Generalized Kinetic Equations and Stochastic Game Theory for Social Systems

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Complexity Features of Social Systems

- Living $\rightarrow$ active entities

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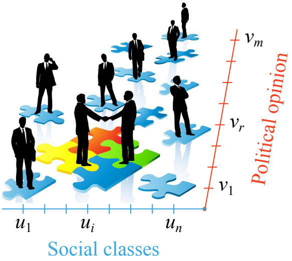
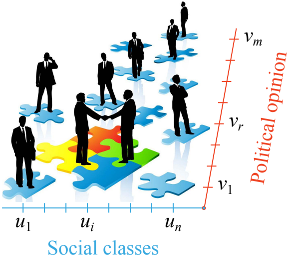
Complexity Features of Social Systems

- Living → active entities
- Behavioral strategies, bounded rationality → randomness of human behaviors
- Heterogeneous distribution of strategies
- Behavioral strategies can change in time
- Self-organized collective behavior can emerge spontaneously:

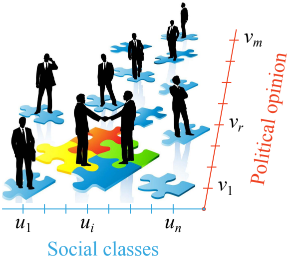
> *A Black Swan is a highly improbable event with three principal characteristics: It is unpredictable; it carries a massive impact; and, after the fact, we concoct an explanation that makes it appear less random, and more predictable, than it was.*
>
> [N. N. Taleb. **The Black Swan: The Impact of the Highly Improbable**, Random House, New York City, 2007]


v_m
v_r
v_1
Political opinion
u_1
u_i
u_n
Social classes



- Social classes: (poor) $u_1 = -1, \ldots, u_i, \ldots, u_n = 1$ (wealthy)

Methods of the Generalized Kinetic Theory for Active Particles



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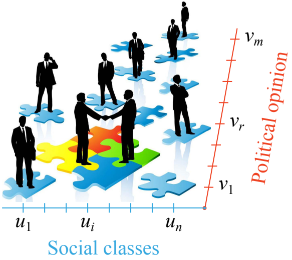

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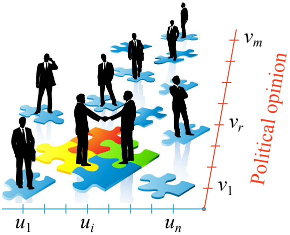

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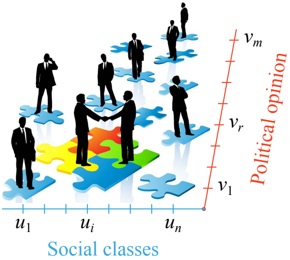
$$\frac{df_i^r}{dt} =$$



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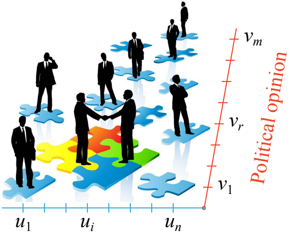
$$\frac{df_i^r}{dt} = \underbrace{\sum_{p,\,q=1}^{m} \sum_{h,\,k=1}^{n} \eta_{hk}^{pq} \mathcal{B}_{hk}^{pq}[\gamma,\,U](i,\,r) f_h^p f_k^q}_{\text{Gain}}$$

$$\mathcal{B}_{hk}^{pq}[\gamma, U](i, r) := \text{Prob}((u_h, v_p) \to (u_i, v_r) | (u_k, v_q), \gamma, U)$$



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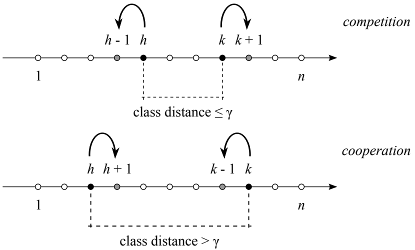
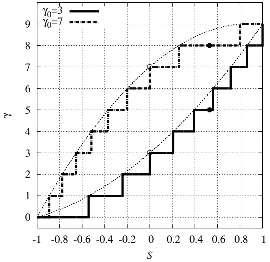
Stochastic Games: Cooperation/Competition + Self-Conviction

