Emergence of flocking and consensus

Sébastien Motsch

Arizona State University

In collaboration with:

- *Pierre-Emmanuel Jabin, Eitan Tadmor* (CSCAMM, univ. Maryland)
- *Alexander Reamy, Ryan Theisen* (ASU), *GuanLin Li* (Georgia Tech)

Partially supported by NSF grant (DMS-1515592).

Transport phenomena in collective dynamics, ETH Zürich

Outline

Outline

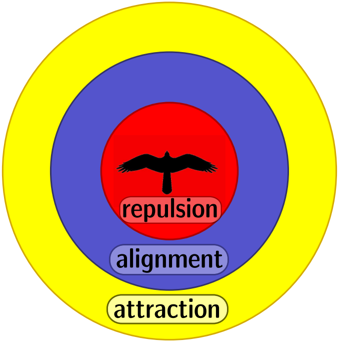
Flocking & Consensus

Flocking and consensus are typical collective behaviors.
They result from **long-term social-interactions**.

Open questions:

- What are the social-interactions? *(inverse problem)*
- Given the rules of interactions, will a flock/consensus emerge? *(direct problem)*

Introduction



Ref.: *Aoki, Huth & Wessel, Reynolds, Couzin...*

Comparison experimental data

- pattern formation (e.g. vortex)
- Bayesian statistics

Ref.: Deneubourg, Theraulaz, Giardina....

Convergence to equilibrium/stability

- analytic study
- energy estimate

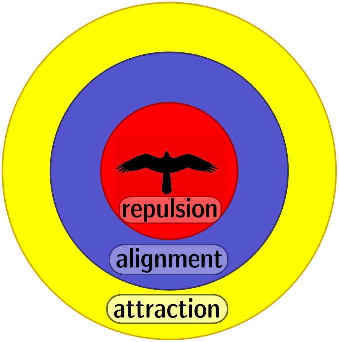
Ref.: Bertozzi, Carrillo, Raoul, Fetecau...

Macroscopic models

- statistical physics
- kinetic equation

Ref.: Degond, Peurichard, Klar, Haskovec...

Introduction



Ref.: *Aoki, Huth & Wessel, Reynolds, Couzin...*

Comparison experimental data

- pattern formation (e.g. vortex)
- Bayesian statistics

Ref.: Deneubourg, Theraulaz, Giardina....

Convergence to equilibrium/stability

- analytic study
- energy estimate

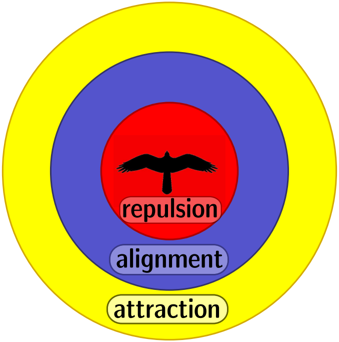
Ref.: Bertozzi, Carrillo, Raoul, Fetecau...

Macroscopic models

- statistical physics
- kinetic equation

Ref.: Degond, Peurichard, Klar, Haskovec...

Introduction



Ref.: *Aoki, Huth & Wessel, Reynolds, Couzin...*

Comparison experimental data

- pattern formation (e.g. vortex)
- Bayesian statistics

Ref.: Deneubourg, Theraulaz, Giardina....

Convergence to equilibrium/stability

- analytic study
- energy estimate

Ref.: Bertozzi, Carrillo, Raoul, Fetecau...

Macroscopic models

- statistical physics
- kinetic equation

Ref.: Degond, Peurichard, Klar, Haskovec...

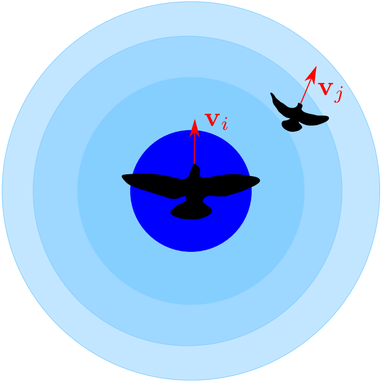
Outline

Cucker-Smale model

N agents (x_i, v_i):

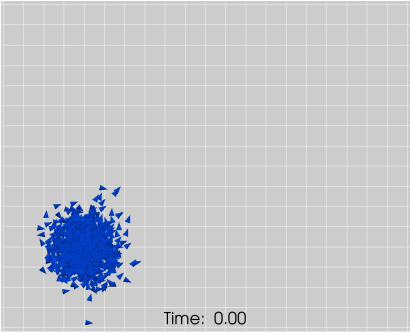
$$\dot{x}_i = v_i,$$
$$\dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \phi_{ij}(v_j - v_i)$$

where $\phi_{ij} = \phi(|x_j - x_i|)$ is the influence function (ϕ decays).

