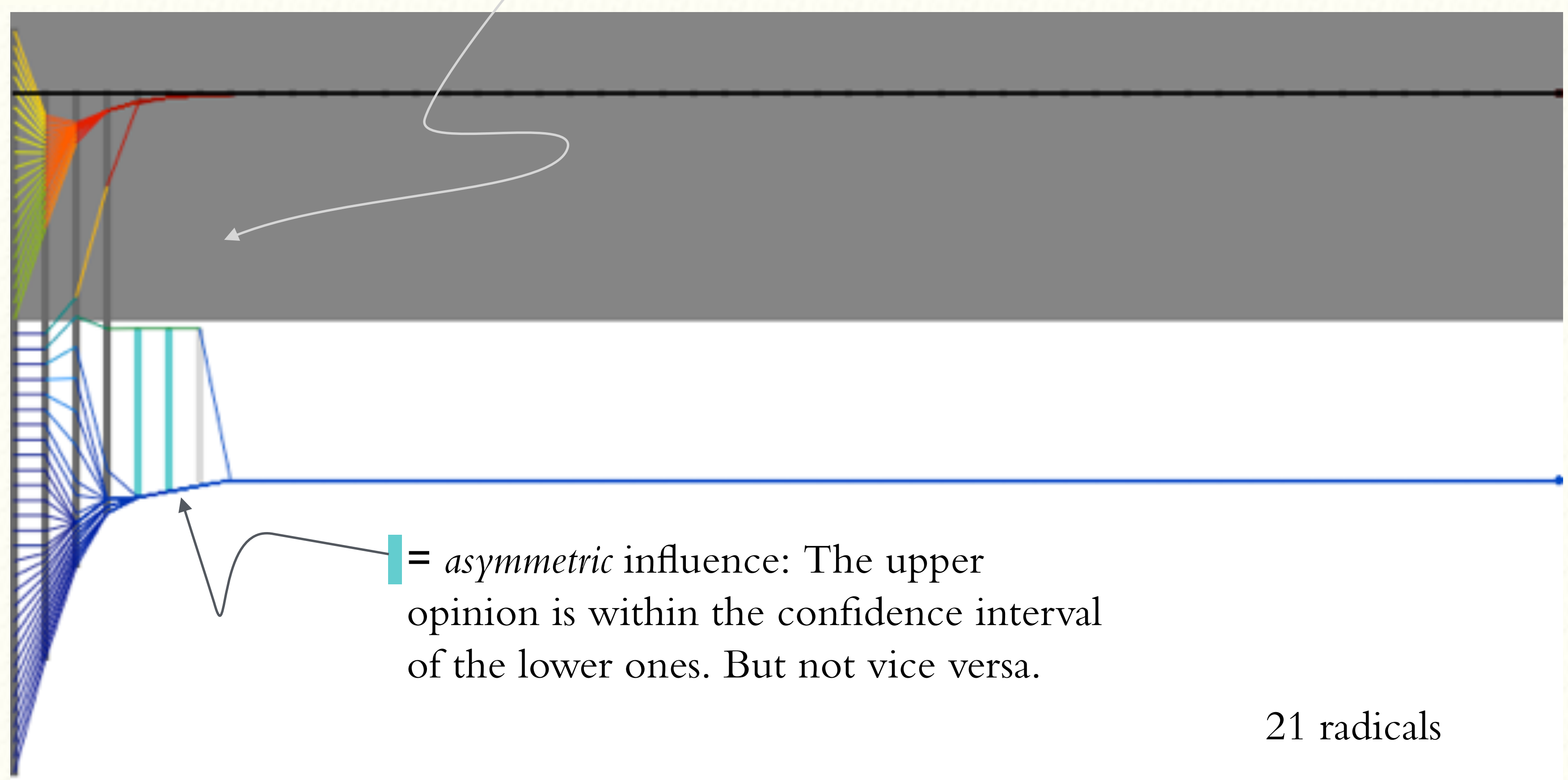
$\#_{normals} = 50$

Colors:

