Stable Swarming Using Adaptive Long-range Interactions

Dan Gorbonos

Based on:

- DG, Reuven Ianconescu , James Puckett, Rui Ni, Nicholas T. Ouellette and Nir S. Gov, NJP, 2016

- DG and Nir S. Gov, in preparation

Transport phenomena in collective dynamics: from micro to social hydrodynamics
03/11/16 ETH Zürich

מכון ויצמן למדע
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Outline

- Motivation
  - The midges and the swarm

  - The adaptive gravity model

- Stable Swarming with Adaptivity
  - Adaptivity as a self-stabilization mechanism
  - Jeans Instability

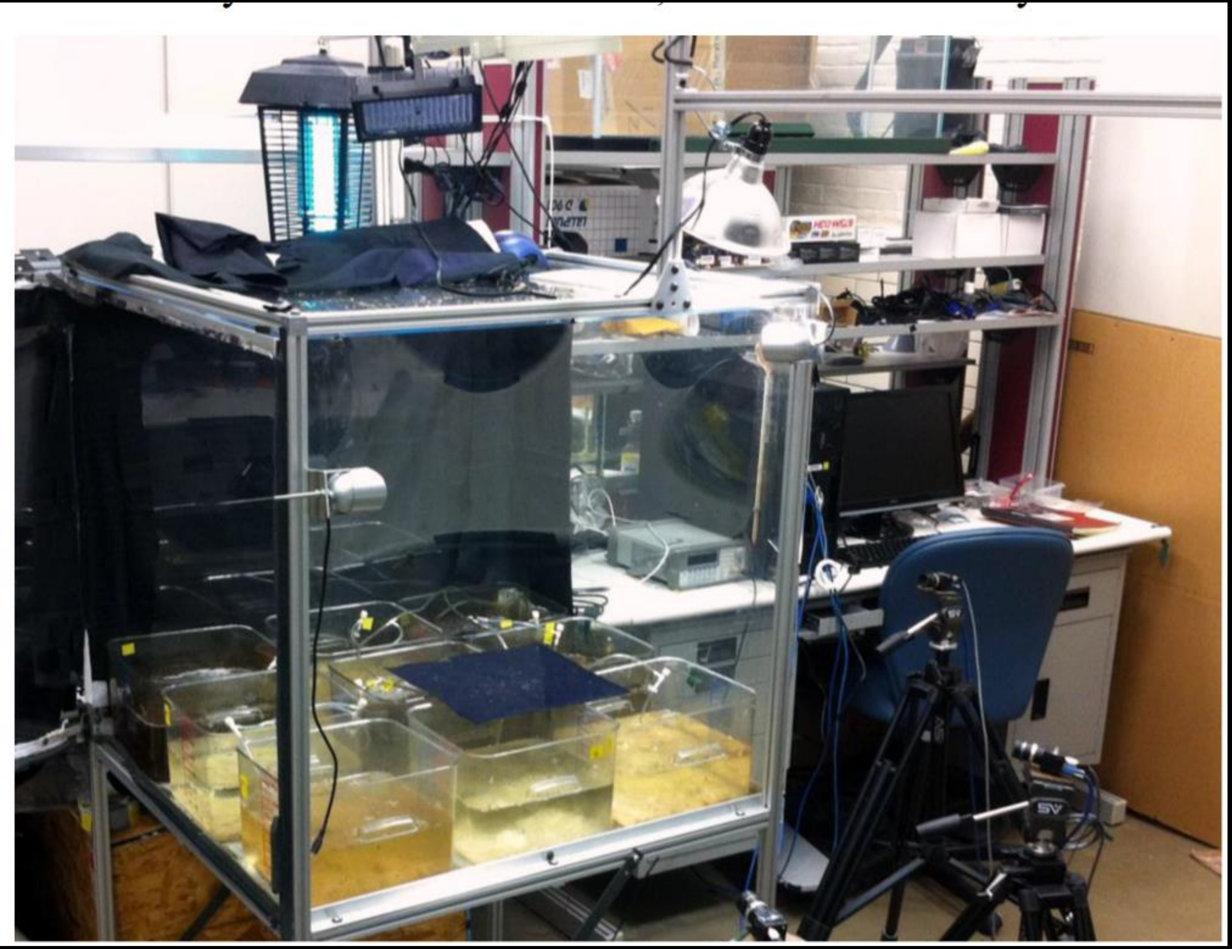
- Conclusions

The midges and the swarm

Chironomid Midge Fly Swarm
(Small White Spots in Image)

