

TOPOLOGICAL PHOTONICS AND PHONONICS IN METAMATERIALS

Andrea Alù

Credits to: D. Sounas, Y. Radi, A. Kord, L. Quan, D. Farfan, Z. Xiao, H. Kwon, G. D'Aguanno

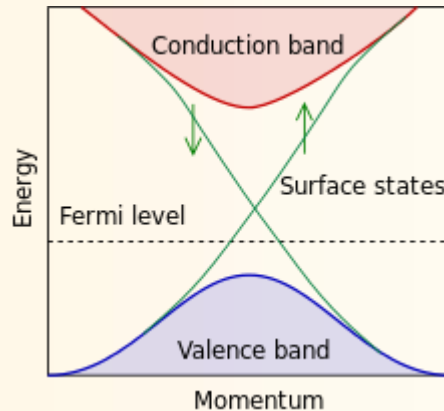
Collaborators: A. Khanikaev (CUNY), H. Krishnaswamy (Columbia), Y. Hadad (TAU), C. Coulais (AMOLF),

Funding: AFOSR, NSF, DARPA, Simons Foundation, Qualcomm, Lockheed Martin

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ELECTRONIC TOPOLOGICAL INSULATORS



A topological insulator is a material with non-trivial topological order, which enables the operation as an *insulator* in the bulk, but that *conducts* on the surface.

The conduction states are *symmetry protected*, and they are associated with unusual phenomena, such as *strong robustness to disorder*, and the *quantum Hall effect*

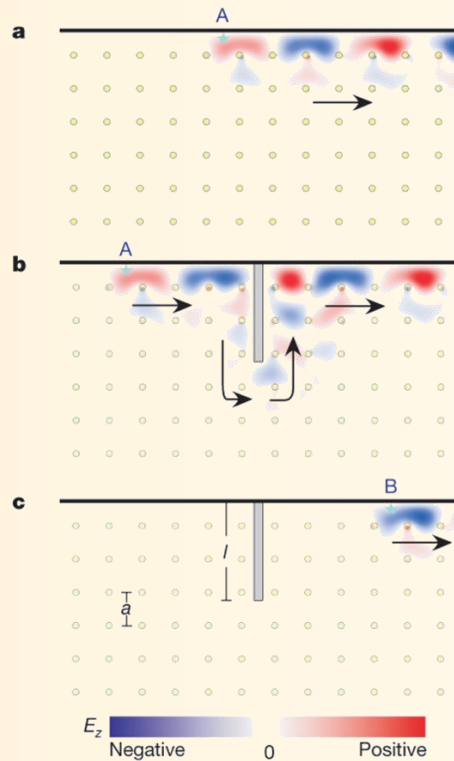
C. Kane, E. Mele, *Phys. Rev. Lett.* **95**, 146802 (2015)

Z. C. Gu, X. G. Wen, *Phys. Rev. B* **85**, 075125 (2009)

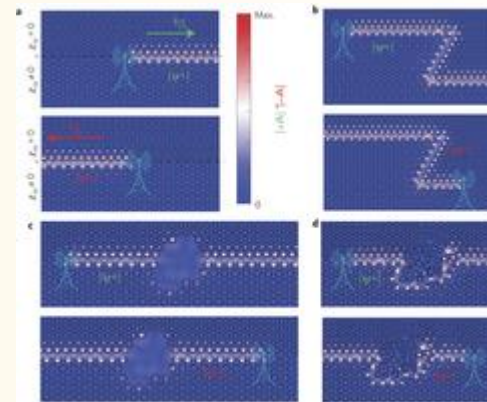
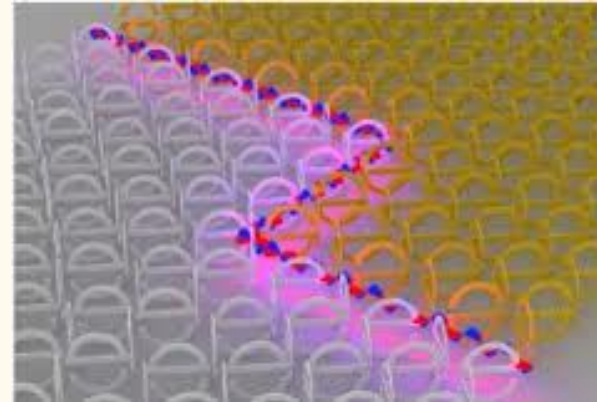
C. Kane, J. Moore, *Phys. World* **24**, 32 (2011)

PHOTONIC AND PHONONIC TOPOLOGICAL INSULATORS

Strong protection (non-reciprocal)



Weak protection (reciprocal)



F. Haldane, S. Raghu, *Phys. Rev. Lett.* **100**, 013904 (2008)

Z Wang, Y. Chong, J. D. Joannopoulos, M. Soljačić, *Nature* **461**, 772 (2009)

A. Khanikaev, S. Mousavi, W. K. Tse, M. Kargarian, A. MacDonald, G. Shvets, *Nat. Mat.* **12**, 233 (2013)

A. P. Slobozhanyuk, A. N. Poddubny, A. E. Miroshnichenko, P. A. Belov, and Y. S. Kivshar, *Phys. Rev. Lett.* **114**, 123901 (2015)

NON-RECIPROCALITY WITH MAGNETIC BIAS

Lorentz reciprocity theorem

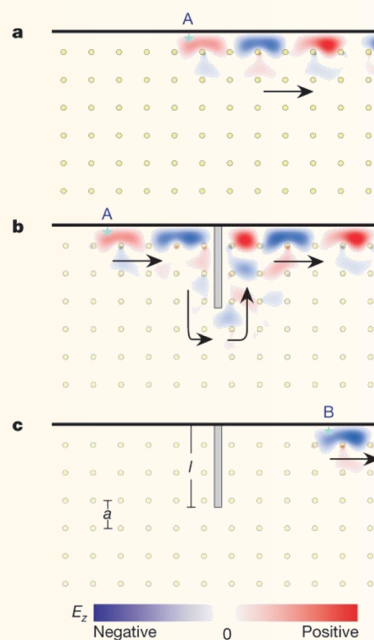
$$\iiint \mathbf{J}_1 \cdot \mathbf{E}_2 dV = \iiint \mathbf{J}_2 \cdot \mathbf{E}_1 dV$$

$$\begin{aligned} \bar{\bar{\epsilon}} &= \bar{\bar{\epsilon}}^T \\ \bar{\bar{\mu}} &= \bar{\bar{\mu}}^T \end{aligned}$$

Time-invariant materials

Linear materials

Static Magnets



Z Wang, Y. Chong, J. D. Joannopoulos, M. Soljačić, *Nature* **461**, 772 (2009)

BREAKING RECIPROCITY CONSTRAINTS

Lorentz reciprocity theorem

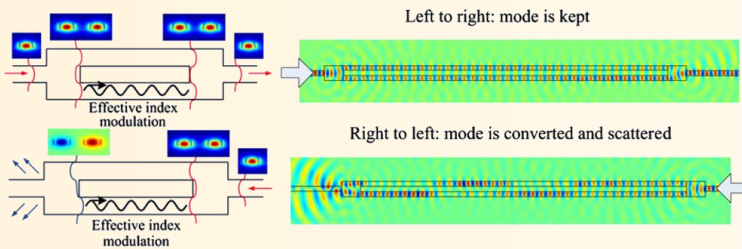
$$\iiint \mathbf{J}_1 \cdot \mathbf{E}_2 dV = \iiint \mathbf{J}_2 \cdot \mathbf{E}_1 dV$$

$$\bar{\bar{\epsilon}} = \bar{\bar{\epsilon}}^T$$

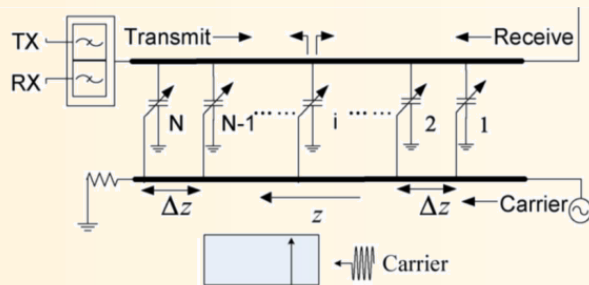
$$\bar{\bar{\mu}} = \bar{\bar{\mu}}^T$$

Time-invariant materials

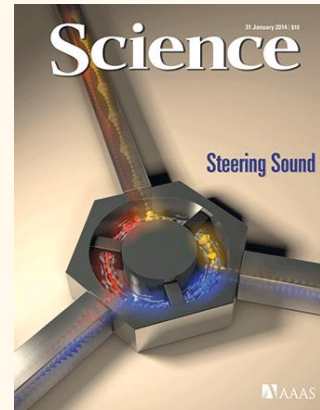
Linear materials



Lira, *PRL* **109**, 033901 (2012)



Qin, *IEEE TAP* **62**, 2260 (2014)



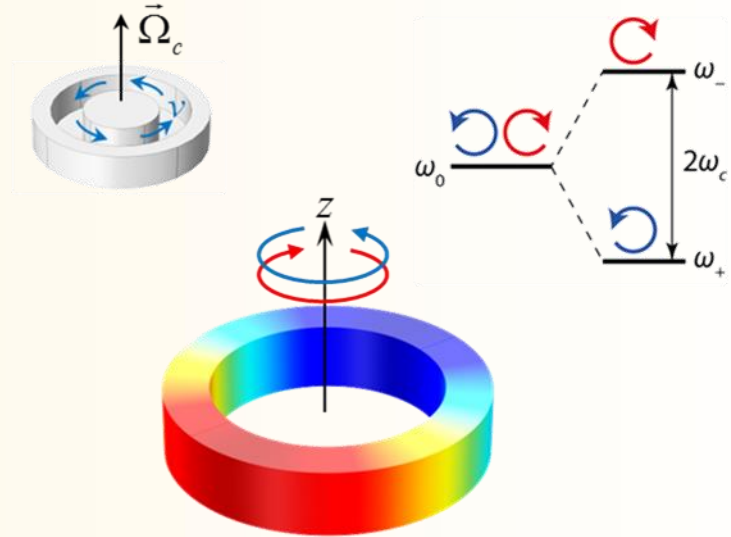
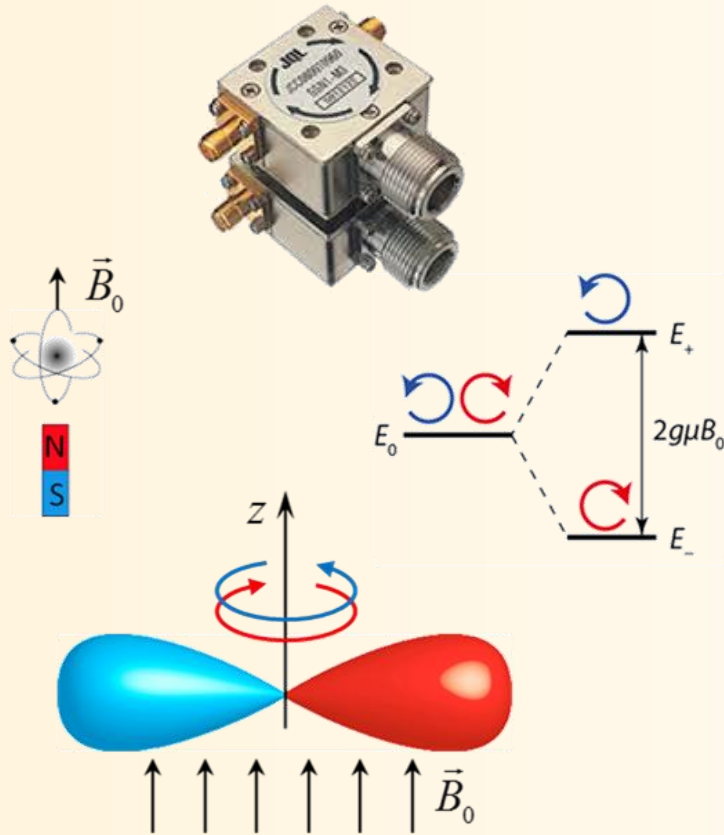
Sounas, *Nature Commun.* **4**, 2407 (2013)

D. L. Sounas, *ACS Photonics* **1**, 198 (2014)

Fleury, *Science* **343**, 516 (2014)

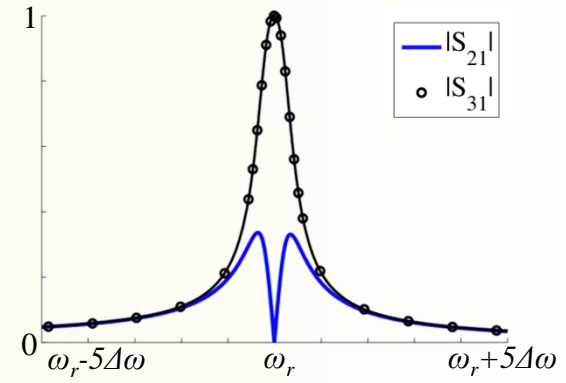
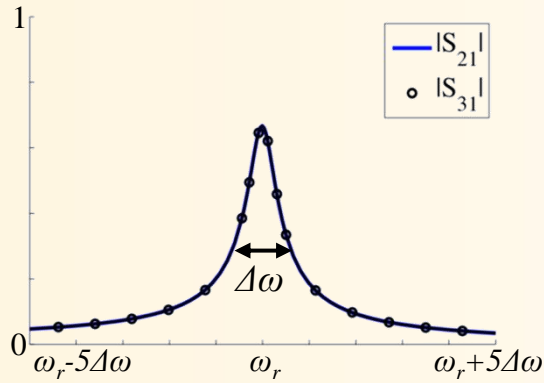
Estep, *Nature Phys.* **10**, 923 (2014)