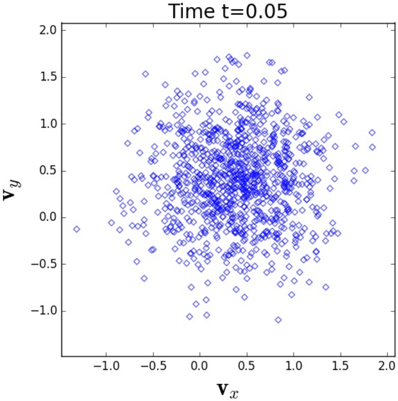


Numerical example

Evolution of the positions x_i



Evolution of the velocities v_i



Energy estimate:

$$\mathcal{H} = \frac{1}{2N^2} \sum_{i,j} |v_j - v_i|^2 \qquad \textit{(kinetic energy)}$$

Using the symmetry $\phi_{ij} = \phi_{ji}$ (e.g. conservation of mean velocity):

$$\frac{d\mathcal{H}}{dt} = -\frac{1}{2N^2} \sum_{i,j} \phi_{ij} |v_j - v_i|^2 \leq -\phi(\max_{ij} |x_i - x_j|) \cdot \mathcal{H}.$$

Energy estimate:

$$\mathcal{H} = \frac{1}{2N^2}\sum_{i,j}|v_j - v_i|^2 \qquad \textit{(kinetic energy)}$$

Using the symmetry $\phi_{ij} = \phi_{ji}$ (e.g. conservation of mean velocity):

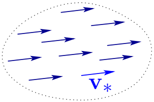
$$\frac{d\mathcal{H}}{dt} = -\frac{1}{2N^2}\sum_{i,j}\phi_{ij}|v_j - v_i|^2 \leq -\phi(\max_{ij}|x_i - x_j|)\cdot\mathcal{H}.$$

Theorem

If the influence function ϕ decays slowly enough, $\int_0^\infty \phi(r)\,dr = +\infty$, then the dynamics converges to a **flock**.

Proof. *Gronwall lemma + linearly growth of* $|x_i - x_j|$*:*

$$\Rightarrow v_i(t) \xrightarrow{t\to\infty} v_* \quad \text{for all} \quad i$$

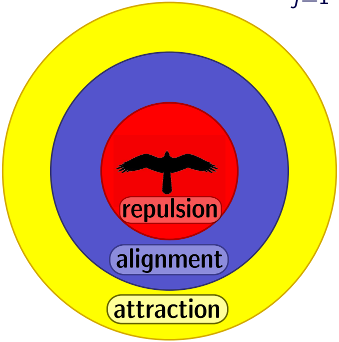


A flock

Ref. Cucker-Smale ('07), Ha-Tadmor ('08), Carrillo-Fornasier-Rosado-Toscani ('09), Ha-Liu ('09)...

3-zones model

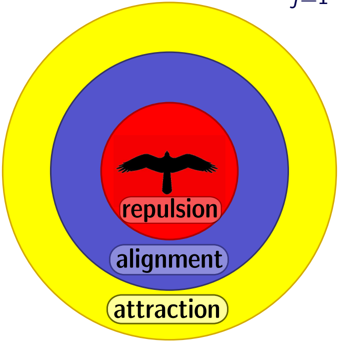
$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \phi_{ij}(v_j - v_i)$$



3-zones model

$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \phi_{ij}(v_j - v_i) - \frac{1}{N}\sum_{j\neq i} \nabla_{x_i} V(|\mathbf{x}_j - \mathbf{x}_i|)$$

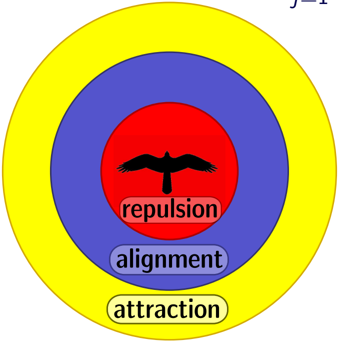
with $V(r)$ potential.

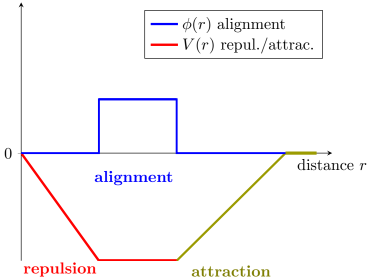


3-zones model

$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \overbrace{\phi_{ij}(v_j - v_i)}^{\text{alignment}} - \frac{1}{N}\sum_{j\neq i} \overbrace{\nabla_{x_i} V(|\mathbf{x}_j - \mathbf{x}_i|)}^{repul./attrac.}$$

with $V(r)$ potential.

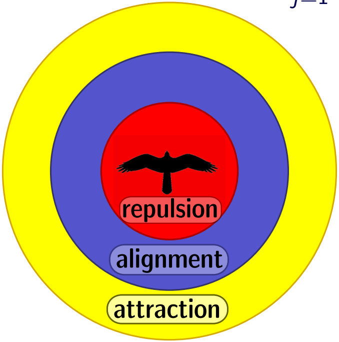


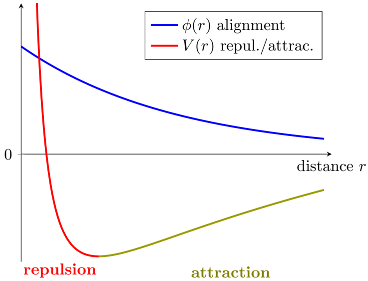


3-zones model

$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \overbrace{\phi_{ij}(v_j - v_i)}^{\text{alignment}} - \frac{1}{N}\sum_{j\neq i} \overbrace{\nabla_{x_i} V(|\mathbf{x}_j - \mathbf{x}_i|)}^{repul./attrac.}$$

with $V(r)$ potential.