The midges and the swarm

In the lab (Stanford U.) :



Nick Ouellette - PI

James Puckett

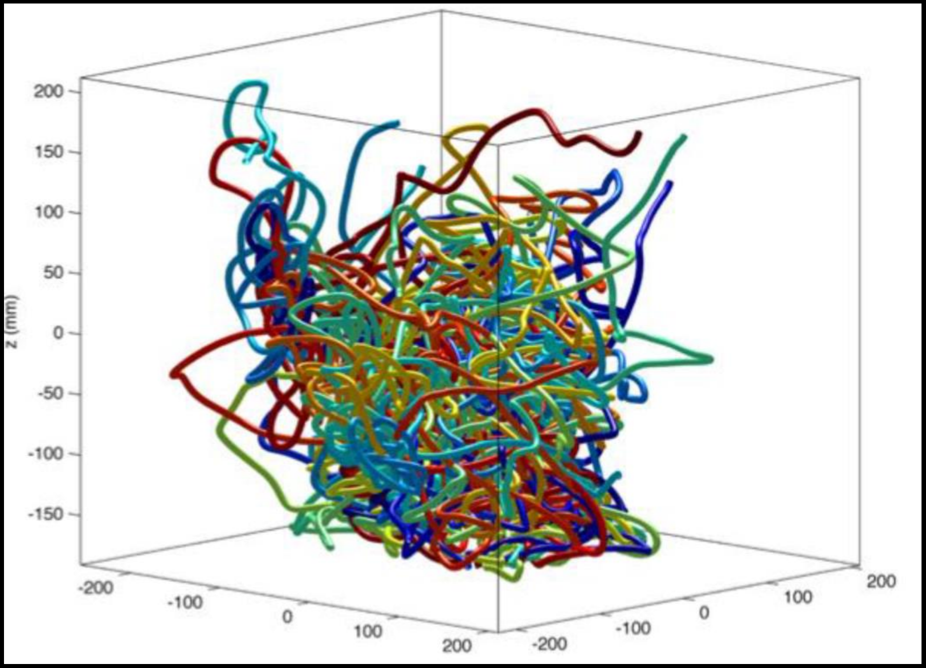
Rui Ni

The midges (Chironomidae)

- Non-biting midges
- Only male swarm (mating ritual)

	nature	lab
• How many ?	$10 - 10^4$	$1 - 100$
• Where ?	stream edges	Black felt “swarm markers”
• When ?	dawn and dusk	Overhead light source – ON/OFF

In the lab (Stanford U.) : Trajectories of midges vs. time



Method:

- High-speed stereo-imaging using three synchronized cameras (100 fps)
- Automated motion tracking algorithm

Measurement:

Kinematics –

$\vec{r}(t), \vec{v}(t), \vec{a}(t)$

The midges and the swarm

In the lab (Stanford U.) :

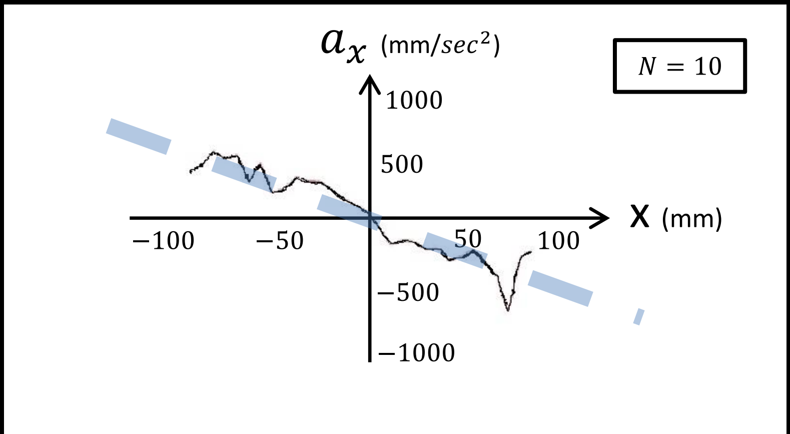
- Long-range Interaction ("force")
- Swarm in the dark
- Not influenced by chemical signals