ANGULAR-MOMENTUM BIASING: A ZEEMAN META-MOLECULE

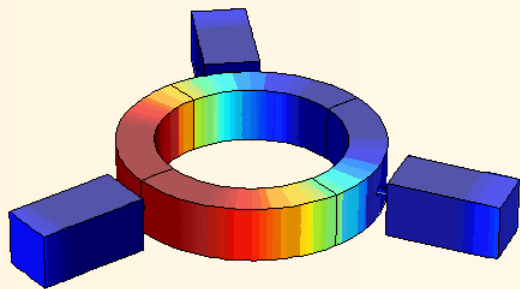


R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* 343, 516 (2014)

AN ACOUSTIC CIRCULATOR



freq(57)=943.75 Surface: Pressure (Pa)

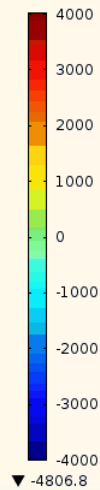


$v = 0 \text{ m/s}$

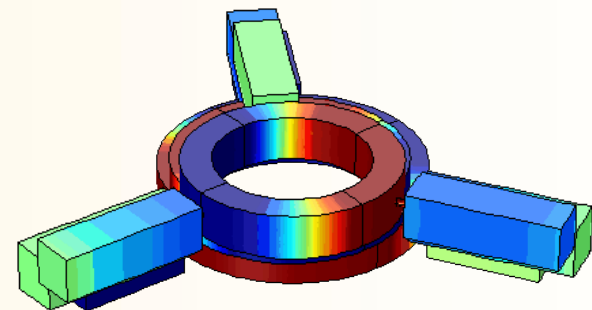


COMSOL MULTIPHYSICS

▲ 4881.4



freq(153)=2955.5 Surface: Pressure (Pa)
freq(153)=2955.5 Surface: Pressure (Pa)
freq(58)=944 Surface: Pressure (Pa)

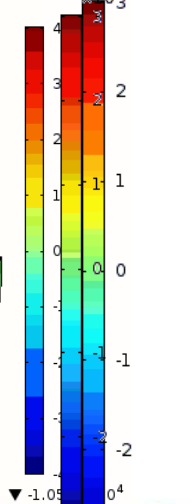


$v = 0.65 \text{ m/s}$

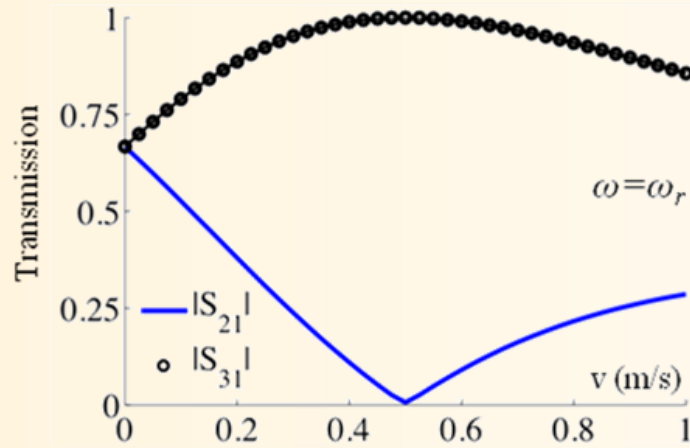


COMSOL MULTIPHYSICS

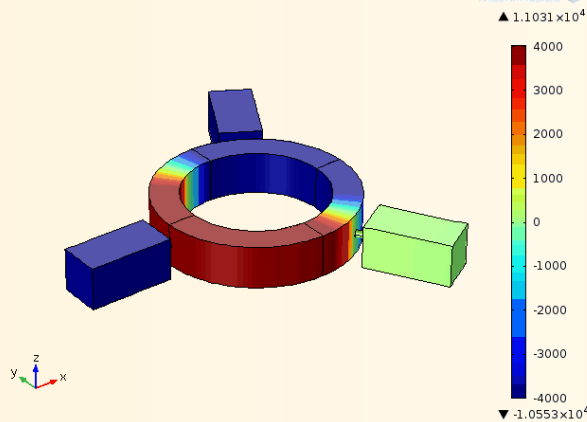
▲ 7.2033 × 10⁷



MODE EVOLUTION



freq(58)=944 Surface: Pressure (Pa)



$$T_{1 \rightarrow 2} = \left| \frac{2}{3} \left(\frac{e^{-i4\pi/3}}{1 - i(\omega - \omega^-) / \gamma^-} + \frac{e^{-i2\pi/3}}{1 - i(\omega - \omega^+) / \gamma^+} \right) \right|^2$$

$$T_{1 \rightarrow 3} = \left| \frac{2}{3} \left(\frac{e^{-i2\pi/3}}{1 - i(\omega - \omega^-) / \gamma^-} + \frac{e^{-i4\pi/3}}{1 - i(\omega - \omega^+) / \gamma^+} \right) \right|^2$$

$$\omega^\pm = \omega_r \pm \gamma / \sqrt{3}$$

$$v_{opt} = \gamma R_{av} / \sqrt{3}$$

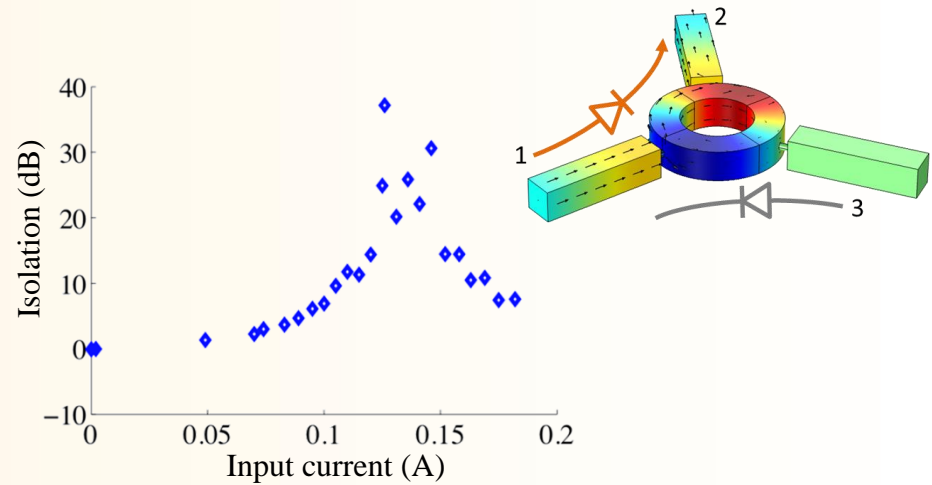
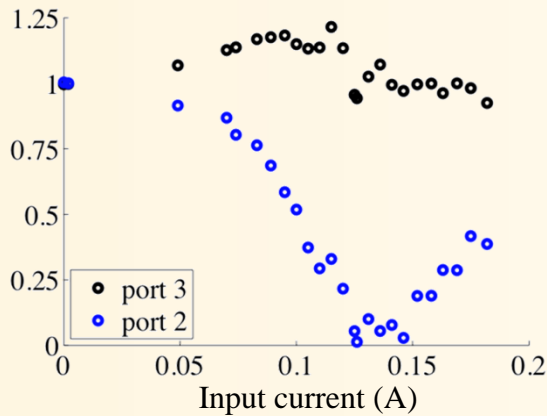
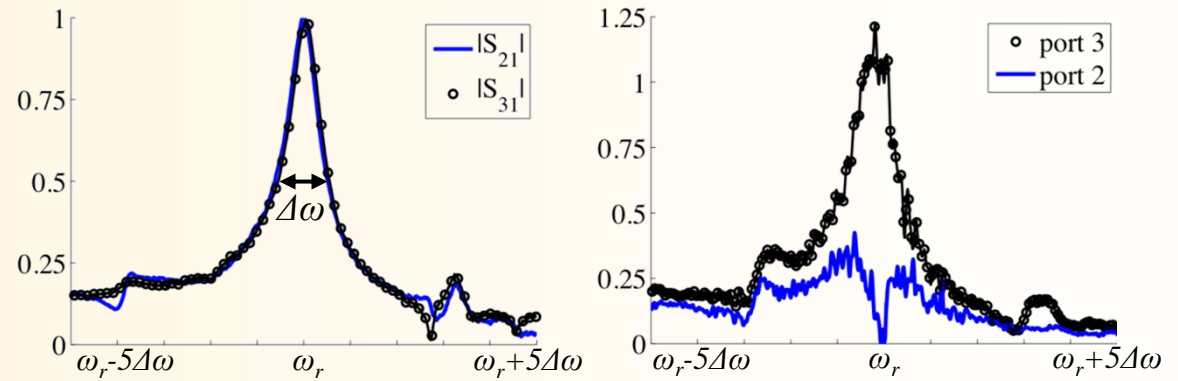
$$\omega_0 = c_0 / R_{av}$$

$$Q = \omega_0 / (2\gamma)$$

$$v_{opt} / c_0 = 1 / (2Q\sqrt{3})$$

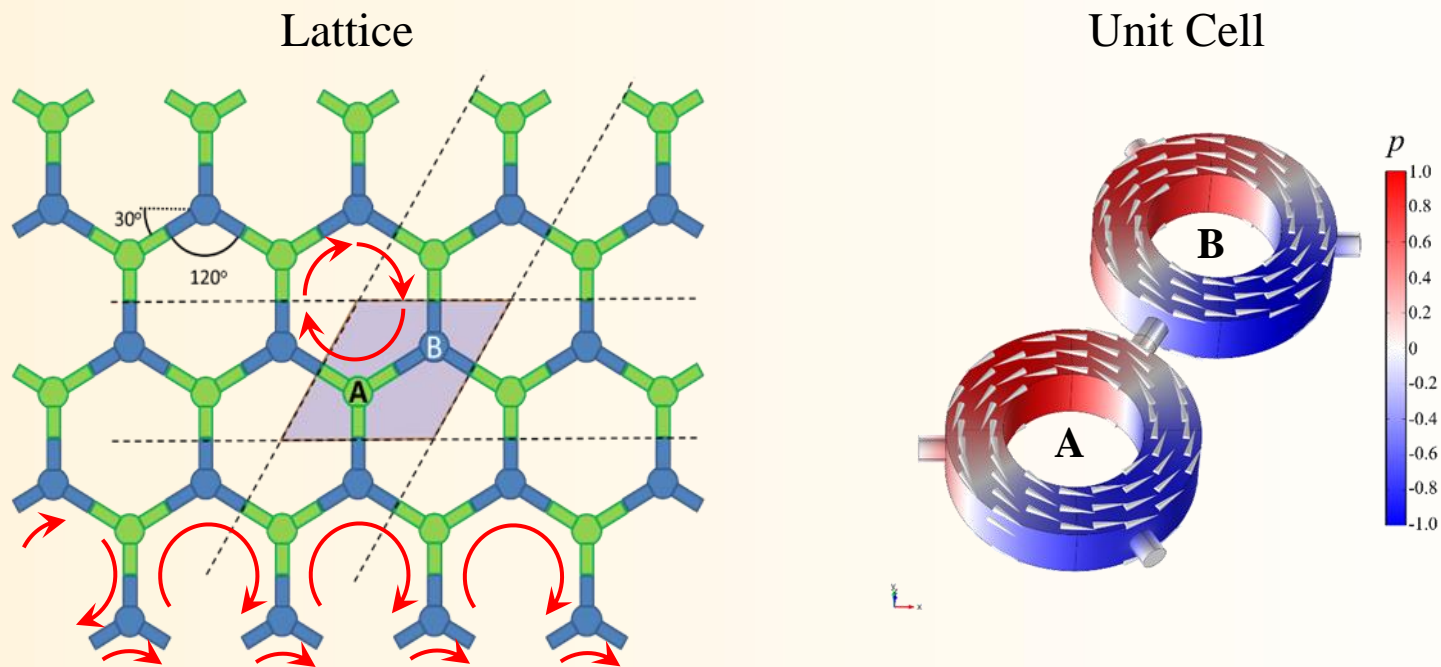
R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* 343, 516 (2014)

AN ACOUSTIC CIRCULATOR



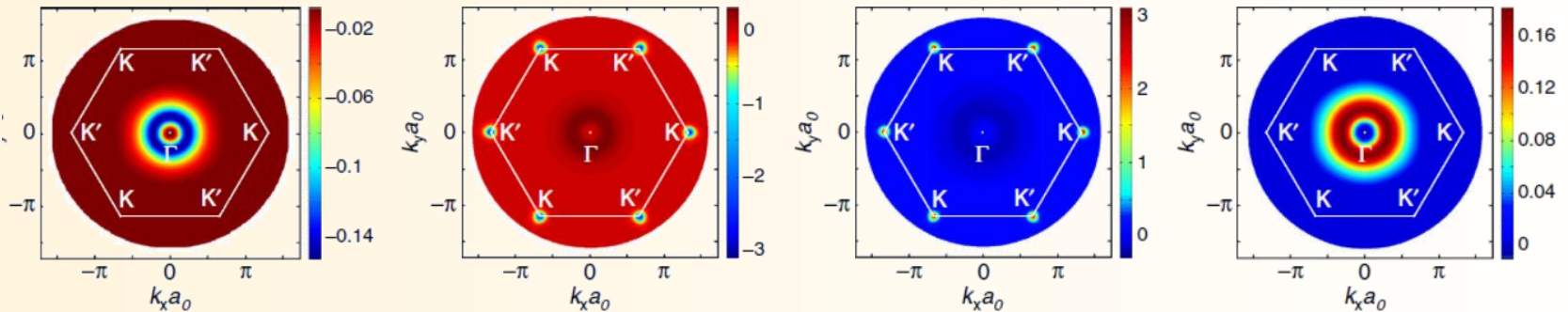
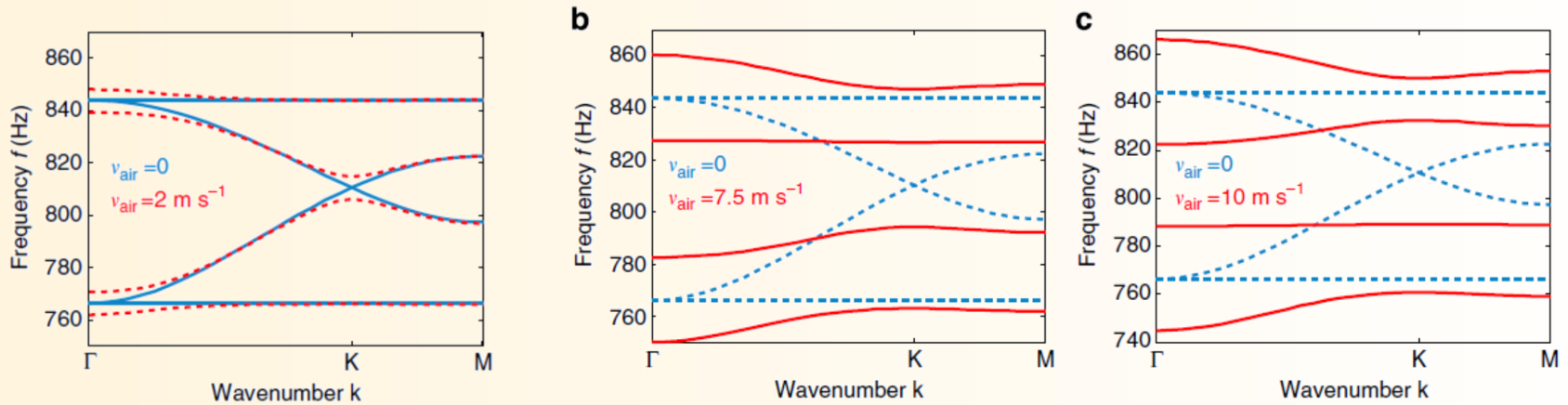
R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* 343, 516 (2014)