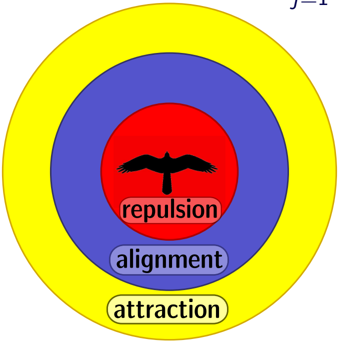


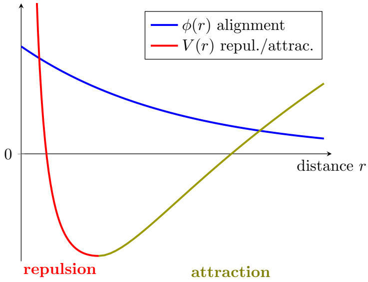


3-zones model

$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \overbrace{\phi_{ij}(v_j - v_i)}^{\text{alignment}} - \frac{1}{N}\sum_{j\neq i} \overbrace{\nabla_{x_i} V(|\mathbf{x}_j - \mathbf{x}_i|)}^{repul./attrac.}$$

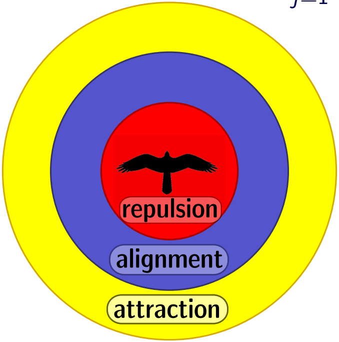
with $V(r)$ potential.





3-zones model

$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \overbrace{\phi_{ij}(v_j - v_i)}^{\text{alignment}} - \frac{1}{N}\sum_{j\neq i} \overbrace{\nabla_{x_i} V(|\mathbf{x}_j - \mathbf{x}_i|)}^{repul./attrac.}$$



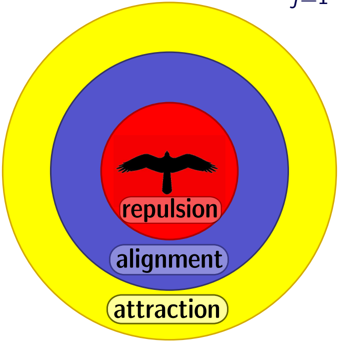
with $V(r)$ potential.

Energy: *(kinetic + potential)*

$$\mathcal{H} = \frac{1}{2N^2}\sum_{i,j}|v_j - v_i|^2 + \frac{1}{2N^2}\sum_{j\neq i} V(|x_j - x_i|)$$

3-zones model

$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \overbrace{\phi_{ij}(v_j - v_i)}^{\text{alignment}} - \frac{1}{N}\sum_{j\neq i} \overbrace{\nabla_{x_i} V(|\mathbf{x}_j - \mathbf{x}_i|)}^{repul./attrac.}$$



with $V(r)$ potential.

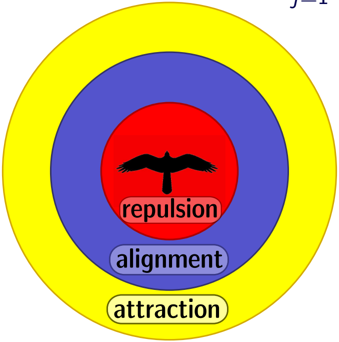
Energy: *(kinetic + potential)*

$$\mathcal{H} = \frac{1}{2N^2}\sum_{i,j}|v_j - v_i|^2 + \frac{1}{2N^2}\sum_{j\neq i} V(|x_j - x_i|)$$

$$\Rightarrow \frac{d\mathcal{H}}{dt} = -\frac{1}{2N^2}\sum_{i,j}\phi_{ij}|v_j - v_i|^2 + 0$$

3-zones model

$$\dot{x}_i = v_i \quad , \quad \dot{v}_i = \frac{1}{N}\sum_{j=1}^{N} \overbrace{\phi_{ij}(v_j - v_i)}^{\text{alignment}} - \frac{1}{N}\sum_{j\neq i} \overbrace{\nabla_{x_i} V(|\mathbf{x}_j - \mathbf{x}_i|)}^{repul./attrac.}$$



with $V(r)$ potential.

Energy: *(kinetic + potential)*

$$\mathcal{H} = \frac{1}{2N^2}\sum_{i,j}|v_j - v_i|^2 + \frac{1}{2N^2}\sum_{j\neq i} V(|x_j - x_i|)$$

$$\Rightarrow \frac{d\mathcal{H}}{dt} = -\frac{1}{2N^2}\sum_{i,j}\phi_{ij}|v_j - v_i|^2 + 0$$

Theorem [Reamy, M, Theisen]

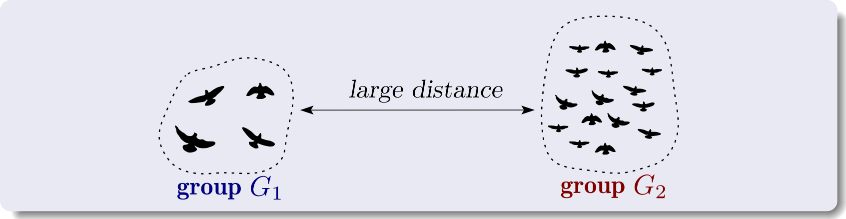
If $\phi > 0$ and $V(r) \stackrel{r\to+\infty}{\longrightarrow} +\infty$ (confinement potential), then the dynamics converges to a **flock**.