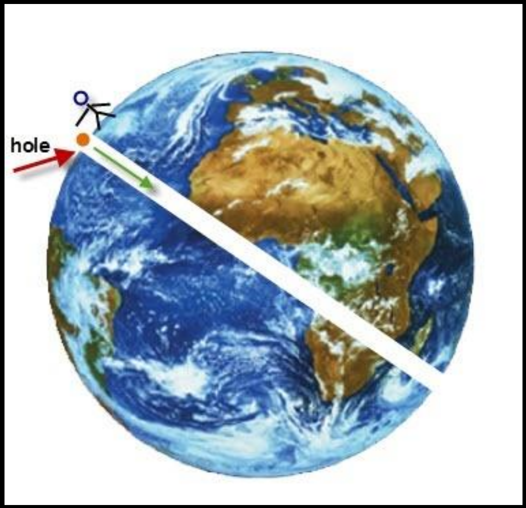
Nick Ouellette - PI

James Puckett (Post-doc)

Rui Ni (Post-doc)

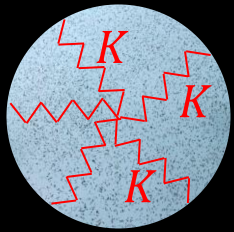
Isotropic Harmonic Oscillator





$$\sum \vec{F} = -K\vec{r}$$

Linear restoring force
– effective spring constant



Assumptions:

- **Long range interaction**
- **Pairwise interaction**
- **uniform density**
- **spherical symmetry**

The only possible force: $F \propto \frac{1}{r^2}$

$$\sum \vec{F} \propto \int \frac{d^3r}{r^2}\hat{r} \propto \vec{r}$$

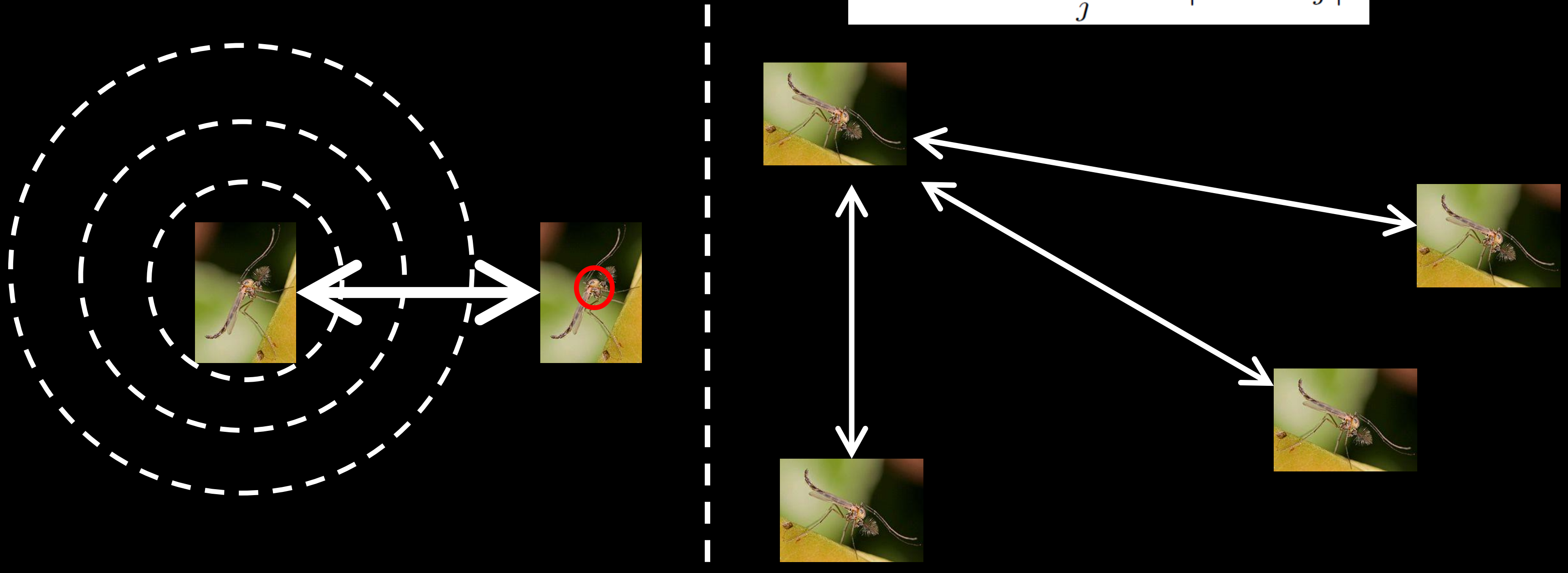
The Model

- Acoustic attraction – Johnston's organ
- Flight sound intensity decays as $\frac{1}{r^2}$