MOLDING THE TOPOLOGY OF THE ACOUSTIC BAND DIAGRAM



A. Khanikaev, R. Fleury, H. Mousavi, and A. Alù, *Nat. Comm.* **6**, 8260 (2015)

NON-TRIVIAL TOPOLOGY OF THE BAND STRUCTURE



$$C = \frac{1}{2\pi} \int_{\text{FBZ}} \Omega(\mathbf{k}) d^2\mathbf{k}$$

$$\Omega(\mathbf{k}) = \partial_{k_x} A_y - \partial_{k_y} A_x$$

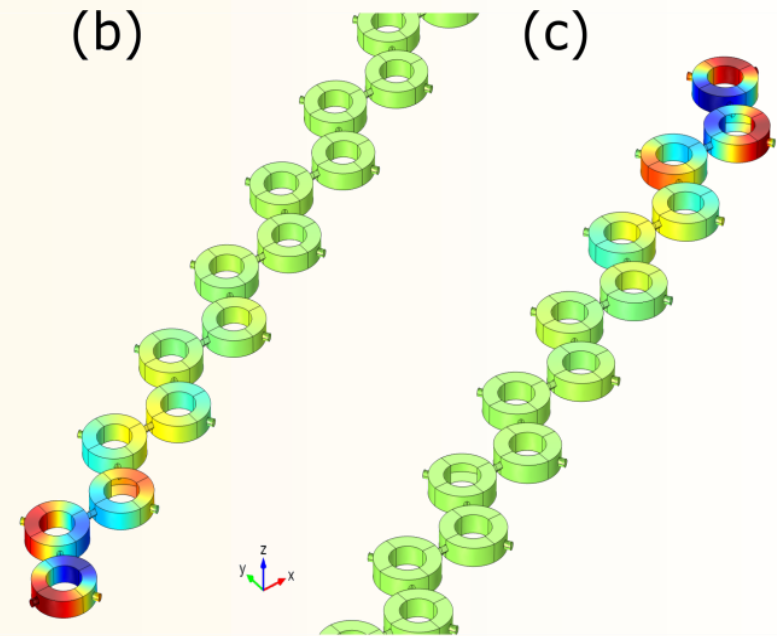
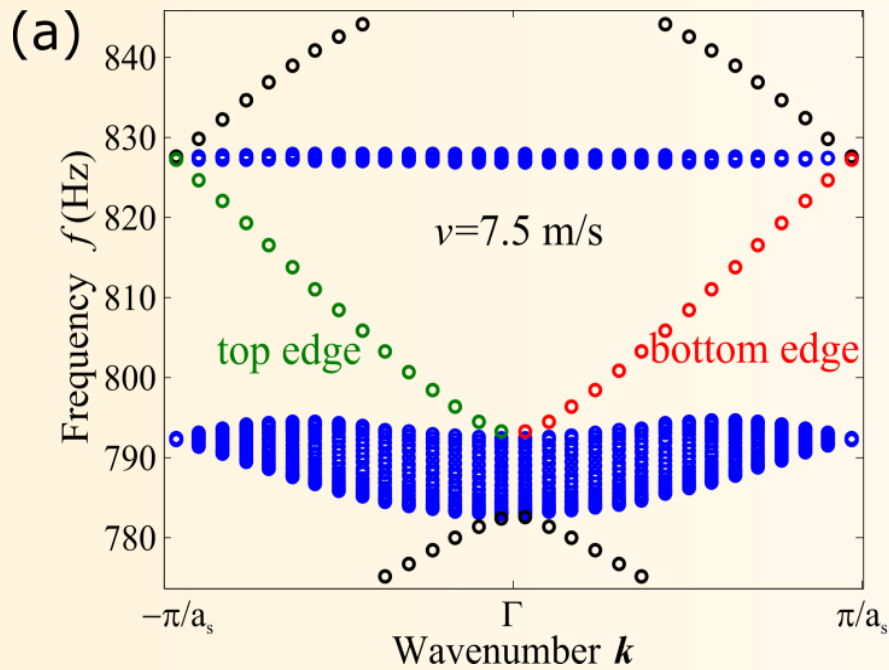
$$\mathbf{A}_{\mathbf{k}} = -i \langle p | \partial_{\mathbf{k}} | p \rangle$$

$C = \{-1, 0, 0, 1\}$ clockwise

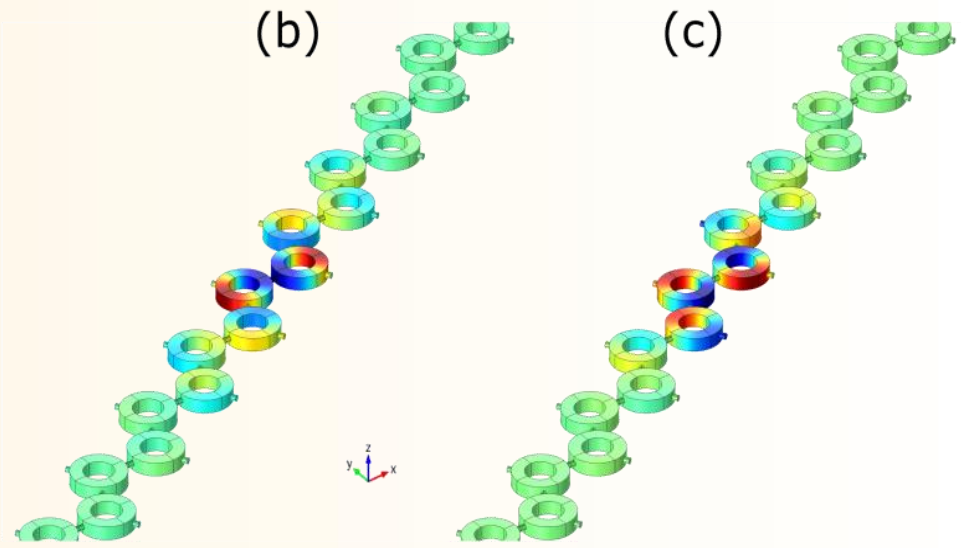
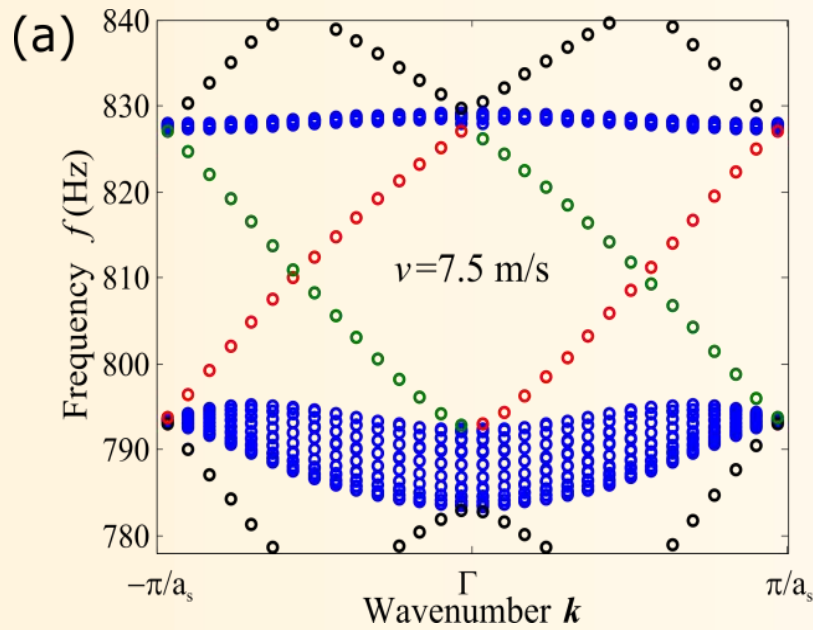
$C = \{1, 0, 0, -1\}$ counterclockwise