- Social dynamics: cooperation vs. competition





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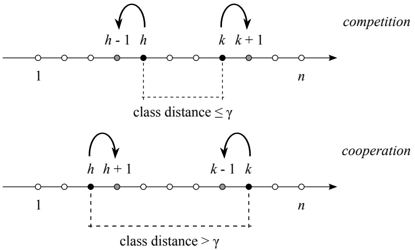

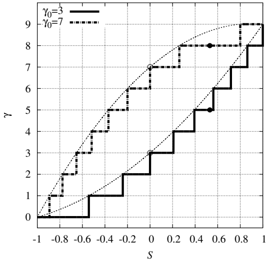

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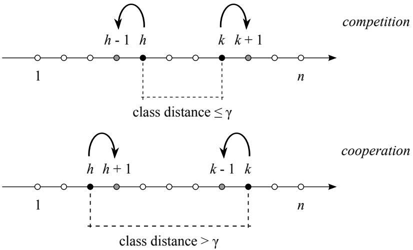
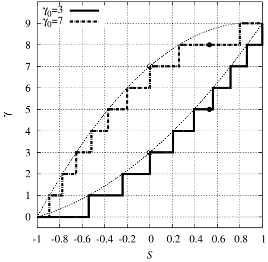
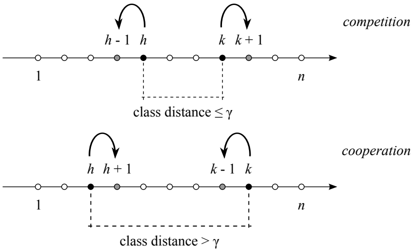
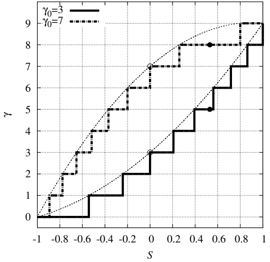




- Opinion dynamics: self-conviction
  - Poor individuals in poor society → distrust

- Social dynamics: cooperation vs. competition





- Opinion dynamics: self-conviction
  - Poor individuals in poor society → distrust
  - Wealthy individuals in a wealthy society → trust

Stochastic Games: Cooperation/Competition + Self-Conviction

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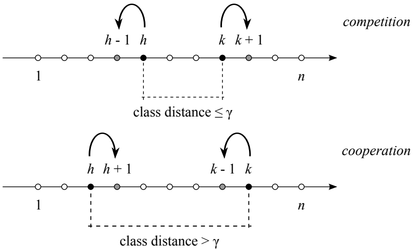

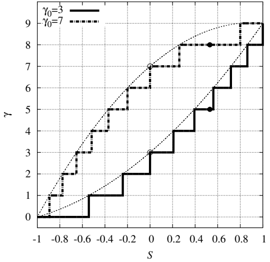

- Opinion dynamics: self-conviction
  - Poor individuals in poor society → distrust
  - Wealthy individuals in a wealthy society → trust
  - Poor individuals in a wealthy society
Wealthy individuals in a poor society } → most uncertain behavior

An example of transition probabilities

- Ansatz:

$$\mathcal{B}_{hk}^{pq}[\gamma,\, U](r,\, i) = \underbrace{\bar{\mathcal{B}}_{hk}[\gamma](i)}_{\text{social dynamics}} \cdot \underbrace{\hat{\mathcal{B}}_{h}^{p}[U](r)}_{\text{opinion dynamics}}$$

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Social dynamics
Cooperation: $|k-h|>\gamma$

- If $h\leq k$:

$$\bar{\mathcal{B}}_{hk}[\gamma](i)=\begin{cases}1-\frac{|k-h|}{n-1} & \text{if } i=h\\ \frac{|k-h|}{n-1} & \text{if } i=h+1\\ 0 & \text{otherwise}\end{cases}$$

- If $h>k$:

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Social dynamics
Cooperation: $|k-h| > \gamma$

- If $h \leq k$:

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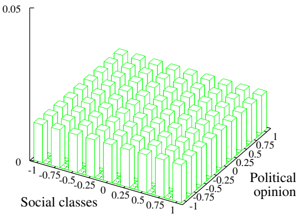
Opinion dynamics
Self-conviction

- Wealthy individual ($u_h \geq 0$) in a poor society ($U < 0$)
- Poor individual ($u_h < 0$) in a wealthy society ($U \geq 0$)