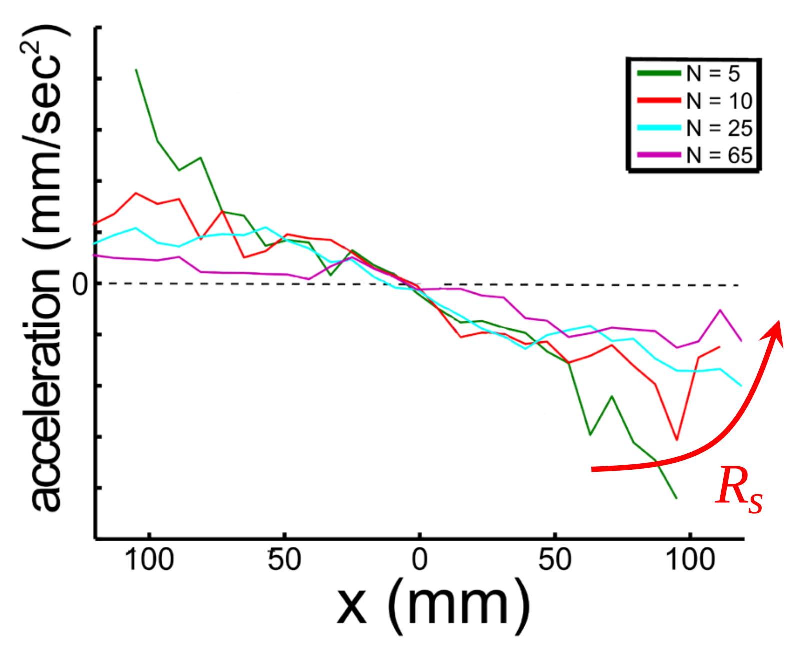
  → Acceleration towards the source $a \sim \frac{1}{r^2}$

  - "Acoustic Gravity"

$$\vec{F}^{i}_{eff} = C \sum_{j} \hat{r}_{ij} \frac{1}{|\vec{r}_i - \vec{r}_j|^2}$$



Another feature (in the lab):



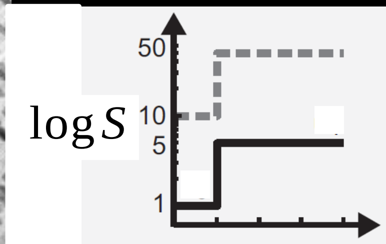
The linear force decreases for larger swarms

What is missing ?

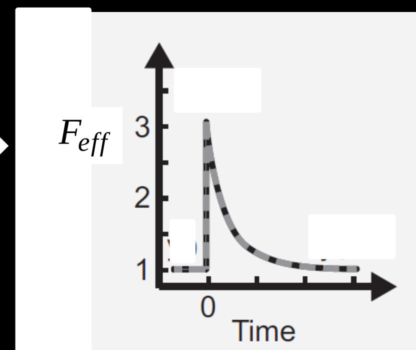
Adaptivity (as a part of the Fold Change Detection Mechanism)

- A typical feature of sensory systems

Input **Stimulus -** $\vec{S}$ → **Output** **response, effective force** $\vec{F}_{eff}$



Two step inputs with the same fold change →



Shoval et al. 2010

Scalar symmetry **For any two stimuli** $\vec{S}$ **and** $p \cdot \vec{S}\ (p > 0)$ **the output is the same**

Acoustic input → **Output**

$$\vec{s}_i = \sum_j \vec{s}_{ij}$$

$$\vec{F}^{\,i}_{eff} = \frac{\vec{s}_i}{\sum_j \left|\vec{s}_{ij}\right|}$$

Motsch & Tadmor 2011

$$\vec{F}_{eff}\,(p \cdot \vec{s}_{11,\dots,}\, p \cdot \vec{s}_{ij,\dots}) = \vec{F}_{eff}\,(\vec{s}_{11,\dots,}\vec{s}_{ij,\dots})$$

Sensitive to directionality but not to the overall amplitude !