A. Khanikaev, R. Fleury, H. Mousavi, and A. Alù, *Nat. Comm.* **6**, 8260 (2015)

TOPOLOGICAL METAMATERIALS

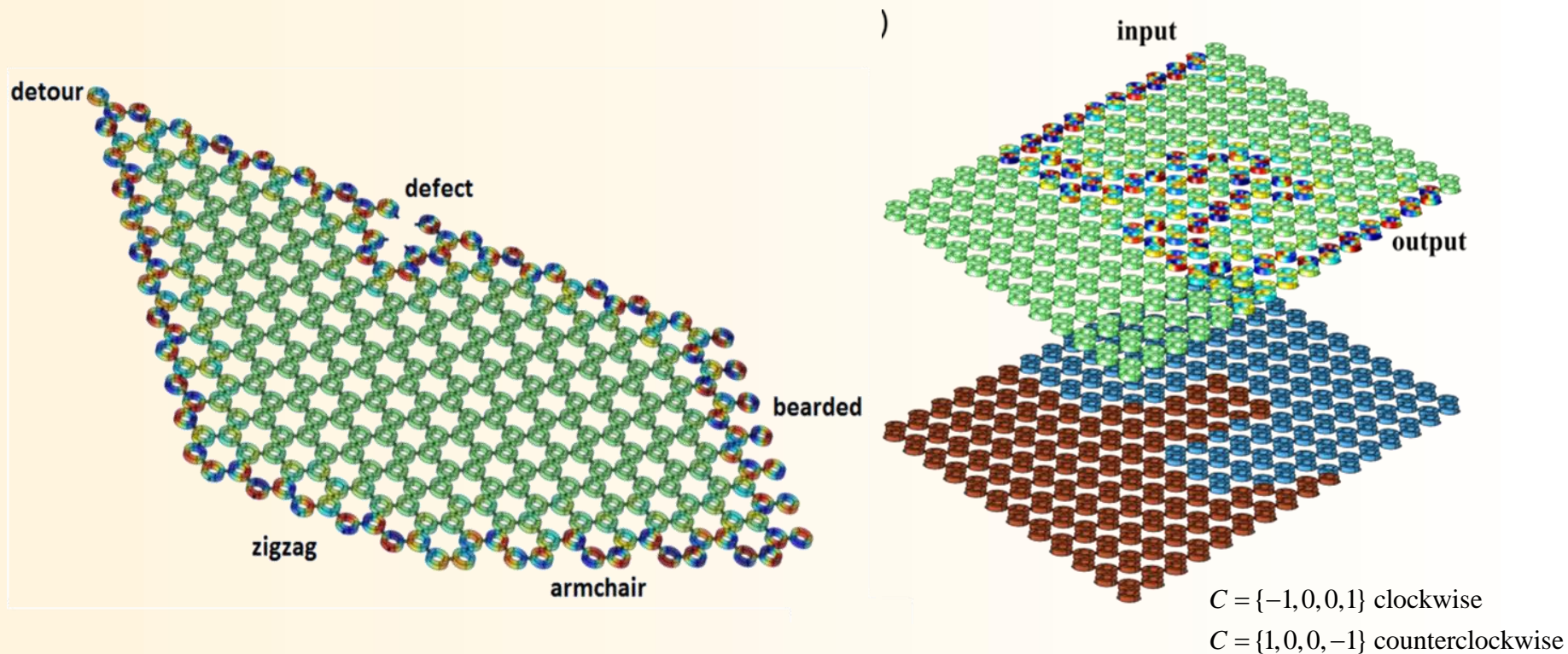


TOPOLOGICAL METAMATERIALS



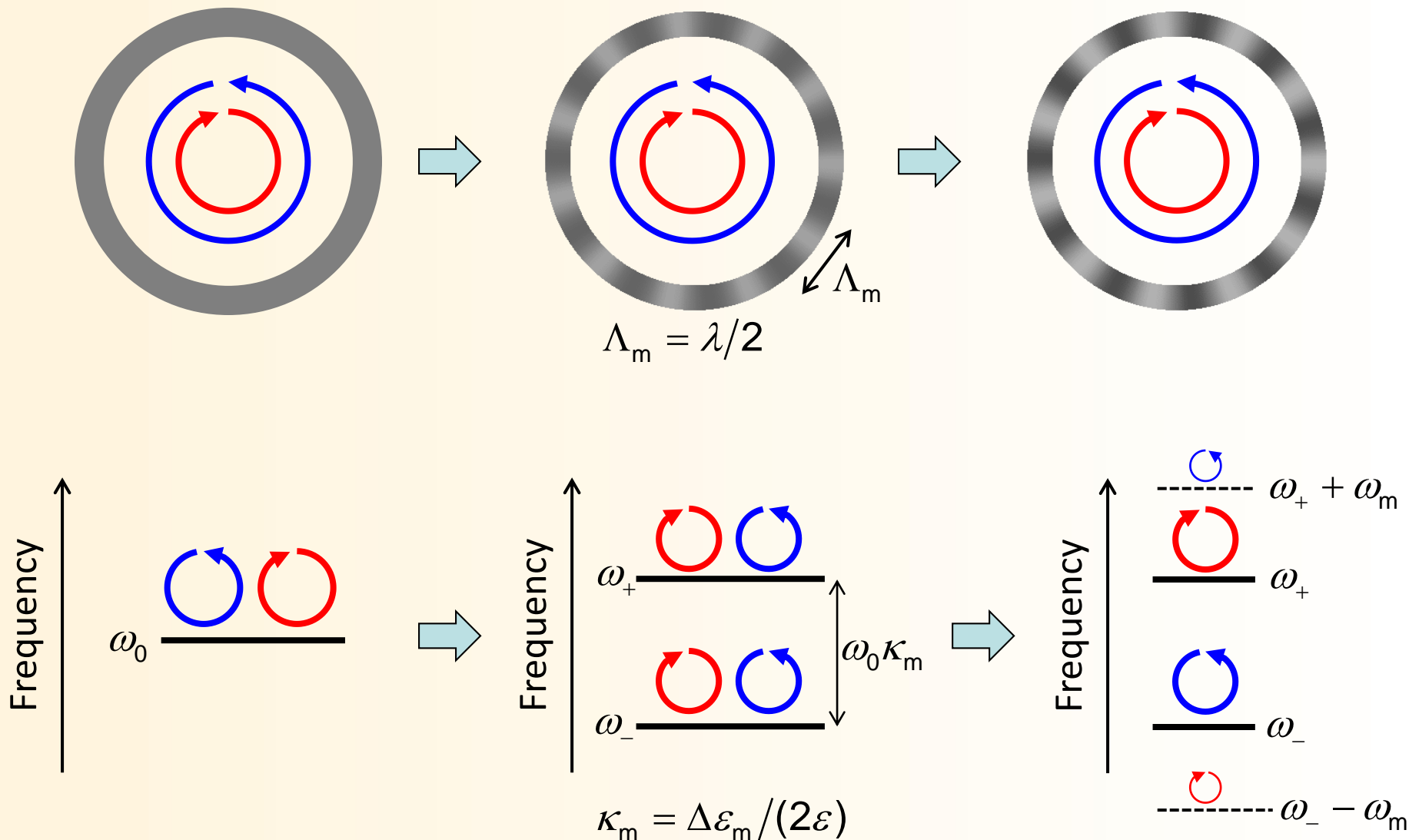
TOPOLOGICAL PHONONIC METAMATERIALS

Reconfigurable waveguides, broadband isolators...



A. Khanikaev, R. Fleury, H. Mousavi, and A. Alù, *Nat. Comm.* **6**, 8260 (2015)

EFFECTIVE SPINNING WITH SPATIOTEMPORAL MODULATION

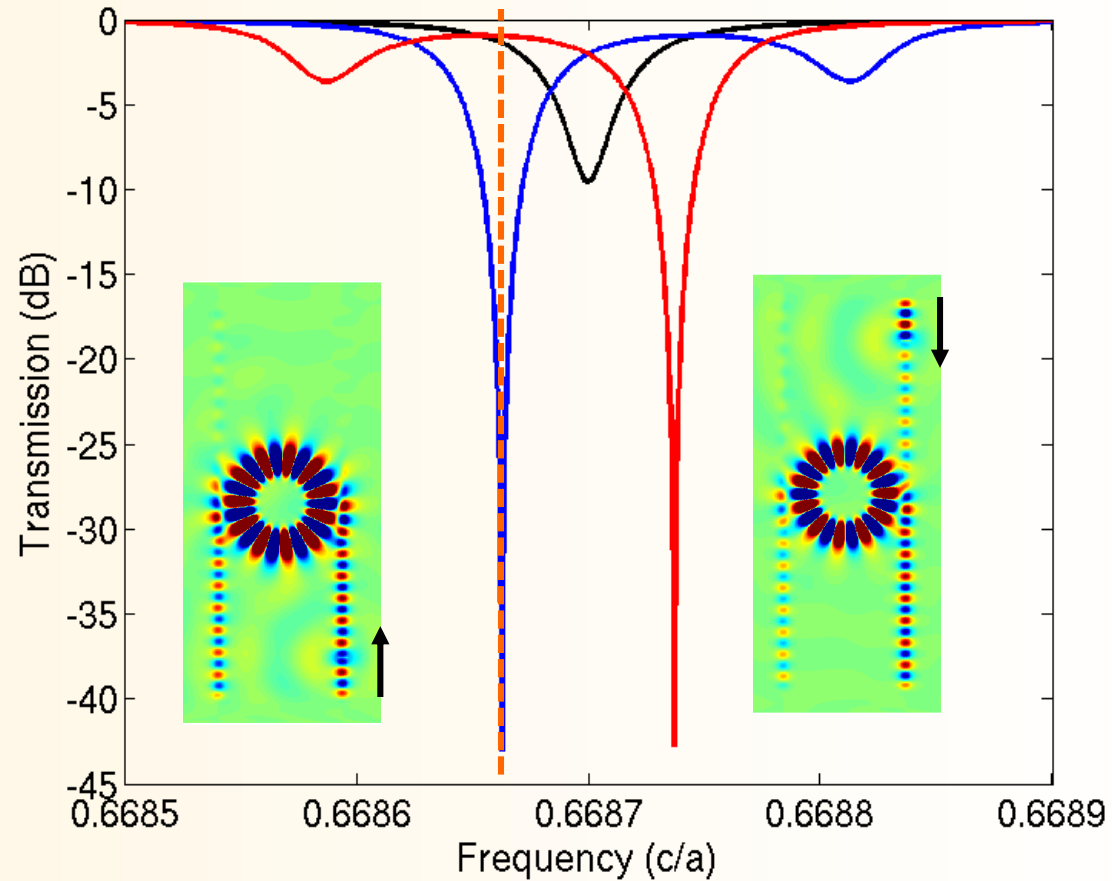


ANGULAR-MOMENTUM-BIASED OPTICAL ISOLATOR

$$\Delta\varepsilon_m = 5 \times 10^{-4} \varepsilon !$$

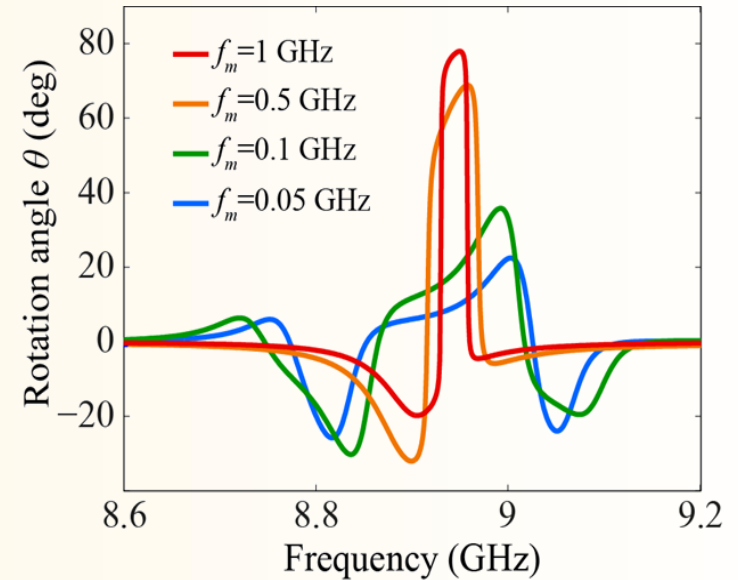
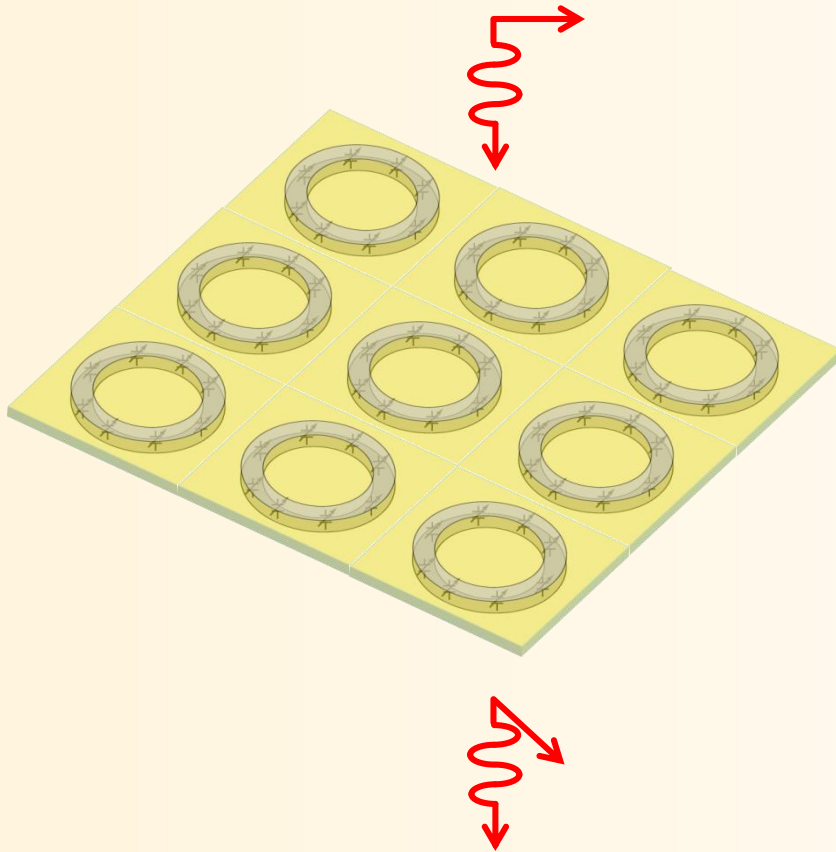
$$Q \sim 7,000$$

$$Q\Delta\varepsilon_m \approx 1$$



D. L. Sounas, A. Alù, *ACS Photonics* 1, 198 (2014)

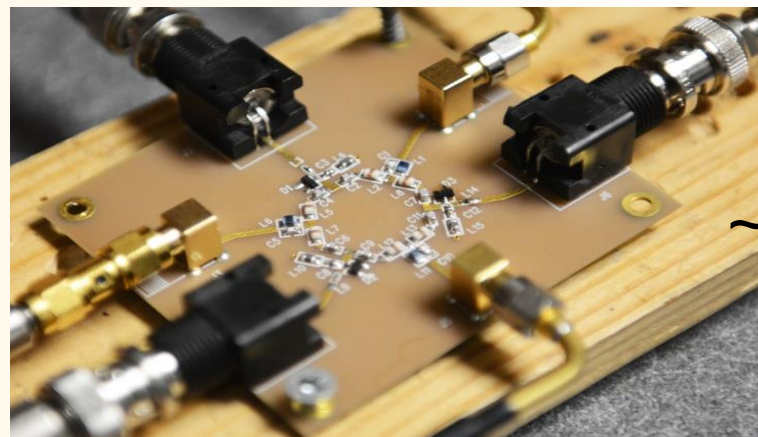
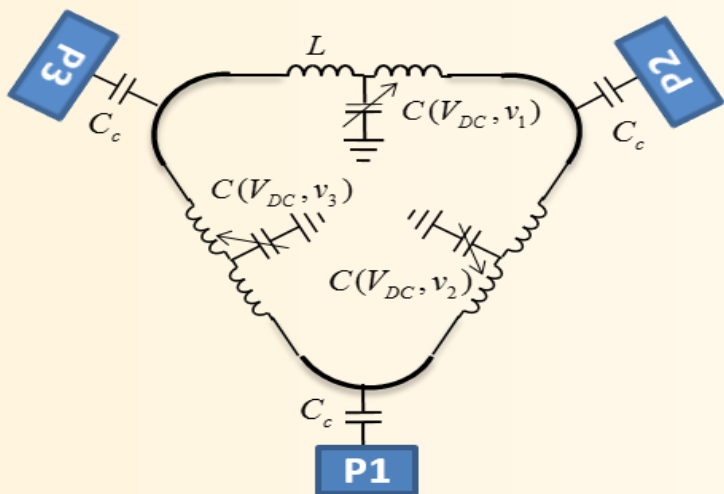
ANGULAR-MOMENTUM NON-RECIPROCAL METASURFACE



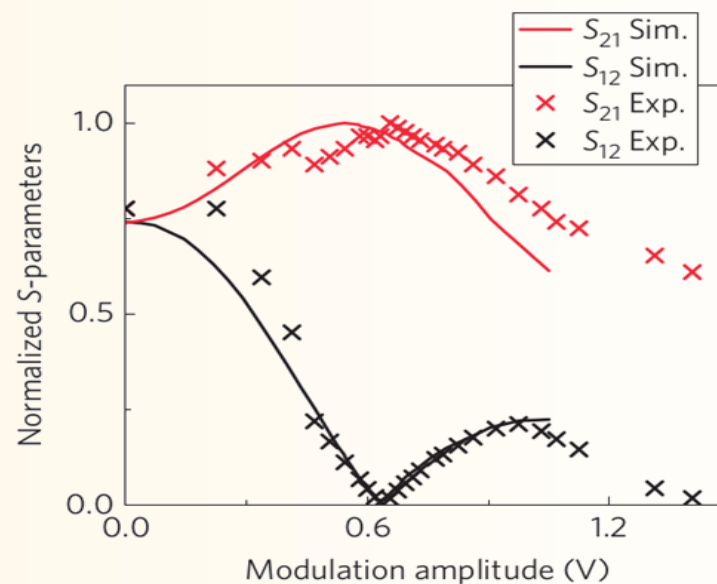
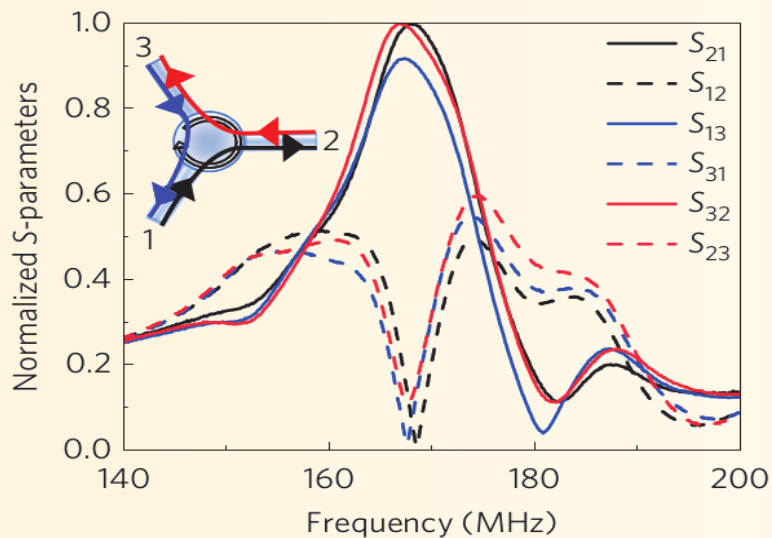
6000 deg/wavelength!