Drawback of the normalization $1/N$

In the "small" group G_1 alone:

$$\dot{v}_i = \frac{1}{N_1} \sum_{j=1}^{N_1} \phi_{ij}(v_j - v_i)$$

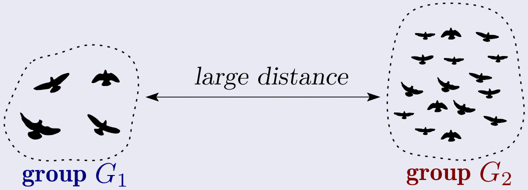
Drawback of the normalization $1/N$



In the "small" group G_1 with the "large" group G_2:

$$\dot{v}_i = \frac{1}{N_1 + N_2} \sum_{j=1}^{N_1+N_2} \phi_{ij}(v_j - v_i)$$

Drawback of the normalization $1/N$



In the "small" group G_1 with the "large" group G_2:

$$\dot{v}_i = \frac{1}{N_1 + N_2} \sum_{j=1}^{N_1+N_2} \phi_{ij}(v_j - v_i) \approx \frac{1}{N_1 + N_2} \sum_{j=1}^{N_1} \phi_{ij}(v_j - v_i) \approx 0!$$

We propose the following dynamical system:

$$\dot{x}_i = v_i, \qquad \dot{v}_i = \frac{1}{\sum_{k=1}^{N} \phi_{ik}} \sum_{j=1}^{N} \phi_{ij} \left(v_j - v_i\right),$$

We weight by **the total influence** $\sum_{k=1}^{N} \phi_{ik}$ rather than N.

We propose the following dynamical system:

$$\dot{x}_i = v_i, \qquad \dot{v}_i = \sum_{j=1}^{N} a_{ij}(v_j - v_i),$$

with $a_{ij} = \frac{\phi_{ij}}{\sum_{k=1}^{N} \phi_{ik}}$, $A = [a_{ij}]$ stochastic matrix ($\sum_j a_{ij} = 1$).

We weight by **the total influence** $\sum_{k=1}^{N} \phi_{ik}$ rather than N.

We propose the following dynamical system:

$$\dot{x}_i = v_i, \qquad \dot{v}_i = \sum_{j=1}^{N} a_{ij}(v_j - v_i),$$

with $a_{ij} = \frac{\phi_{ij}}{\sum_{k=1}^{N} \phi_{ik}}$, $A = [a_{ij}]$ stochastic matrix $(\sum_j a_{ij} = 1)$.

We weight by **the total influence** $\sum_{k=1}^{N} \phi_{ik}$ rather than N.

Consequences: ◦ non-symmetric interaction: $a_{ij} \neq a_{ji}$

◦ momentum $\overline{v}$ **not preserved**: $\frac{d}{dt}\overline{v} \neq 0$.

Question: *Can we prove flocking for this dynamics?*

Flocking: ℓ^∞ approach

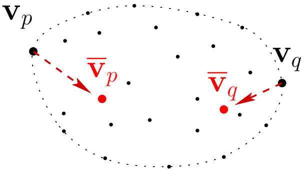
- **Trick:** $\dot{v}_i = \sum_j a_{ij}(v_j - v_i)$

Flocking: ℓ^∞ approach

- **Trick:** $\dot{v}_i = (\quad \overline{v}_i \quad - \; v_i)$ with $\overline{v}_i = \sum_j a_{ij} v_j$

Flocking: ℓ^∞ approach

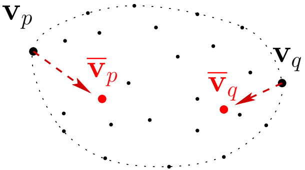
- **Trick:** $\dot{v}_i = (\quad \overline{v}_i \quad - \; v_i)$ with $\overline{v}_i = \sum_j a_{ij} v_j$



Flocking: ℓ^∞ approach

- **Trick:** $\dot{v}_i = (\quad \overline{v}_i \quad - \; v_i)$ with $\overline{v}_i = \sum_j a_{ij} v_j$
- Let $[\mathbf{v}] = \max_{p,q} |v_p - v_q|$ the velocity *diameter*

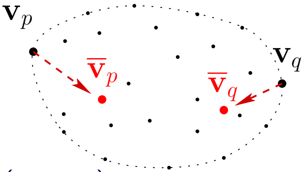
$$\frac{d}{dt}[\mathbf{v}] \;\leq\; |\overline{v}_p - \overline{v}_q| - [\mathbf{v}] \;\leq\; 0.$$



Flocking: ℓ^∞ approach

- **Trick:** $\dot{v}_i = (\quad \overline{v}_i \quad - \; v_i)$ with $\overline{v}_i = \sum_j a_{ij} v_j$
- Let $[\mathbf{v}] = \max_{p,q} |v_p - v_q|$ the velocity *diameter*

$$\frac{d}{dt}[\mathbf{v}] \;\leq\; |\overline{v}_p - \overline{v}_q| - [\mathbf{v}] \;\leq\; 0.$$



- **Lemma.** Let A stochastic matrix, then

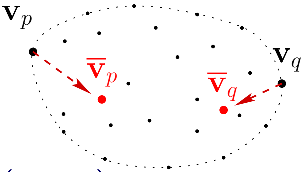
$$[A\mathbf{v}] \leq (1-\lambda)[\mathbf{v}], \quad \lambda = \min_{p,q} \sum_i \min(a_{pi}, a_{qi}).$$

λ is a measure of the **connectivity** of A.