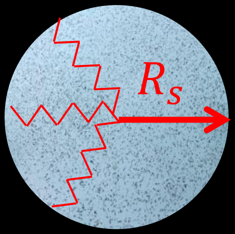
Isotropic Harmonic Oscillator

with adaptivity

$$\vec{F}^{\,i}_{eff} = C \frac{\sum_j \frac{\hat{r}_{ij}}{\left|\vec{r}_i - \vec{r}_j\right|}}{\sum_j \frac{1}{\left|\vec{r}_i - \vec{r}_j\right|}}$$

Uniform density
& spherical symmetry

$$\sum_j \frac{1}{\left|\vec{r}_i - \vec{r}_j\right|^2} \to \int^{R_s} \frac{d^3 r}{\left|\vec{r}_i - \vec{r}_j\right|^2} \sim R_s$$



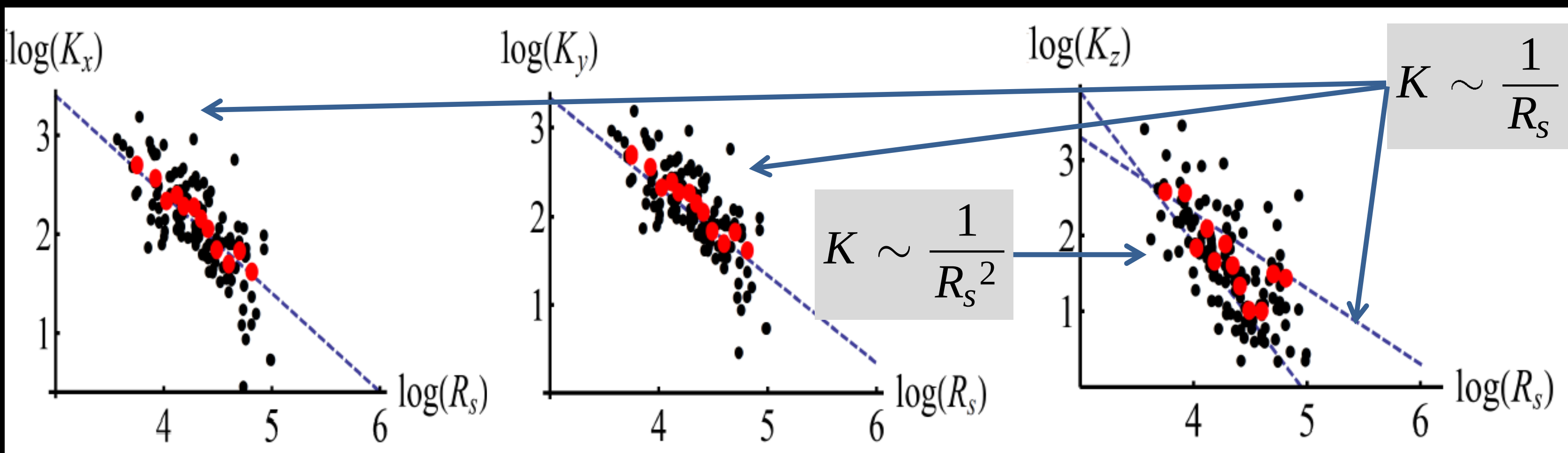
$$K \sim \frac{1}{R_s}$$

$$\vec{F} = K\vec{r}$$

effective spring constant

Adaptive Gravity – Evidence

Supported by data – 122 swarms !

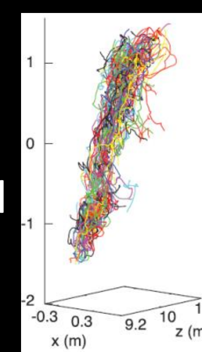


Black – raw data

Red – Binned average

Blue – (-1) slope (spherical) /(-2) slope (cylindrical)

Large swarms are elongated along the vertical axis



Dependence of The Effective Force on The Density (Uniform)