size of $\varepsilon_i(t)$ in % of $\varepsilon_i(t_0)$

100 %

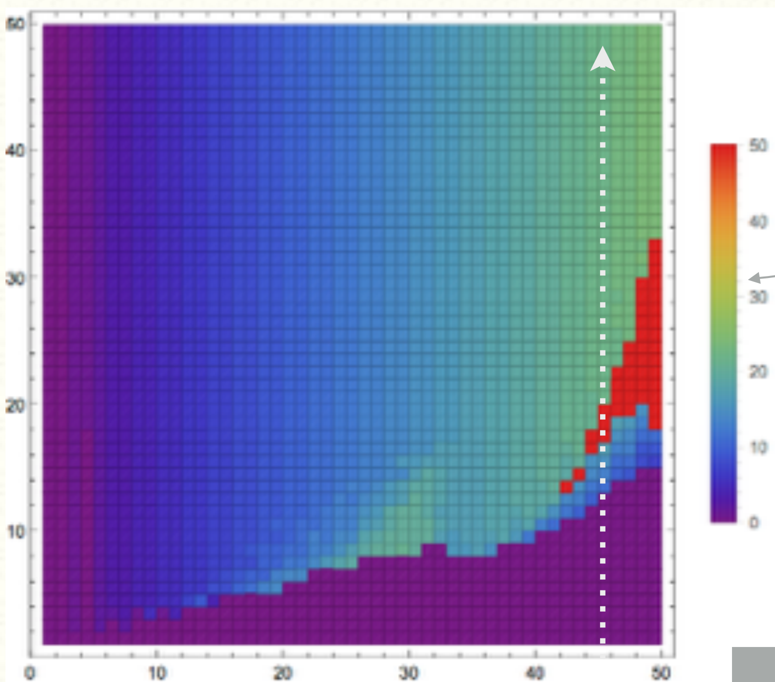
0 %



With CD: Normals that end up radical

[$R = 1.0$ $\#_{\text{normals}} = 50$]

$\#_{radicals}$



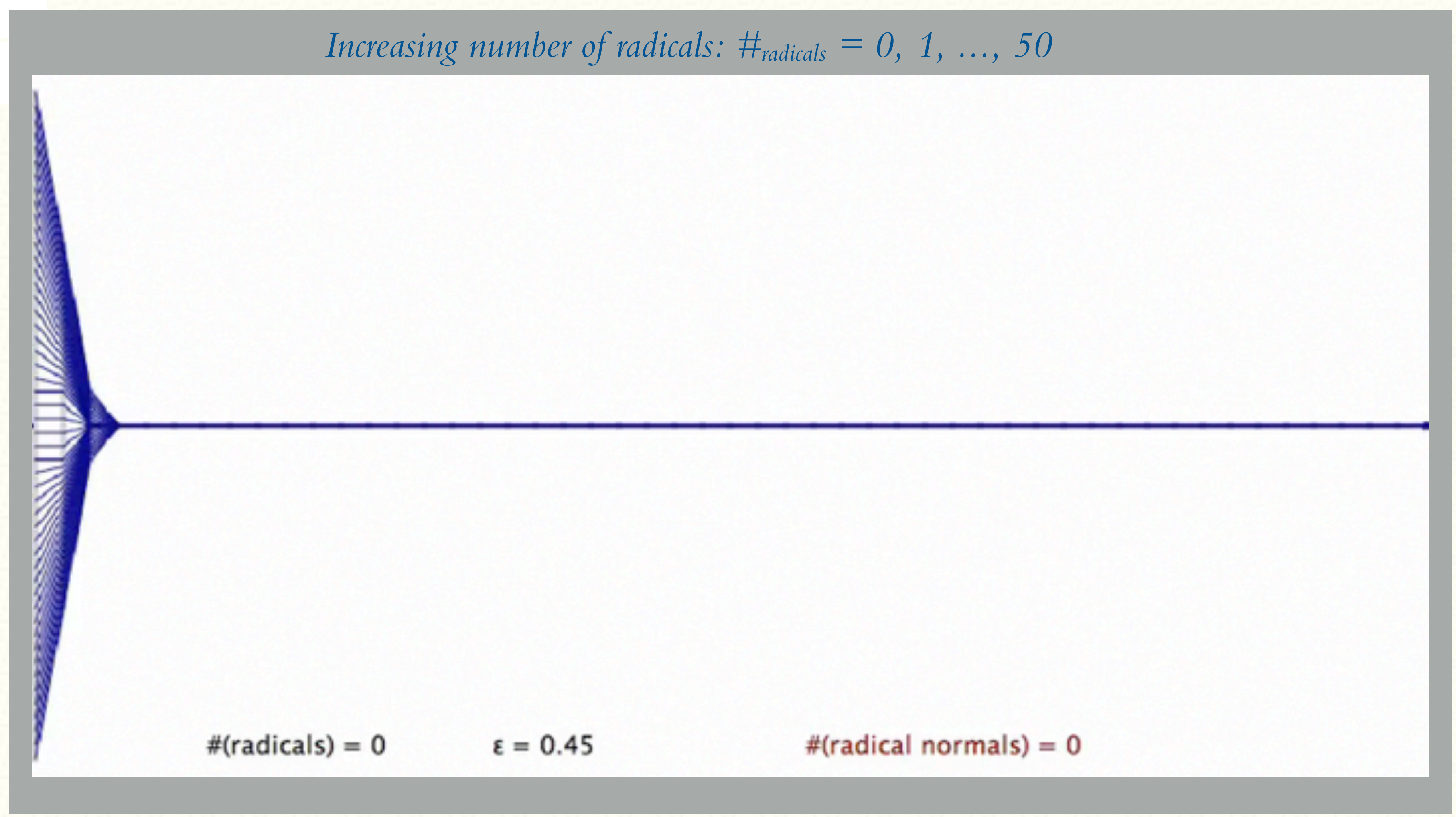
Colors:
number of normals that end up at the radicals' position R= 1.0