Flocking: ℓ^∞ approach

- **Trick:** $\dot{v}_i = (\quad \overline{v}_i \quad - \quad v_i)$ with $\overline{v}_i = \sum_j a_{ij} v_j$
- Let $[\mathbf{v}] = \max_{p,q} |v_p - v_q|$ the velocity *diameter*

$$\frac{d}{dt}[\mathbf{v}] \;\leq\; |\overline{v}_p - \overline{v}_q| - [\mathbf{v}] \;\leq\; 0.$$



- **Lemma.** Let A stochastic matrix, then

$$[A\mathbf{v}] \leq (1-\lambda)[\mathbf{v}], \quad \lambda = \min_{p,q} \sum_i \min(a_{pi}, a_{qi}).$$

λ is a measure of the **connectivity** of A.

- Here, $\lambda \geq \phi([\mathbf{x}])$, where $[\mathbf{x}]$ is the diameter of positions. Thus,

$$\frac{d}{dt}[\mathbf{x}] \leq [\mathbf{v}] \quad , \quad \frac{d}{dt}[\mathbf{v}] \leq -\phi([\mathbf{x}])[\mathbf{v}].$$

Flocking: non-symmetric interactions

Using a Lyapunov functional (Ha-Liu), we deduce:

Theorem [M,Tadmor]

If the influence function ϕ decays slowly enough, $\int_0^\infty \phi(r)\,dr = +\infty$, then the dynamics converges to a **flock**.

Remarks.

- Extensions for various non-symmetric model
  $\Rightarrow$ *add leaders*
- The asymptotic velocity v_* is unknown:
  $\Rightarrow$ *emergent quantity*

Flocking: non-symmetric interactions

Using a Lyapunov functional (Ha-Liu), we deduce:

Theorem [M,Tadmor]

If the influence function ϕ decays slowly enough, $\int_0^\infty \phi(r)\,dr = +\infty$, then the dynamics converges to a **flock**.

Remarks.

- Extensions for various non-symmetric model
  $\Rightarrow$ *add leaders*
- The asymptotic velocity v_* is unknown:
  $\Rightarrow$ *emergent quantity*
- We need long-range interaction
  $\mathrm{Supp}(\phi) = [0, +\infty)$

Flocking: non-symmetric interactions

Using a Lyapunov functional (Ha-Liu), we deduce:

Theorem [M,Tadmor]

If the influence function ϕ decays slowly enough, $\int_0^\infty \phi(r)\,dr = +\infty$, then the dynamics converges to a **flock**.

Remarks.

- Extensions for various non-symmetric model
  $\Rightarrow$ *add leaders*
- The asymptotic velocity v_* is unknown:
  $\Rightarrow$ *emergent quantity*
- We need long-range interaction
  $\text{Supp}(\phi) = [0, +\infty)$
- Extension to kinetic equation:
  Ref.: *Karper-Mellet-Trivisa, Kang-Vasseur...*

Outline

Consensus model

Opinions are represented by a vector $x_i \in \mathbb{R}^d$

$$\dot{x}_i = \sum_j a_{ij}(x_j - x_i), \qquad a_i = \frac{\phi_{ij}}{\sum_k \phi_{ik}},$$

with $\phi_{ij} = \phi(|x_j - x_i|^2)$ and ϕ has a **compact support** in $[0, 1]$.

Consensus model

Opinions are represented by a vector $x_i \in \mathbb{R}^d$

$$\dot{x}_i = \sum_j a_{ij}(x_j - x_i), \qquad a_i = \frac{\phi_{ij}}{\sum_k \phi_{ik}},$$

with $\phi_{ij} = \phi(|x_j - x_i|^2)$ and ϕ has a **compact support** in $[0, 1]$.

Discretization: $$\boxed{x_i^{n+1} = \frac{\sum_j \phi_{ij} x_j^n}{\sum_k \phi_{ik}}}$$

Hegselmann-Krause model.

Ref. Blondel, Hendricks, Tsitsiklis...

Consensus model

Opinions are represented by a vector $x_i \in \mathbb{R}^d$

$$\dot{x}_i = \sum_j a_{ij}(x_j - x_i), \qquad a_i = \frac{\phi_{ij}}{\sum_k \phi_{ik}},$$

with $\phi_{ij} = \phi(|x_j - x_i|^2)$ and ϕ has a **compact support** in $[0, 1]$.

Discretization: $$x_i^{n+1} = \frac{\sum_j \phi_{ij} x_j^n}{\sum_k \phi_{ik}}$$ *Hegselmann-Krause model.*

Ref. Blondel, Hendricks, Tsitsiklis...

Question I: *do we have formation of consensus?*

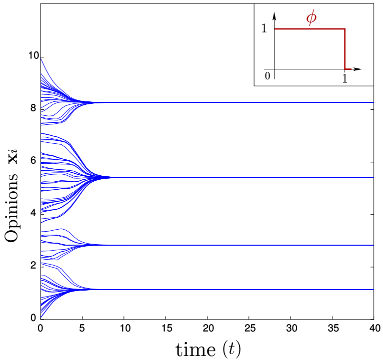
$$x_i(t) \stackrel{t\to\infty}{\longrightarrow} x_*.$$

Short answers:

- yes *if* $|x_i(0) - x_j(0)| < 1$ for all i, j ($\Rightarrow$ global interaction)
- otherwise, it depends...