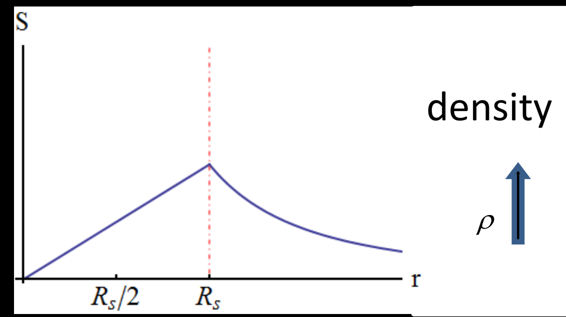
Regular gravity

$$\frac{1}{r^2}$$

$$\vec{s}_{ij} = C\frac{\hat{r}_{ij}}{r_{ij}^2}$$

$$\vec{s}_i = \sum_j \vec{s}_{ij}$$

At the center: $S(r) = -\frac{4\pi C\rho}{3}r$



Increasing density

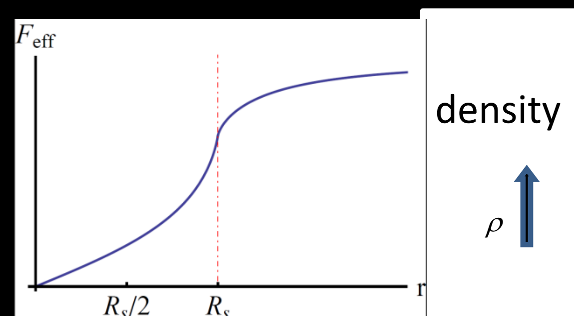
Stronger pull

Adaptive gravity

$$\frac{1}{r^2}$$

$$\vec{F}^i_{eff} = \frac{\vec{s}_i}{\sum_j |\vec{s}_{ij}|}$$

At the center: $F_{eff}(r) = -\frac{\tilde{C}}{3R_s}r + O(\frac{r^2}{R_s^2})$



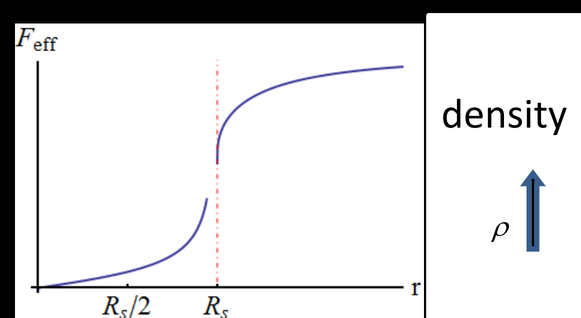
Marginal

Adaptive forces

$$\frac{1}{r^n} \quad (n > 2)$$

$$\vec{s}_{ij} = C\frac{\hat{r}_{ij}}{r_{ij}^n}$$

$$\vec{F}^i_{eff} = \frac{\vec{s}_i}{\sum_j |\vec{s}_{ij}|}$$



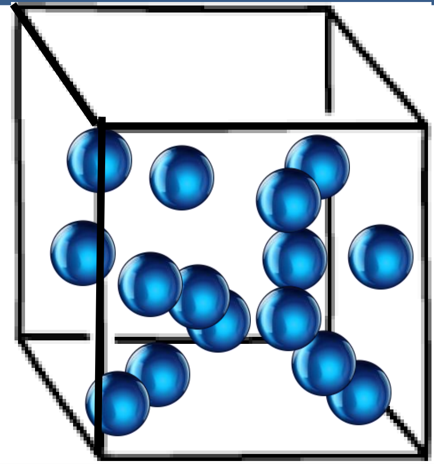
Increasing density

Weaker pull

At the center: $F_{eff}(r) = -\frac{2\tilde{C}}{R_s \ln(R_s{}^6\rho^2)}r + O(\frac{r^2}{R_s^2}) \quad n = 3$

$$F_{eff}(r) = -\frac{\tilde{C}|n-3|}{3R_s(\rho^{\frac{n-3}{3}}R_s{}^{n-3} - 1)}r + O(\frac{r^2}{R_s^2}) \quad n > 3$$

Jeans Instability (Gravity)



- Balance: gravitational pull ↔ random velocities
- $\rho > \rho^G_{Jeans}$ ⇨ collapse (minimal density for collapse)
- If $t_{esc} > t_{col}$