D. L. Sounas, C. Caloz, and A. Alù, *Nature Commun.* **4**, 2407 (2013)

RF MAGNET-LESS INTEGRATED CIRCULATOR



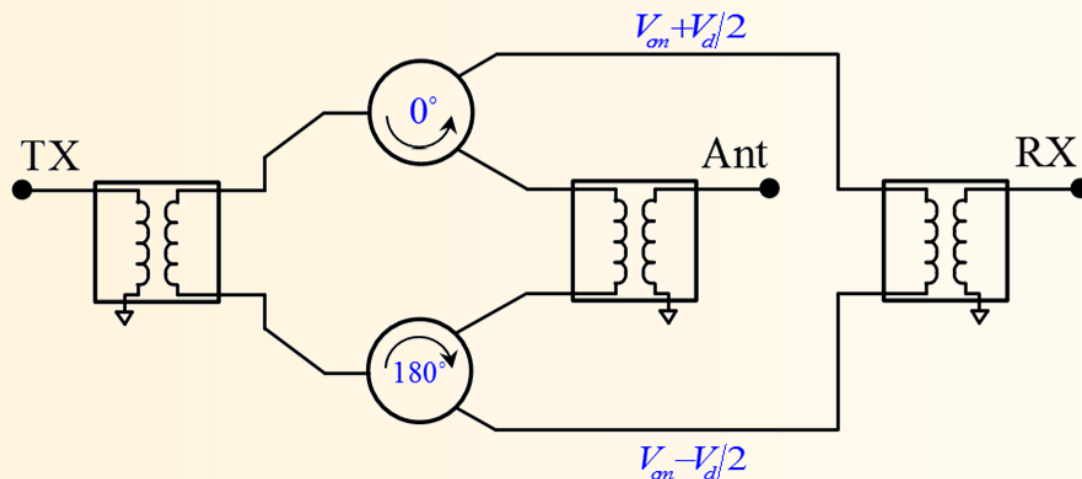
$\sim \lambda/70$



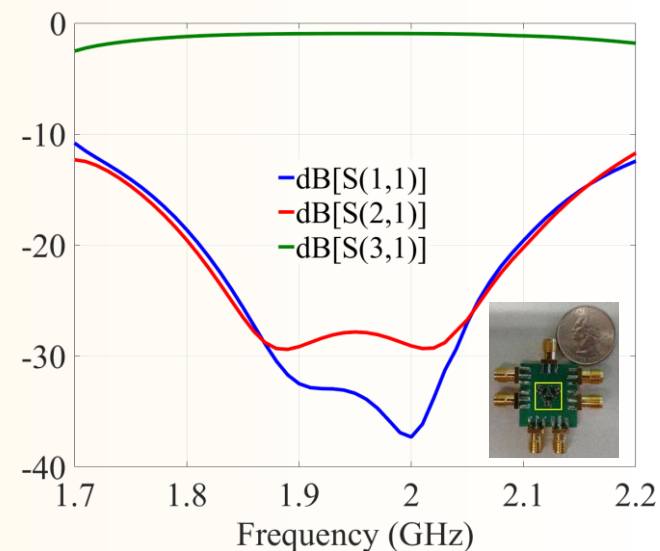
N. A. Estep*, D. L. Sounas*, J. Soric, and A. Alù, *Nature Phys.*, **10**, 923 (2014)

RF CIRCULATOR WITH COUPLED MODULATED RESONATORS

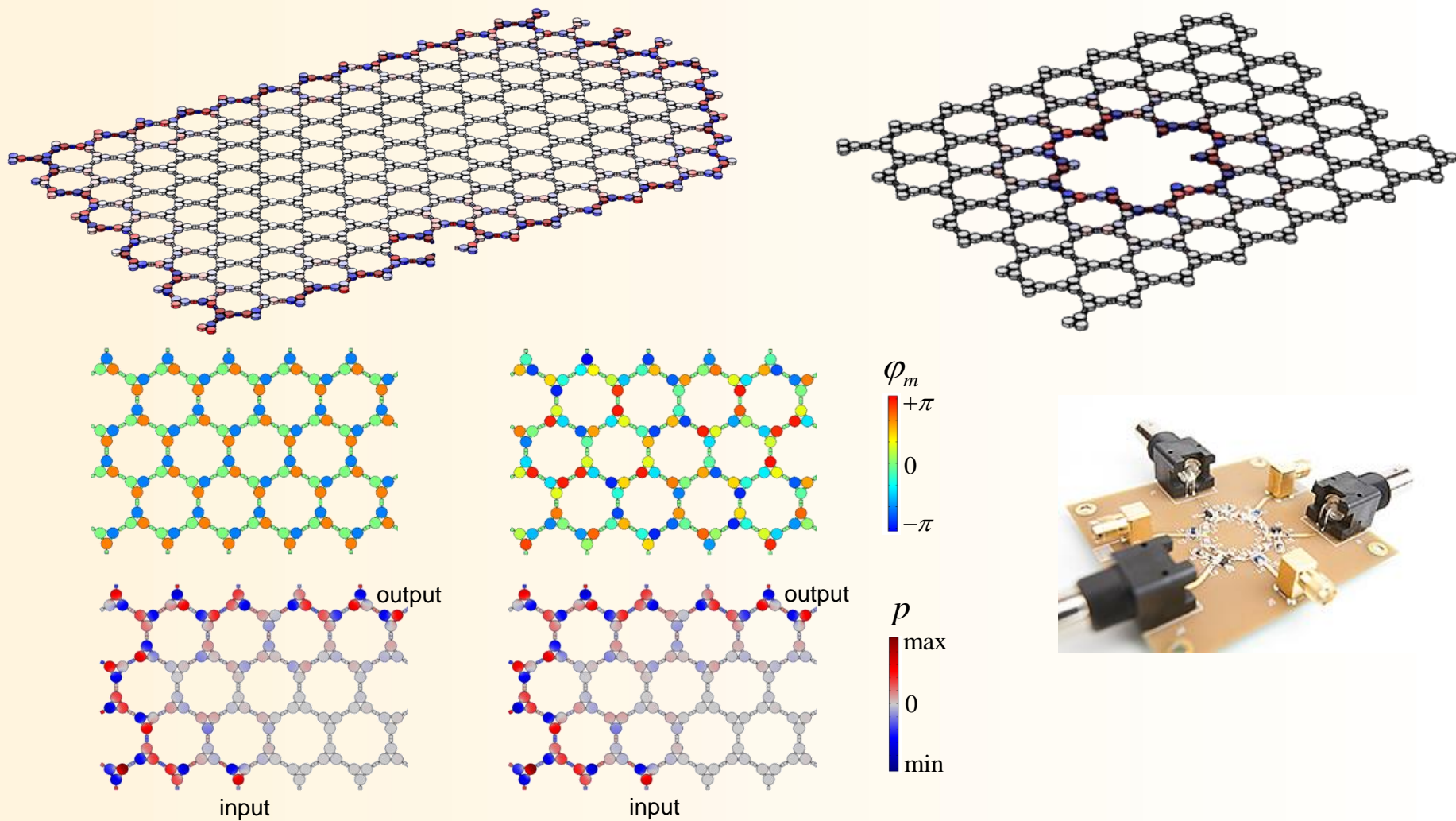
Differential 1st Order Bandpass Delta



- Reduction of all IM products (entire cancellation without random mismatches)
- Improves several metrics including IL and BW
- Reduces modulation frequency and power consumption
- Allows the use of less-scaled technologies to improve power handling



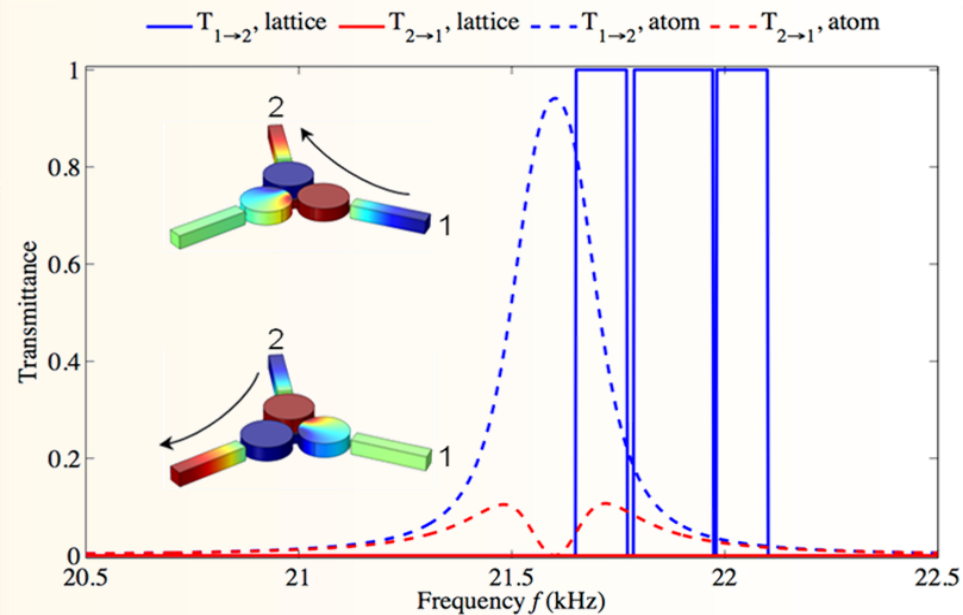
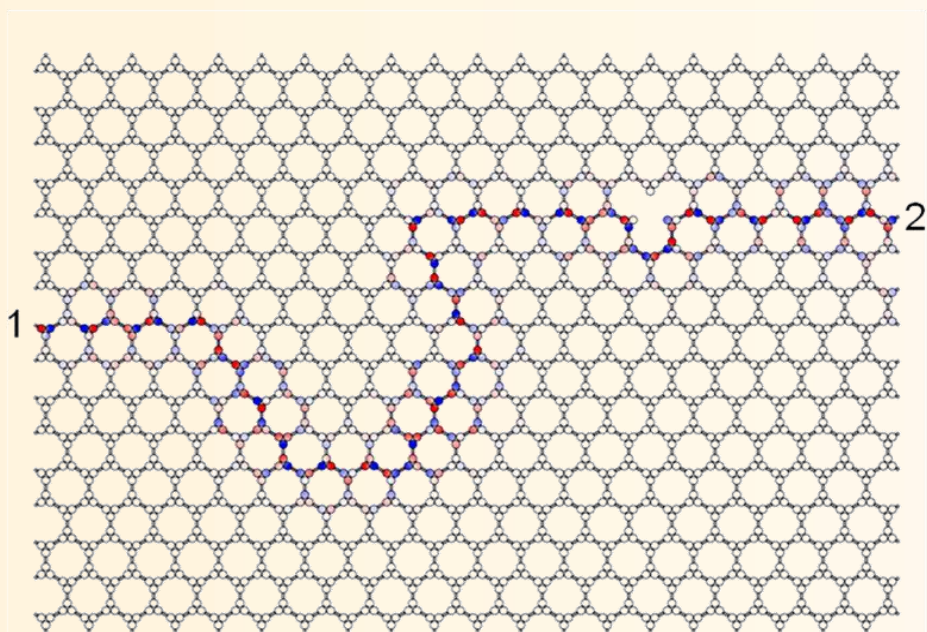
FLOQUET TOPOLOGICAL INSULATORS FOR EM WAVES