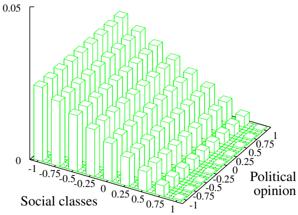
$$\hat{\mathcal{B}}^{p}_{h}[U](r) = \begin{cases} \beta & \text{if } r = p-1 \\ 1-2\beta & \text{if } r = p \\ \beta & \text{if } r = p+1 \\ 0 & \text{otherwise} \end{cases}$$

$$0 \leq \beta \leq \frac{1}{2}$$

- Initial conditions

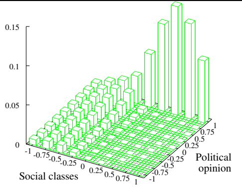
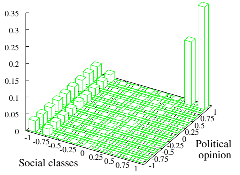


Society "neutral" on average
Mean wealth: 0



Society poor on average
Mean wealth: -0.4

- Society which is "economically neutral" on average

$\gamma = 3$ cooperative	$\gamma = 7$ competitive
	

- Society which is poor on average

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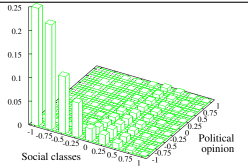
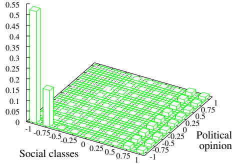
<tr><td>constant γ</td><td>
</td><td>
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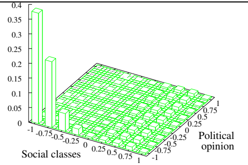
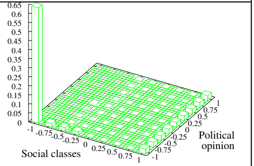
<tr><td>variable γ</td><td>
</td><td>
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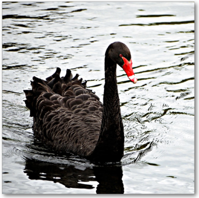
What About Black Swans?



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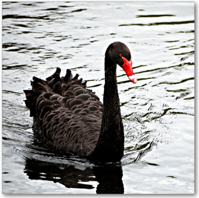

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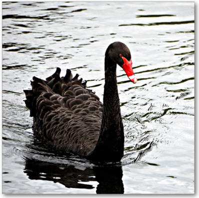

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- Need for macroscopically detectable indicators of sudden changes
- Identify early-warning signals preceding radicalization

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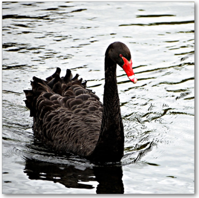

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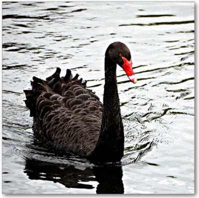
Example

Let $\tilde{\mathbf{f}}$ be a phenomenologically guessed/expected asymptotic distribution:

$$d_{\mathsf{BS}}(t) := \|\tilde{\mathbf{f}} - \mathbf{f}(t)\|_{\mathbb{R}^{n \times m}}$$



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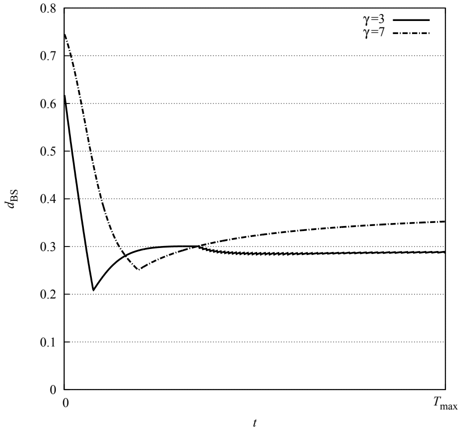
What About Black Swans?



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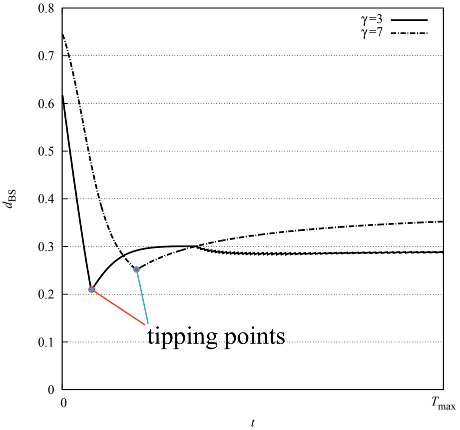

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