Escape time (random velocities)

$$t_{esc} = \frac{R_s}{\sqrt{\overline{v^2}}}$$

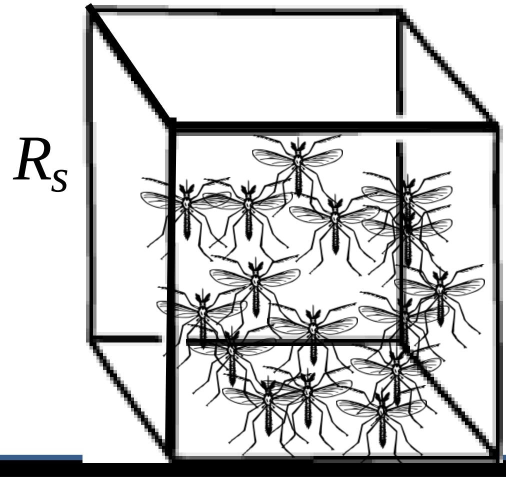
$$\vec{F} = -k\vec{r}$$

Time for collapse

$$t_{col} = \frac{\pi}{2\sqrt{k}}$$

$$k = \frac{4\pi C}{3}\rho \quad \Rightarrow \quad \rho^G_{Jeans} = \frac{3\pi \overline{v^2}}{16 R_s^2 C}$$

Jeans Instability (Adaptive Gravity)



- No critical density ρ^{G}_{Jeans} !

- If $t_{esc} > t_{col}$

Escape time (random velcoities)

$$t_{esc} = \frac{R_s}{\sqrt{\overline{v^2}}}$$

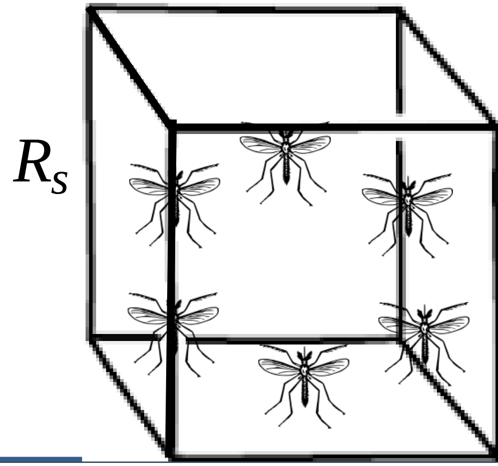
$$\vec{F} = -k\vec{r}$$

Time for collapse

$$t_{col} = \frac{\pi}{2\sqrt{k}}$$

$$k = \frac{\overline{C}}{3R_s} \quad \Rightarrow \quad \frac{4R_s\overline{C}}{3\pi^2} > \overline{v^2}$$

Jeans Instability (Adaptive Forces $\frac{1}{r^n}$ $(n > 2)$)



- Stabilization at a particular density ρ^A_{Jeans}
- If $t_{esc} > t_{col}$

Escape time (random velcoities)