N. Estep, D. Sounas, A. Alù, *Nature Physics* **10**, 923 (2014)

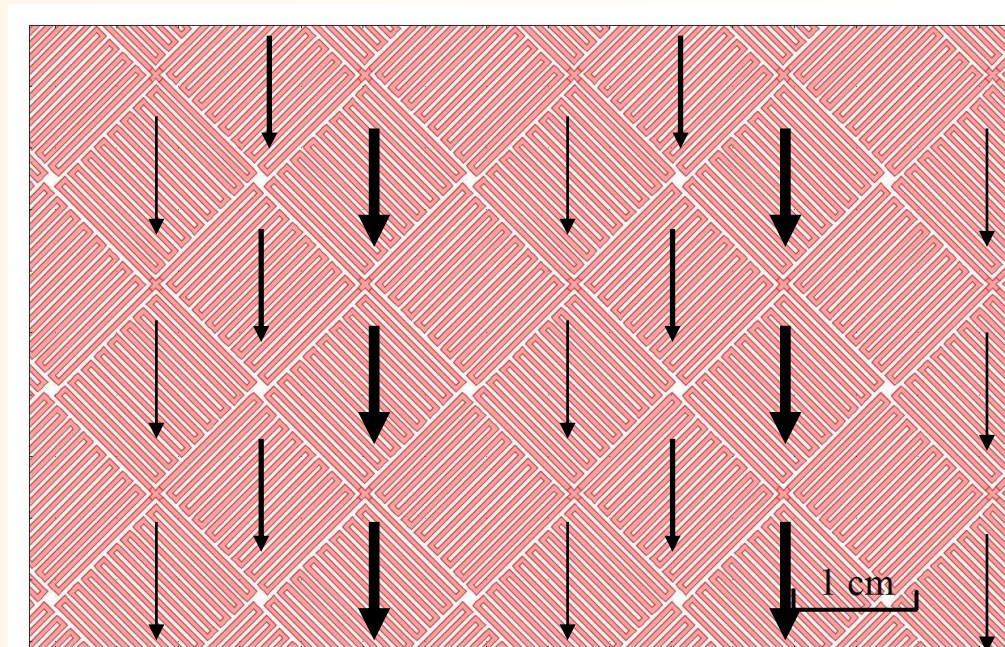
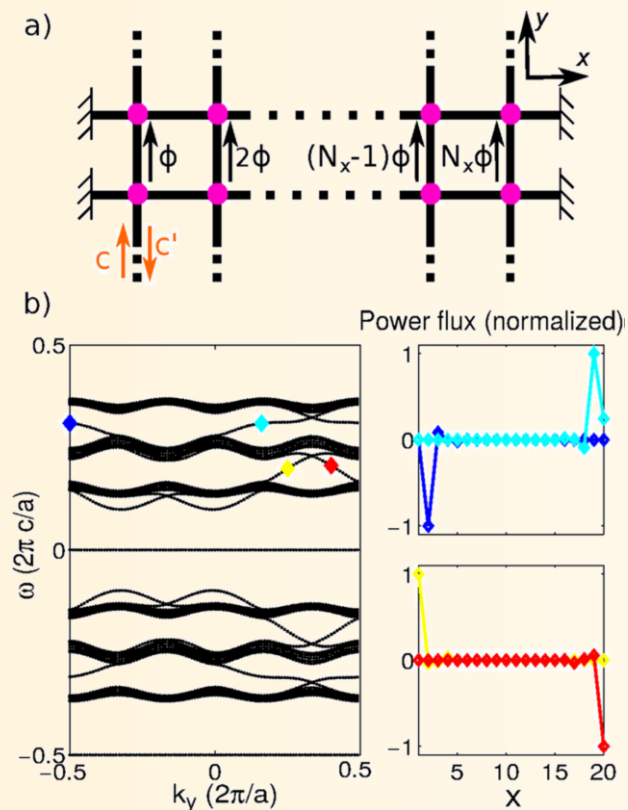
R. Fleury, A.B. Khanikaev and A. Alù, *Nature Communications*, **7**, 11744 (2016)

CONTINUOUS BANDWIDTH OF ISOLATION



R. Fleury, A.B. Khanikaev and A. Alù, *Nature Communications*, 7, 11744 (2016)

PHASE GRADIENTS TO INDUCE TOPOLOGICAL PROTECTION



Q Lin, S. Fan, *New J. Phys.* **17**, 075008 (2015)

EFFECTIVE GAUGE FIELD BIAS

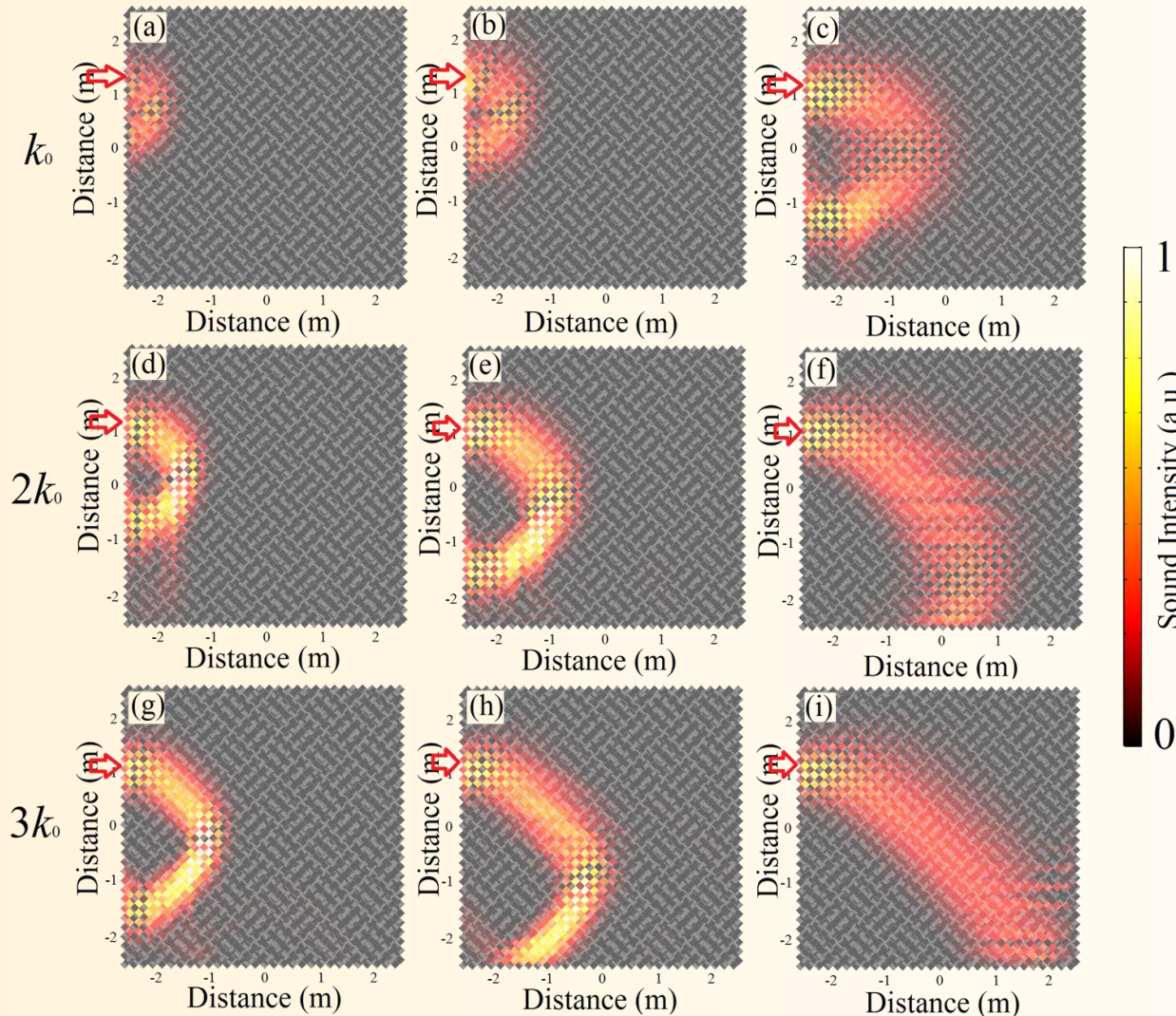
$3\varphi_0$

$2\varphi_0$

φ_0

$\varphi_0 = 5 \text{ deg}$

$v_{air} < 6 \text{ m/s}$

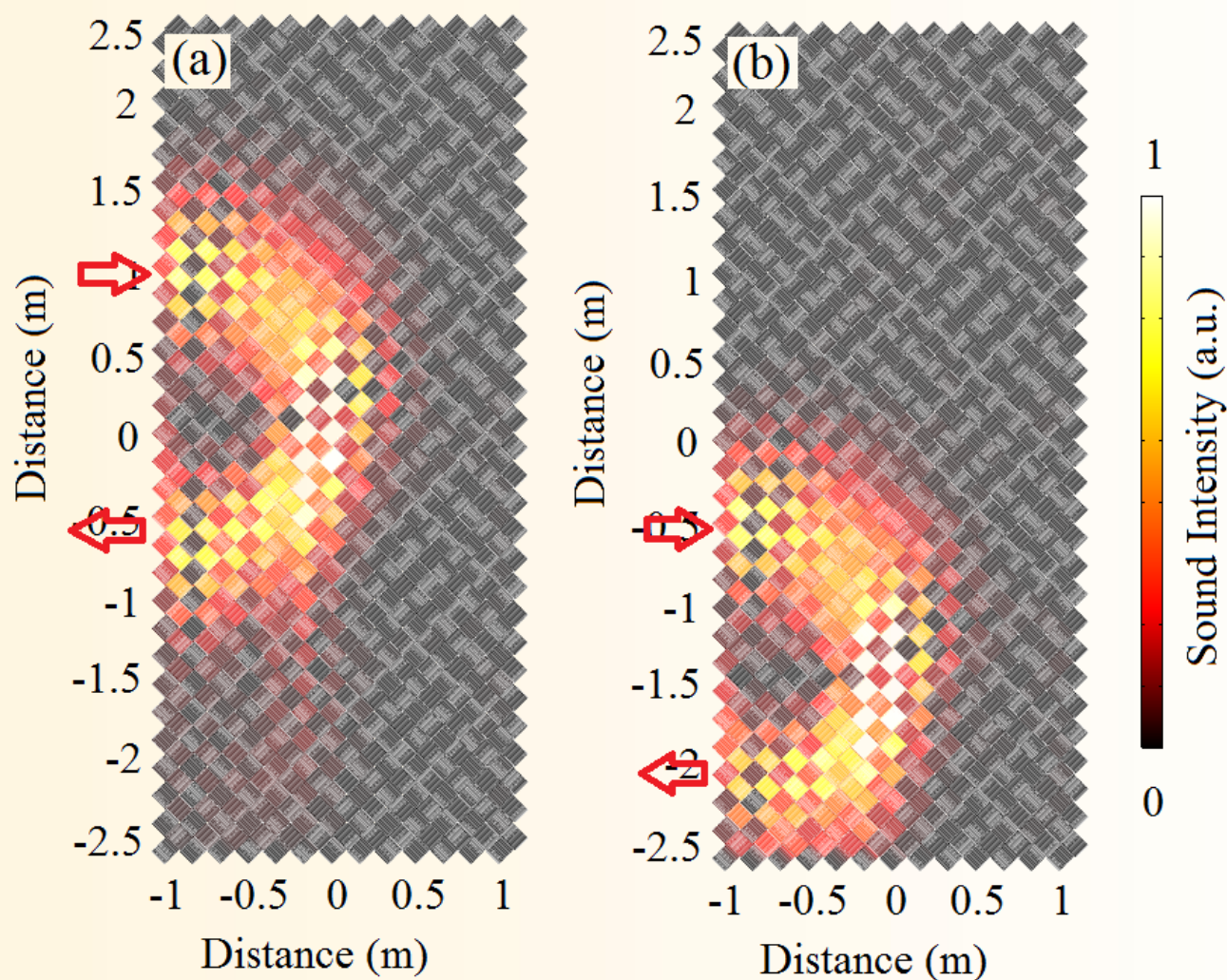


$$F = qvB = mv^2/R$$

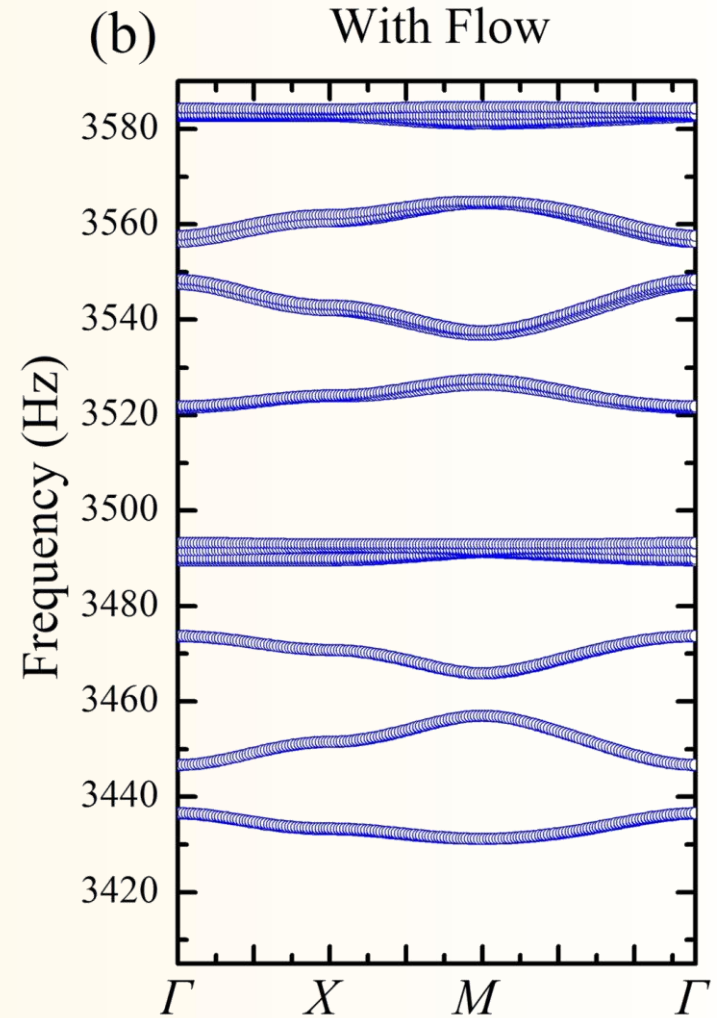
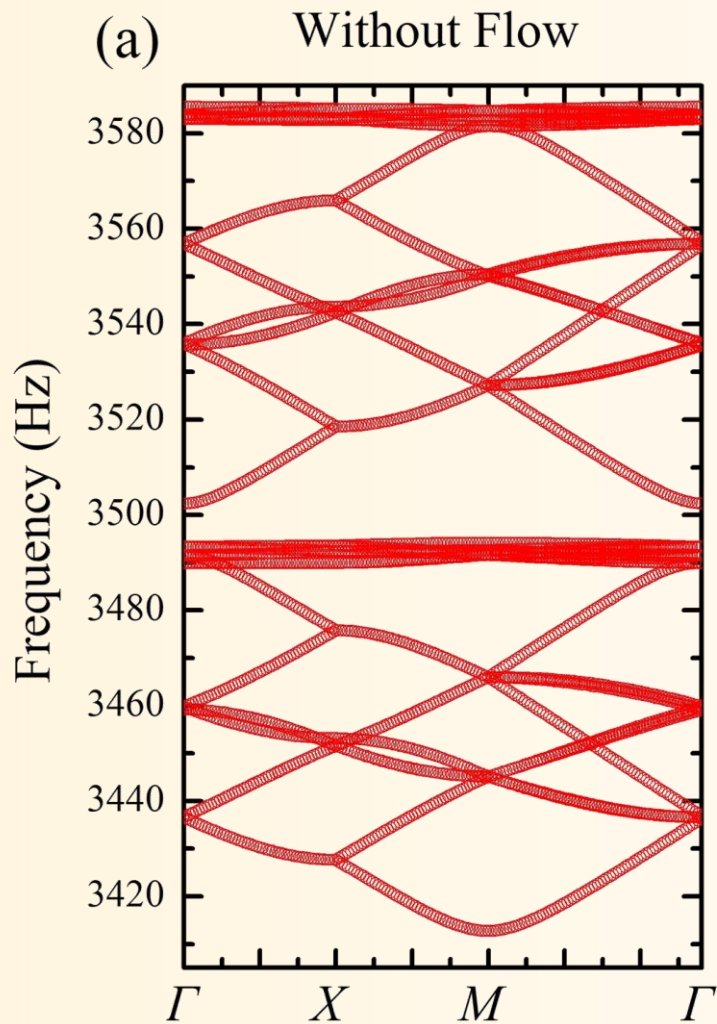