$\varepsilon = 0.45$

100 %

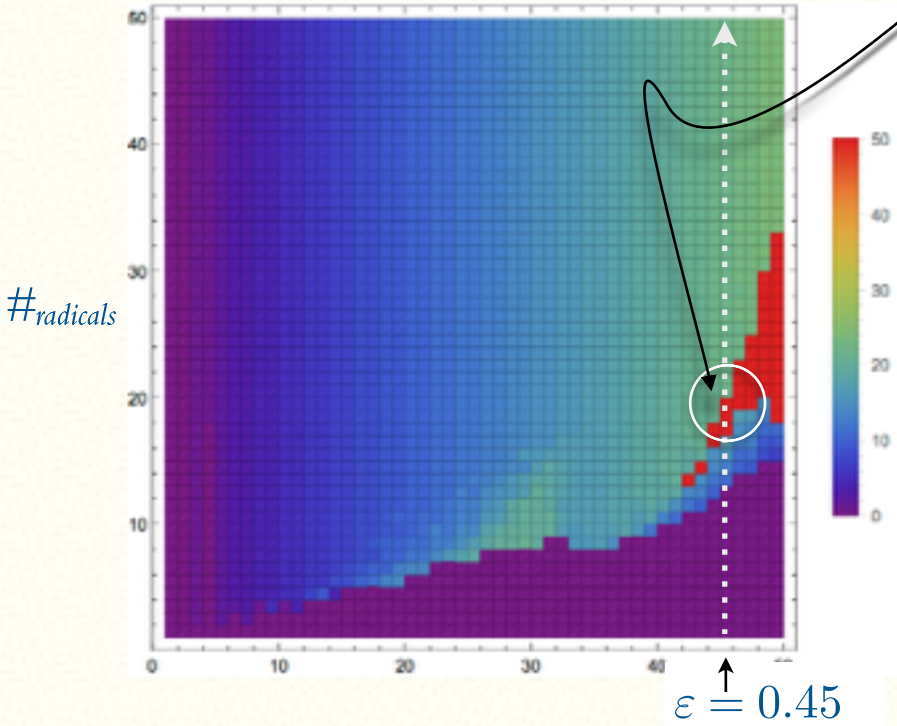
0 %

<u>Colors:</u>
size of $\varepsilon_i\,(t)$ in % of $\varepsilon_i\,(t_0)$



Explanandum 5

Colors:
number of normals that end up at the radicals' position R= 1.0