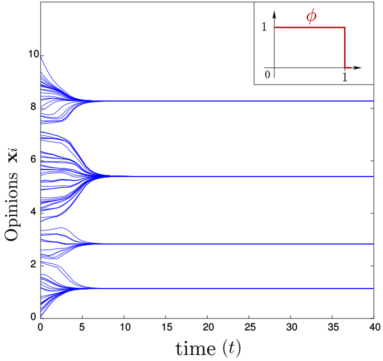
Numerical examples

Simulation 1D

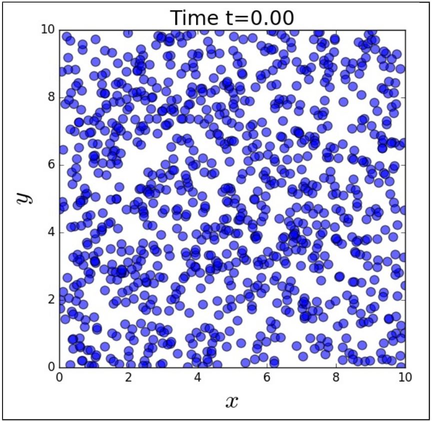


Numerical examples

Simulation 1D



Simulation 2D

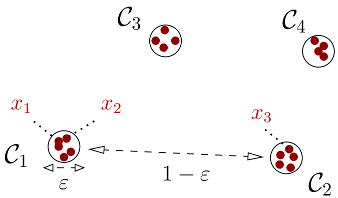


Convergence to a stationary state

We observe the formation of **clusters**.

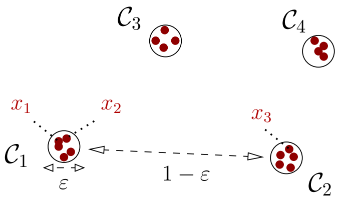
Question II: *do the dynamics always converge? i.e.*

$$x_i(t) \xrightarrow{t\to\infty} \overline{x}_i.$$



Convergence to a stationary state

We observe the formation of **clusters**.



Question II: *do the dynamics always converge? i.e.*

$$x_i(t) \xrightarrow{t\to\infty} \overline{x}_i.$$

Theorem [Jabin, M]

Suppose the interaction function ϕ satisfies $|\phi'(r)|^2 \leq C\phi(r)$, then the dynamics converges.

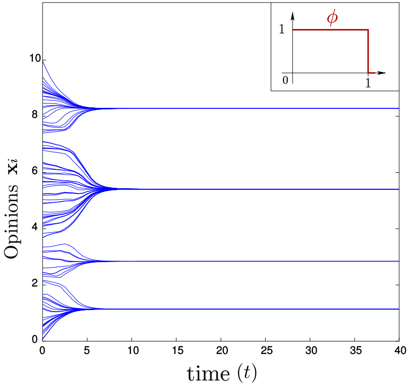
Consensus formation

Question III: *how can we 'enhance' consensus formation?*
Which interaction function ϕ is more likely to lead to a consensus?

Consensus formation

Question III: *how can we 'enhance' consensus formation?*
Which interaction function ϕ is more likely to lead to a consensus?

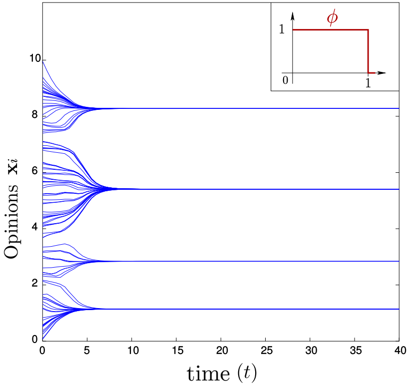
We investigate several influence functions ϕ.

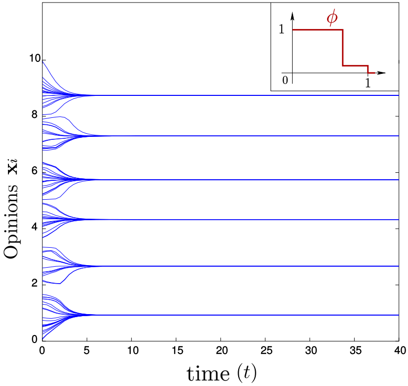


Consensus formation

Question III: *how can we 'enhance' consensus formation?*
Which interaction function ϕ is more likely to lead to a consensus?

We investigate several influence functions ϕ.

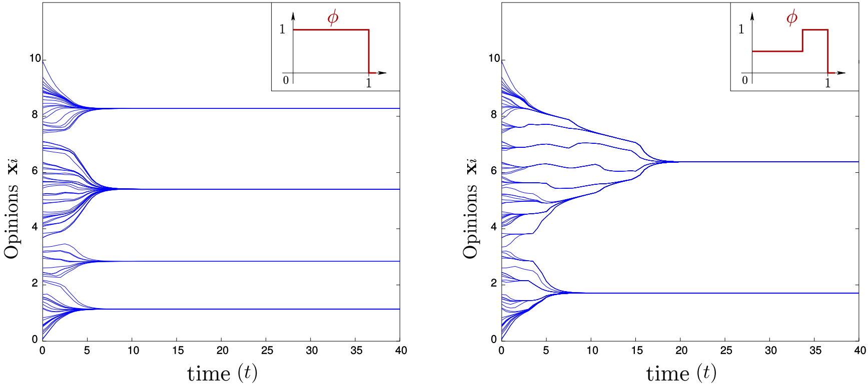




Consensus formation

Question III: *how can we 'enhance' consensus formation?*
Which interaction function ϕ is more likely to lead to a consensus?