$$R = mv/qB$$

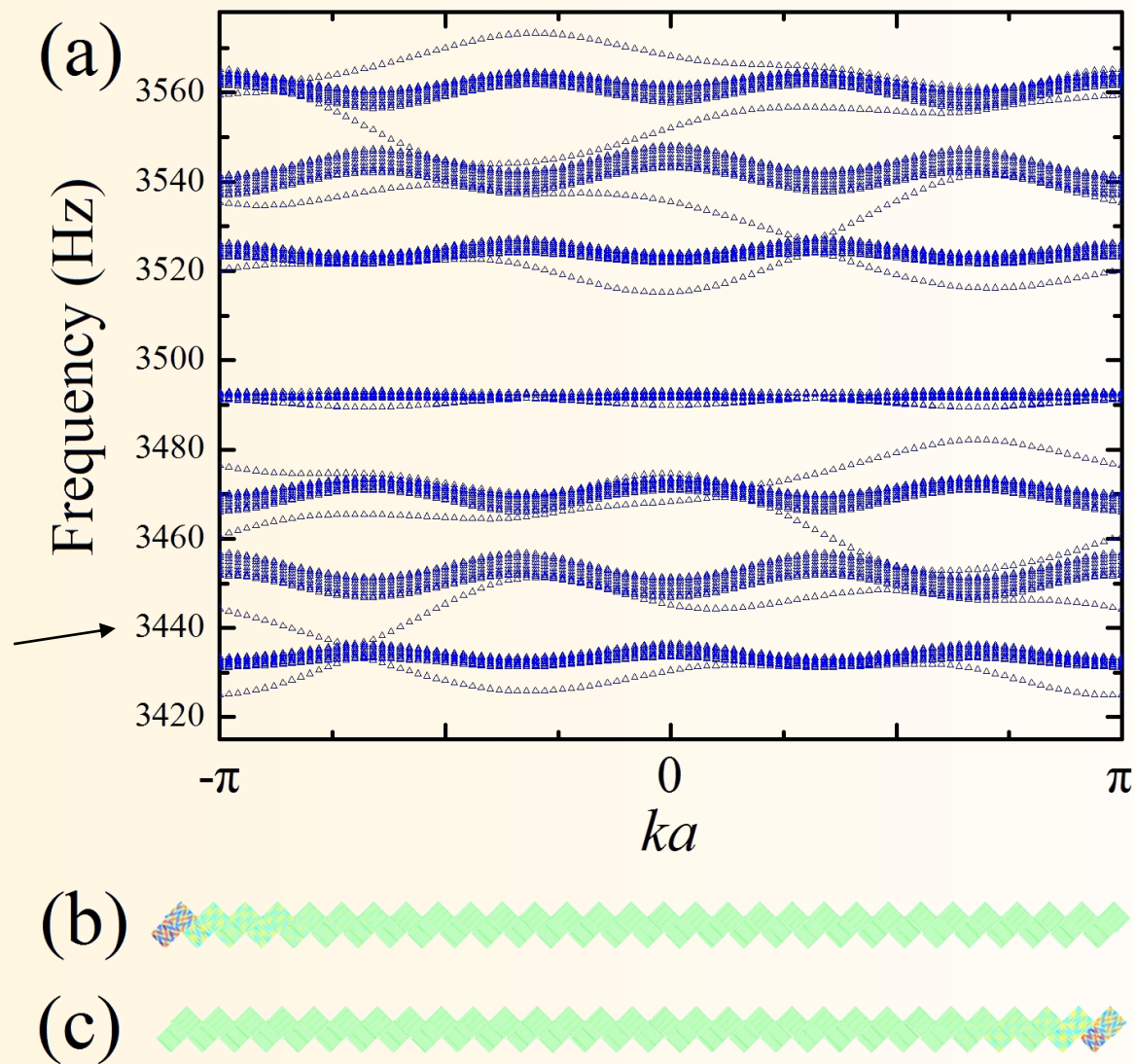
NON-RECIPROCAL TRANSMISSION



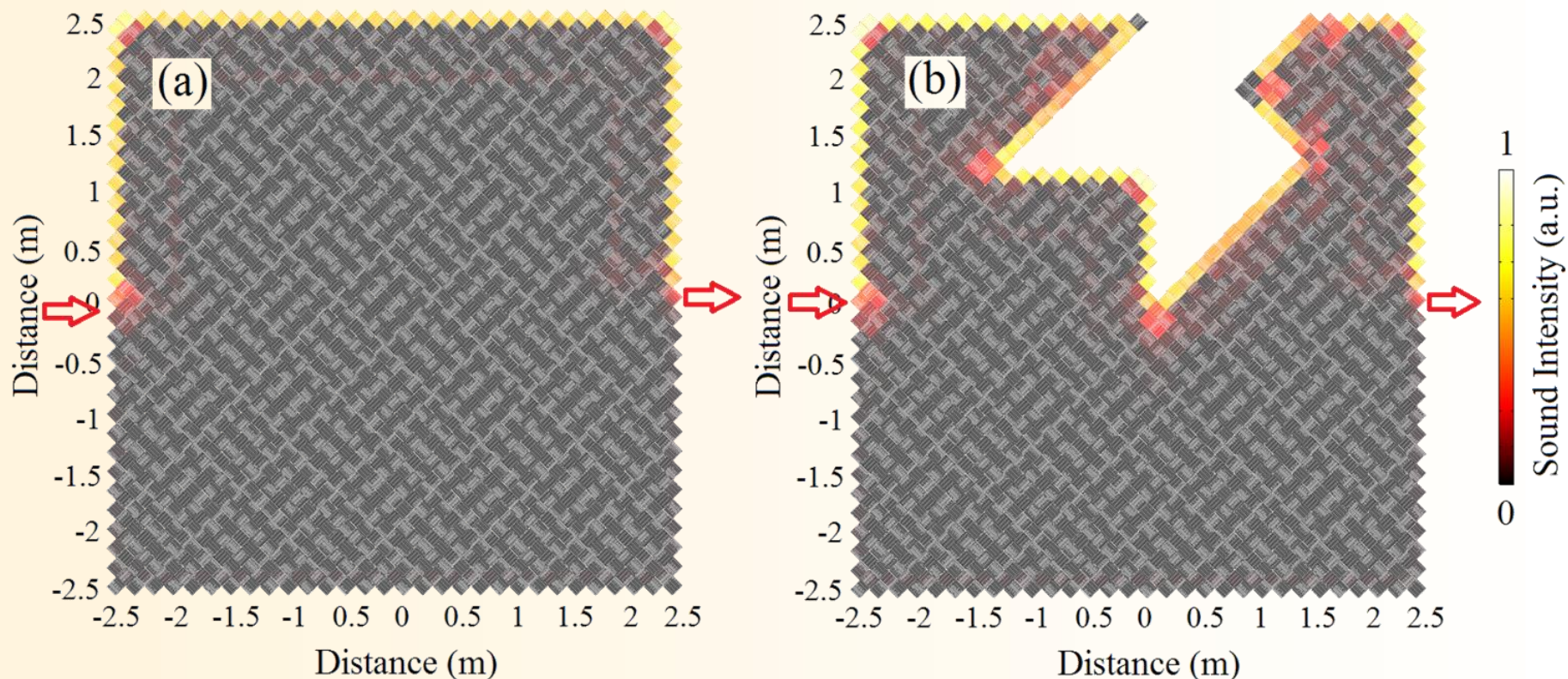
TOPOLOGICAL PHONONICS



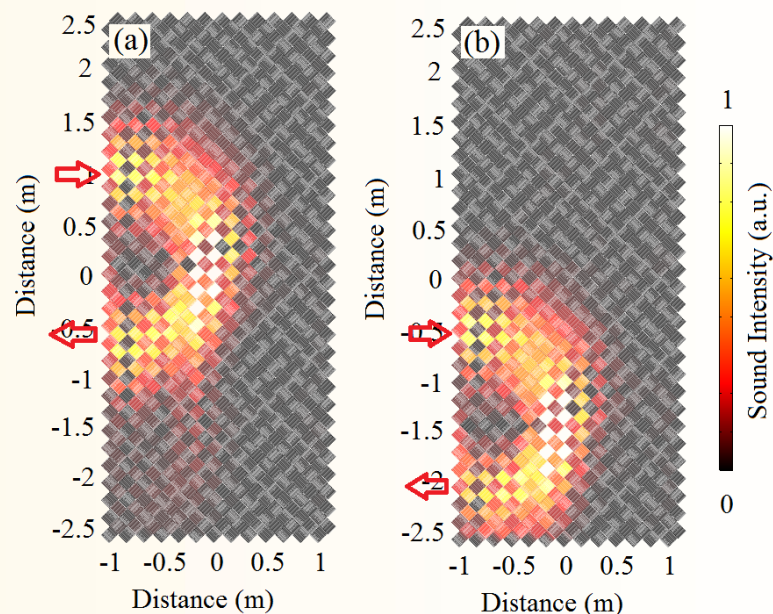
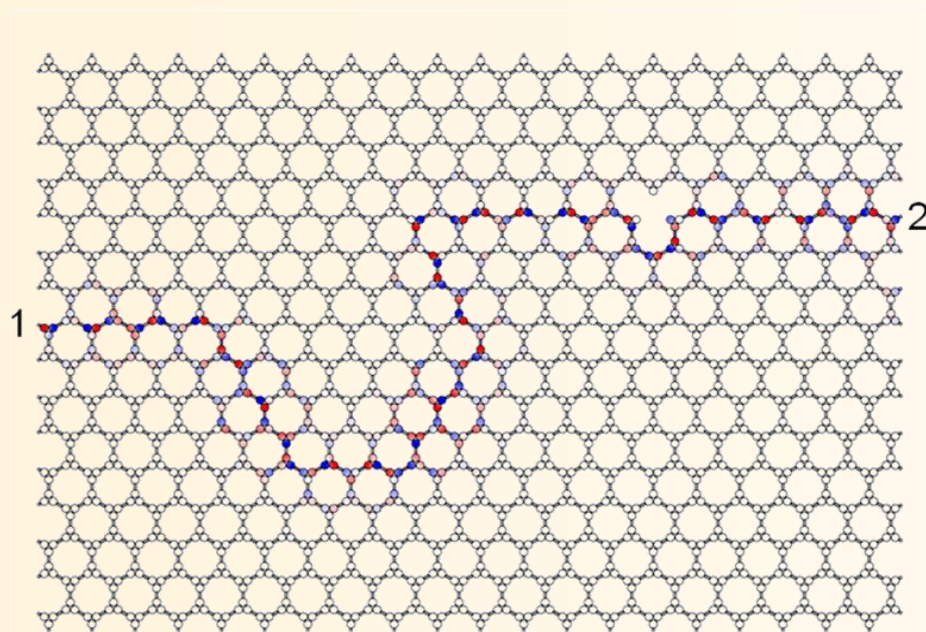
EDGE MODE DISPERSION