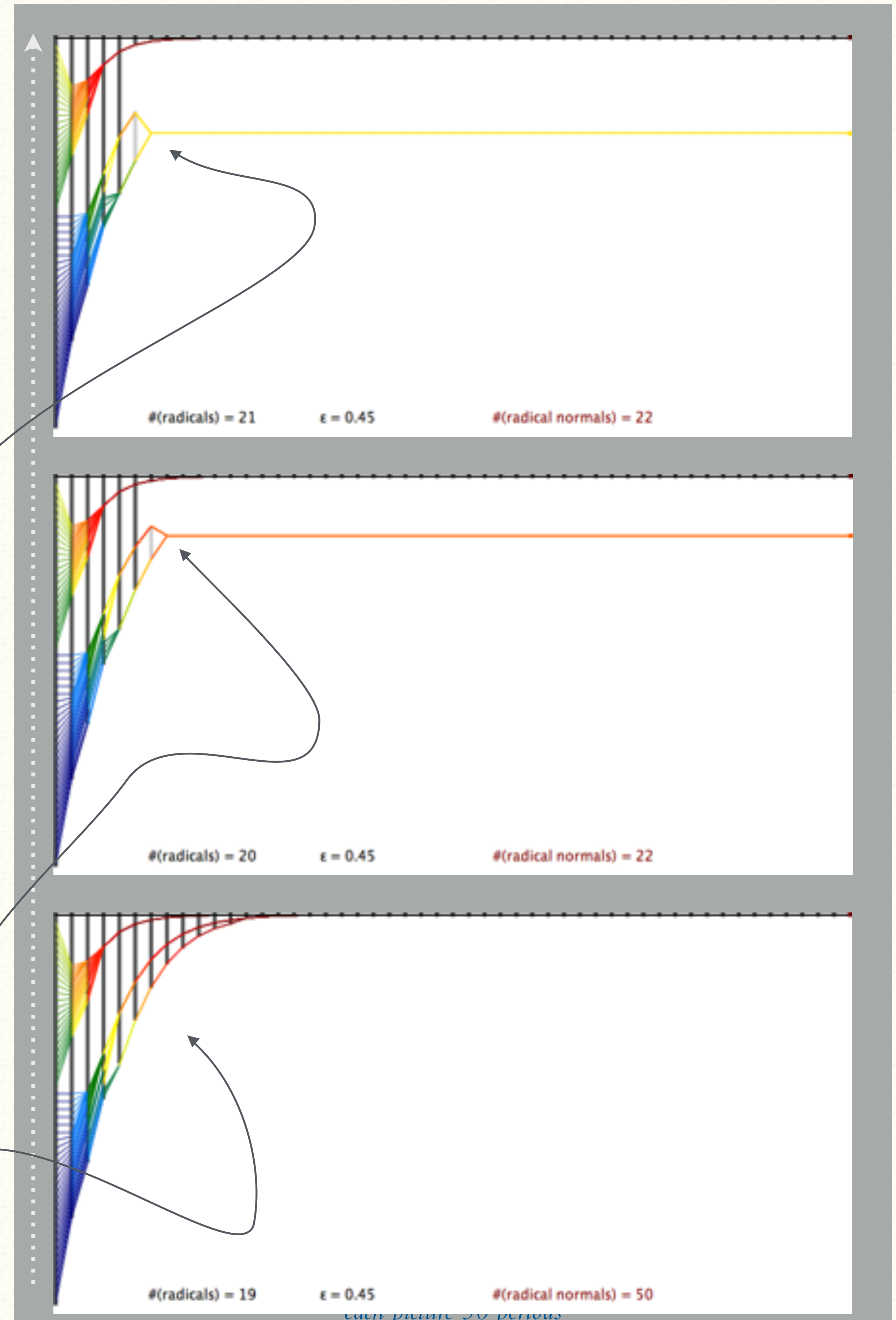


Idea for an explanation:

With a further radical the chain breaks even earlier. The position of the non radical normals is more moderate.