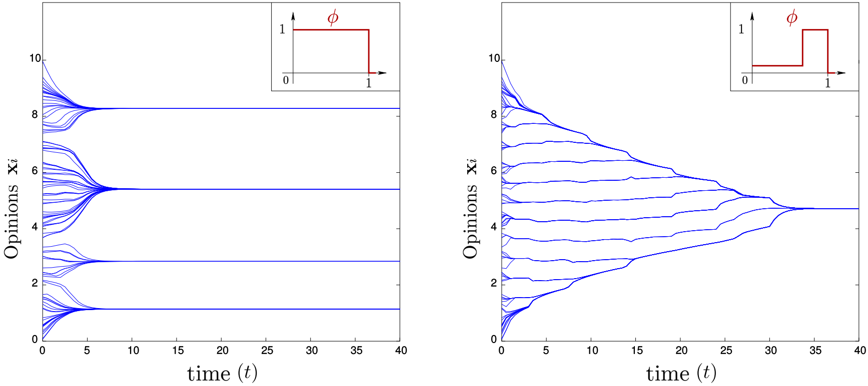
We investigate several influence functions ϕ.



Consensus formation

Question III: *how can we 'enhance' consensus formation?*
Which interaction function ϕ is more likely to lead to a consensus?

We investigate several influence functions ϕ.



Key Observation:

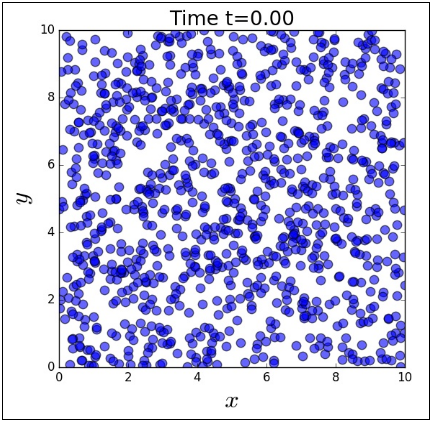
the stronger the influenced of 'close' neighbors,
the less likely a consensus will form.

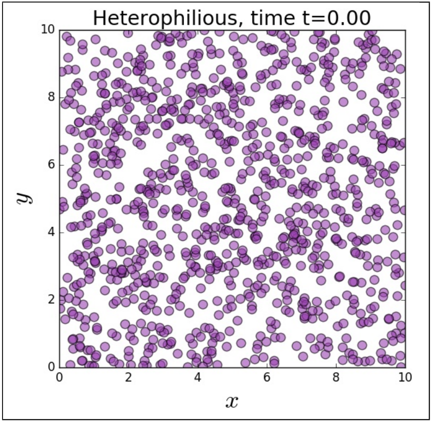
$\Rightarrow$ **heterophily** (*love of the different*) enhances consensus.

Key Observation:

the stronger the influenced of 'close' neighbors,
the less likely a consensus will form.

$\Rightarrow$ **heterophily** (*love of the different*) enhances consensus.





Heterophilious dynamics

Analytic study is challenging

$\Rightarrow$ *trace the connectivity of the graph A (e.g. eigenvalues)*

Simplified model: nearest-neighbor interactions

$$\dot{x}_i = \sum_{i-1,i+1} \phi_{ij}(x_j - x_i).$$

Theorem [M,Tadmor]

if ϕ increases (on its support) then the connectivity is preserved:

$\Rightarrow$ if $\{x_i(0)\}_i$ connected, then it converges to a **consensus**.

Proof. Let $\Delta_i = x_{i+1} - x_i$ and $\Delta_p = \max_i \Delta_i$:

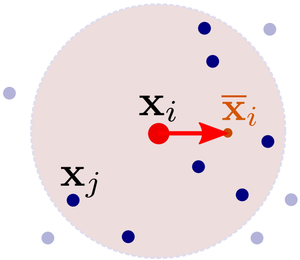
$$\frac{d}{dt}|\Delta_p|^2 \leq \left(\phi_{(p-1)p} - 2\phi_{p(p+1)} + \phi_{(p+1)(p+2)}\right)|\Delta_p|^2 \leq 0. \qquad \square$$

"*No-one left behind*" dynamics

Consensus dynamics

$$\dot{x}_i = \overline{x}_i - x_i$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$



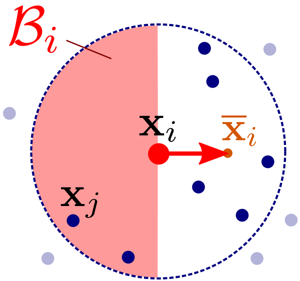
"No-one left behind" dynamics

Consensus dyn. *no-one left behind*

$$\dot{x}_i = \mu_i(\overline{x}_i - x_i)$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

$$\mu_i = \begin{cases} 0 & \text{if } x_j \in B_i \\ 1 & \text{otherwise} \end{cases}$$