$$t_{esc} = \frac{R_s}{\sqrt{\overline{\mathrm{v}^2}}}$$

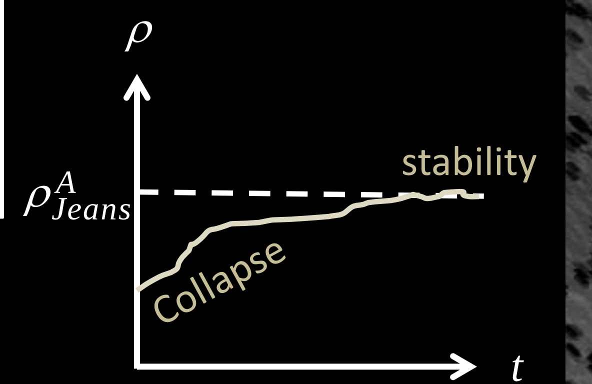
$$\vec{F} = -k\vec{r}$$

Time for collapse

$$t_{col} = \frac{\pi}{2\sqrt{k}}$$

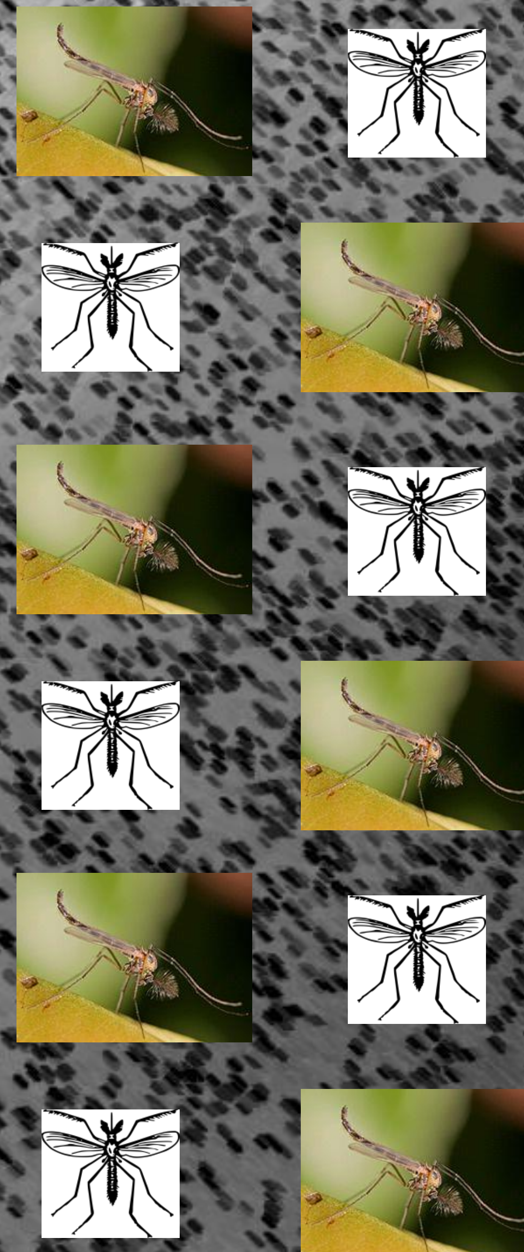
$$n = 3 \quad k = \frac{2\bar{C}}{R_s \ln(R_s{}^6 \rho^2)} \quad \Rightarrow \quad \rho^A_{Jeans} = \frac{1}{R_s^3} Exp(\frac{4\bar{C}R_s}{\pi^2 \overline{\mathrm{v}^2}})$$

$$n > 3 \quad k = \frac{\bar{C}|n-3|}{3R_s(\rho^{\frac{n-3}{3}} R_s{}^{n-3} - 1)} \quad \Rightarrow \quad \rho^A_{Jeans} = \frac{1}{R_s^3}\left(1 + \frac{4\bar{C}|n-3|R_s}{3\pi^2 \overline{\mathrm{v}^2}}\right)^{\frac{3}{n-3}}$$



Conclusions

- Midge swarm dynamics is dominated by long range acoustic interactions
- The interactions are adaptive - weaker when the background intensity is higher.
- Adaptivity, for general power-law interactions, stabilizes the swarm against collapse
- A prediction: A Selection of a particular density for higher power law interactions $(n > 2)$



Acknowledgements

Nir Gov – PI

Reuven Ianconescu (Research Assistant)

Experimental group (Yale U.):

Nick Ouellette - PI

James Puckett

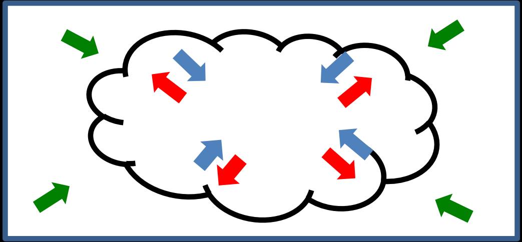
Rui Ni

THE END

Some Results...

Extended Virial Theorem

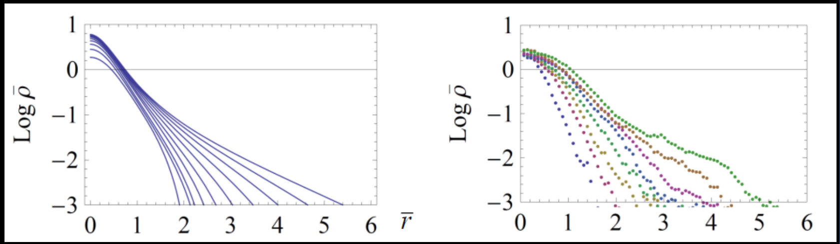
$$2T + W - \oint p\vec{r} \cdot d\vec{s} = 0$$



Surface Pressure
– keeps the swarm together

Poisson-Boltzmann equation
w/ cut-off

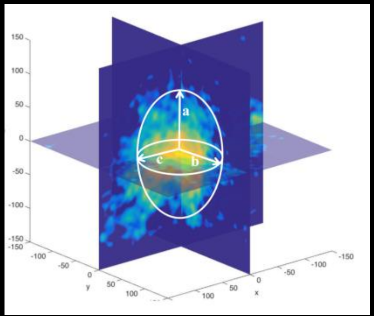
Density profile



Theoretical

Data

Ellipsoidal Approximation



Boundary **closer** to the center
– **Stiffer** effective spring