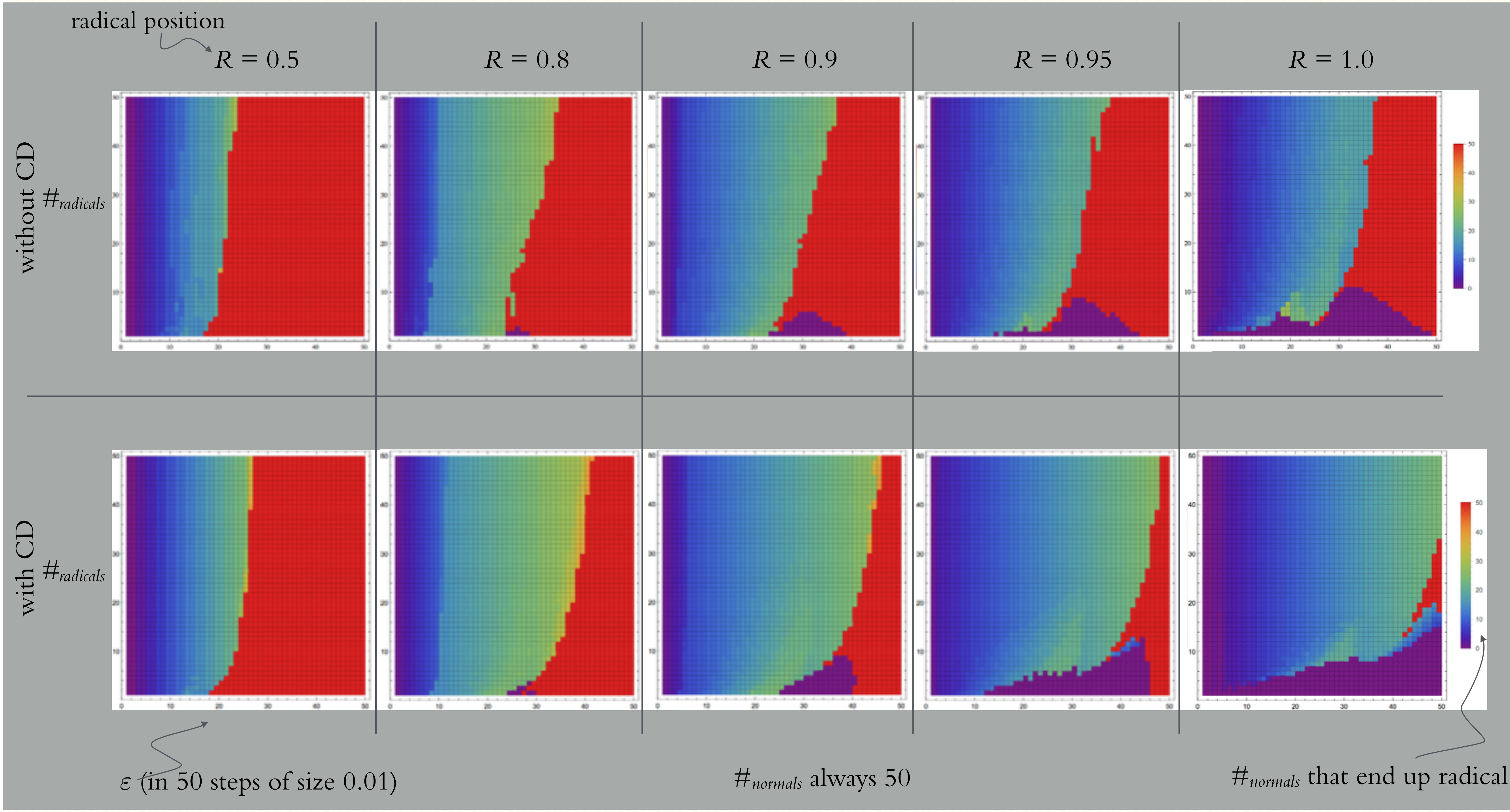
With one more radical the size of ε decreases too fast and the chain breaks.

The size of ε decreases in all parts of the profile. There are two bridging groups. The chain of direct or indirect influence of radicals never breaks.



Colors:
size of $\varepsilon_i(t)$ in % of $\varepsilon_i(t_0)$

Normals that end up at the radical position



- With *or* without a confidence dynamics, the dynamics is very wild if *R* is extreme. It becomes less and less wild as the radicals' position *R* moves direction centre.
- *With* a confidence dynamics the sudden jump to a situation in which all normals become radicals, occurs only for much higher initial confidence levels: In some parts of the parameter space becoming less open-minded protects normals from becoming radicals.

§5
Next steps

Finding the answers to some problems:

1. Are our expected value start distributions really ‚*representative*‘ ?
2. Do *absolut numbers* of normals and radicals matter? Or is it only the *ratio* that matters?
3. Where in the opinion space do the non-radical normals end up (mean, median, minimal distance to radicals etc.)?
4. What if $\varepsilon > 0.5$? (I obviously stopped the computations too early in the case with a confidence dynamics).
5. What precisely are *bridging groups or opinions*? How to apply and adapt network centrality measures in and to our context?
6. Taking the *control* perspective: When and how to ‚build or destroy bridges‘?

Decisive task: Understanding bridging

Many thanks for your attention!