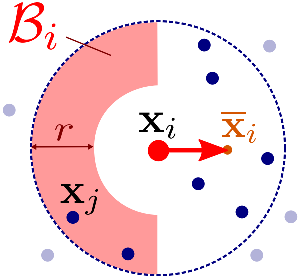
"*No-one left behind*" dynamics

Consensus dyn. *no-one left behind*

$$\dot{x}_i = \mu_i(\overline{x}_i - x_i)$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

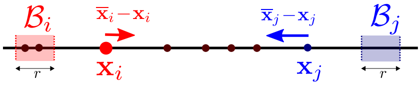
$$\mu_i = \begin{cases} 0 & \text{if } x_j \in B_i \\ 1 & \text{otherwise} \end{cases}$$



"*No-one left behind*" dynamics

Consensus dyn. *no-one left behind*

$$\dot{x}_i = \mu_i(\overline{x}_i - x_i)$$



with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

$$\mu_i = \begin{cases} 0 & \text{if } x_j \in B_i \\ 1 & \text{otherwise} \end{cases}$$

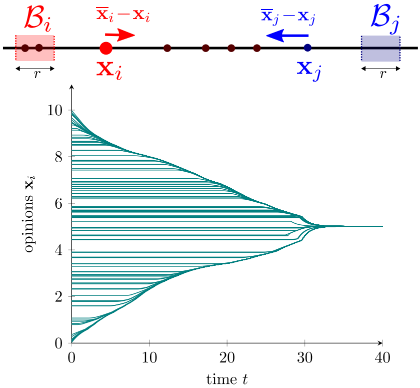
"*No-one left behind*" dynamics

Consensus dyn. *no-one left behind*

$$\dot{x}_i = \mu_i(\overline{x}_i - x_i)$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

$$\mu_i = \begin{cases} 0 & \text{if } x_j \in B_i \\ 1 & \text{otherwise} \end{cases}$$



Theorem [Li, M]

In $\mathbb{R}^1$, if $\{x_i(0)\}_i$ connected, then $[\mathbf{x}](t) \xrightarrow{t\to+\infty} 0$ with explicit decay.

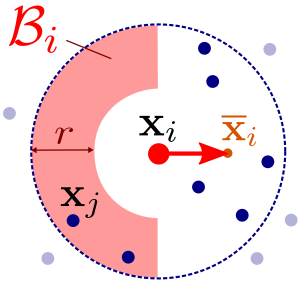
"No-one left behind" dynamics

Consensus dyn. *no-one left behind*

$$\dot{x}_i = \mu_i(\overline{x}_i - x_i)$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

$$\mu_i = \begin{cases} 0 & \text{if } x_j \in B_i \\ 1 & \text{otherwise} \end{cases}$$



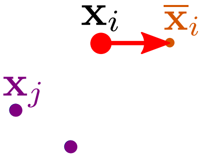
"*No-one left behind*" dynamics

Consensus dyn. *no-one left behind*

$$\dot{x}_i = \mu_i(\overline{x}_i - x_i)$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

$$\mu_i = \begin{cases} 0 & \text{if } x_j \in B_i \\ 1 & \text{otherwise} \end{cases}$$



"*No-one left behind*" dynamics

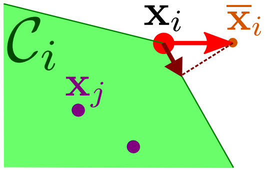
Consensus dyn. *no-one left behind*

$$\dot{x}_i = P_{\mathcal{C}_i}(\overline{x}_i - x_i)$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

$P_{\mathcal{C}_i}$ orthogonal projection on

$\mathcal{C}_i = \{v \mid \langle v, x_j - x_i \rangle \geq 0,\ \forall x_j \in \mathcal{B}_i\}$.



"No-one left behind" dynamics

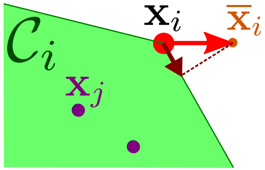
Consensus dyn. *no-one left behind*

$$\dot{x}_i = P_{\mathcal{C}_i}(\overline{x}_i - x_i)$$

with $\overline{x}_i = \sum_j a_{ij}(x_j - x_i)$ and

$P_{\mathcal{C}_i}$ orthogonal projection on

$\mathcal{C}_i = \{v \mid \langle v, x_j - x_i \rangle \geq 0, \forall x_j \in \mathcal{B}_i\}$.



Theorem [Li, M]

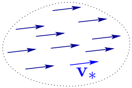
In $\mathbb{R}^d$, if $\{x_i(0)\}_i$ connected, then $[\mathbf{x}](t) \xrightarrow{t\to+\infty} 0$ ~~with explicit decay~~.

Outline

Summary/Perspectives

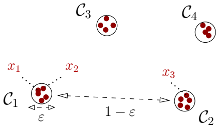
Summary

- **Large time behavior** for model of **flocking**
  - ⇒ flocking for the 3-zones model
  - ⇒ method for non-symmetric models



A flock

- **Opinion formation**: cluster and consensus
  - ⇒ convergence to cluster formation
  - ⇒ *enhancing* consensus (*"heterophilia"*)
  - ⇒ *enforcing* consensus (*"no-one left behind"*)



Perspectives

- *Control* the dynamics
  - ⇒ *M. Caponigro, N. Pouradier-Duteil, B. Piccoli...*
- Mixing behaviors (heterogeneity)
  - ⇒ *Daniel Weser*