ROBUST EDGE MODE PROPAGATION



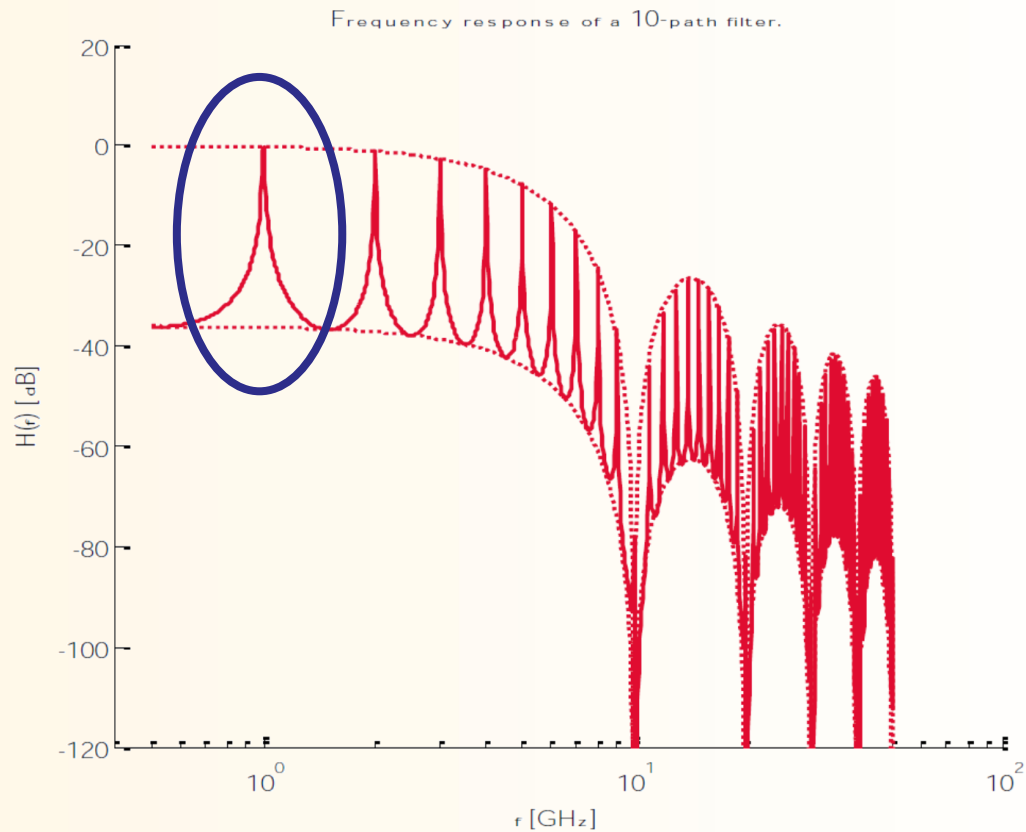
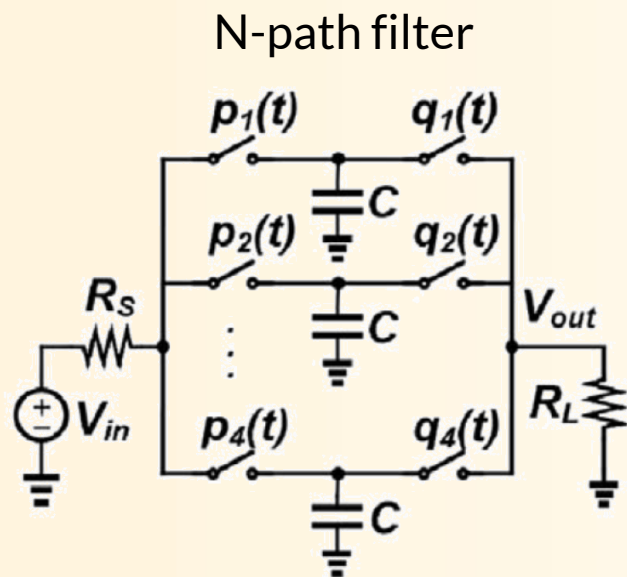
INTRODUCING PHASE DELAYS IN A SMALL FOOTPRINT



Topological propagation requires large phase delays across unit cells
Can we induce large phase delays within deeply subwavelength footprints?

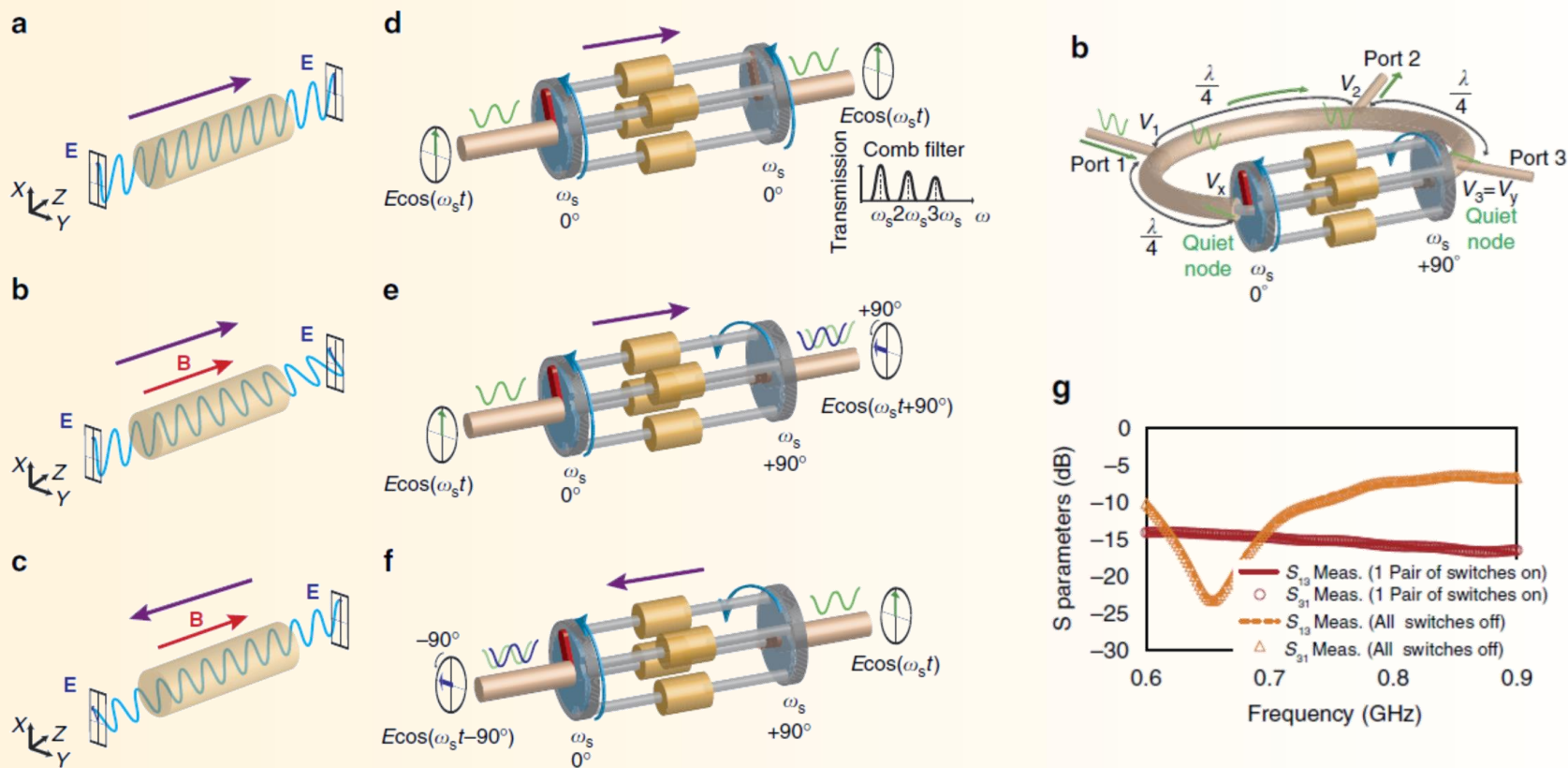
INTRODUCING PHASE DELAYS IN A SMALL FOOTPRINT

In a totally different field, the integrated circuit community has been solving this problem using time modulation!



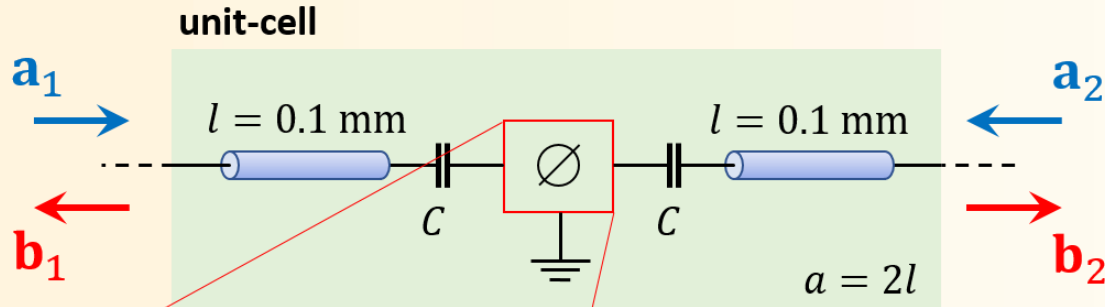
M. Darvishi, R. van der Zee, B. Nauta, IEEE JSSC 48, 2962 (2013)
N. Reiskarimian, et al., IEEE T-CS-II 63, 728 (2016)

NON-RECIPROCAL PHASE SHIFTER BASED ON N-PATH FILTERS



N. Reiskarimian, H. Krishnaswamy, *Nature Comm.* 7, 11217 (2016)

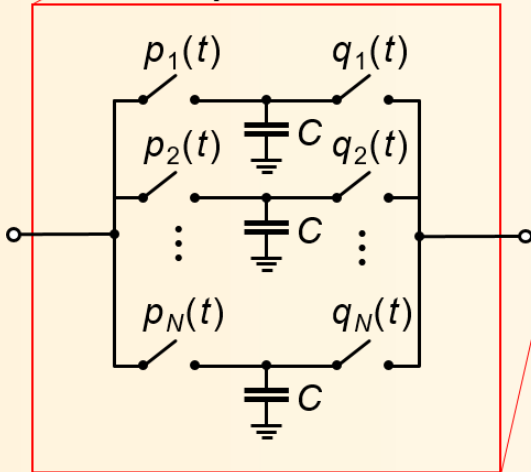
PROPAGATION ALONG A LATTICE OF N-PATH FILTERS



$$V_i = \frac{\mathbf{a}_i + \mathbf{b}_i}{\sqrt{Z_0}}$$

$$I_i = \frac{\mathbf{a}_i - \mathbf{b}_i}{\sqrt{Z_0}}$$

N-path filter



Floquet S-matrix

$$\begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix}$$



Bloch periodicity

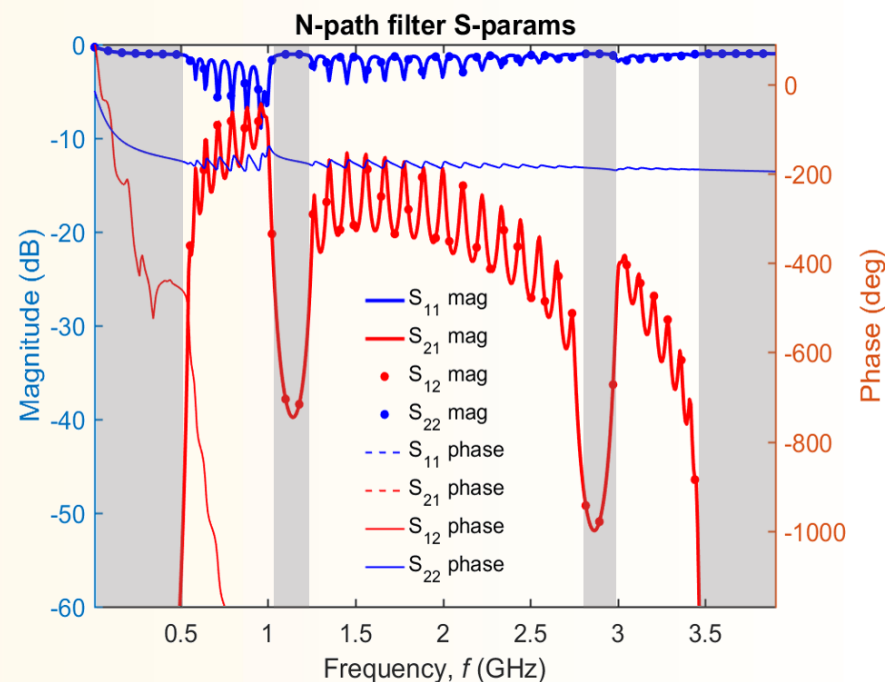
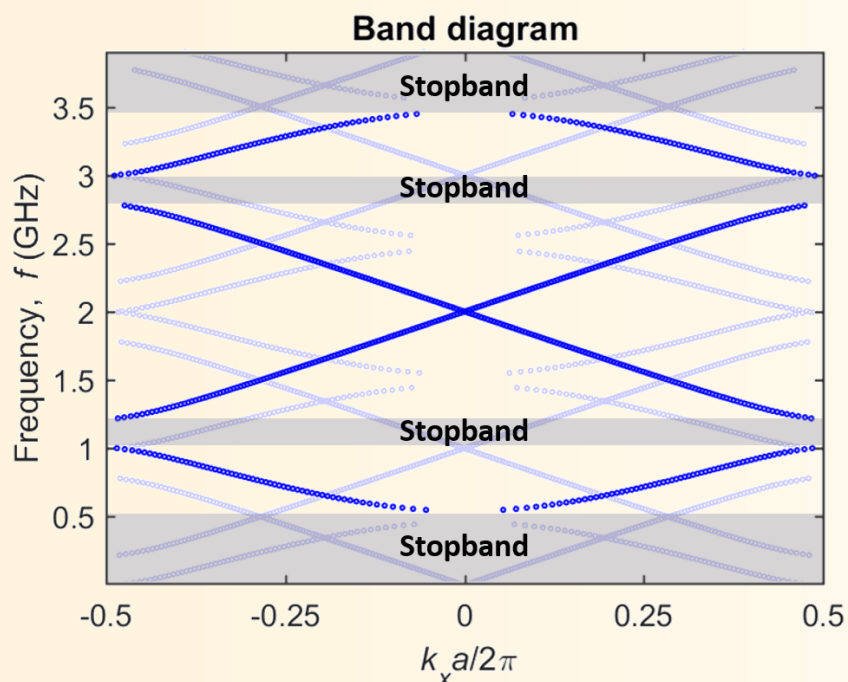
$$\begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix} = \begin{bmatrix} e^{-jk_x a} & \\ & e^{jk_x a} \end{bmatrix} \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix}$$

≡ Can be formulated* as an eigenvalue problem!

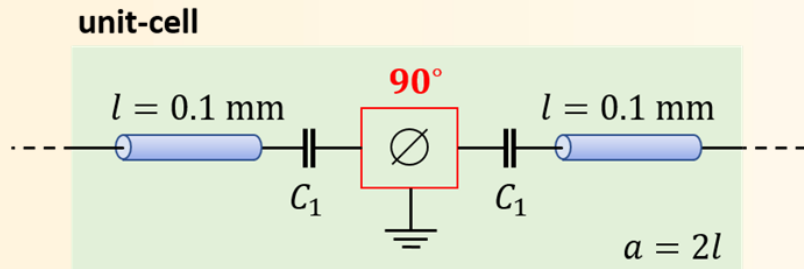
$$\begin{bmatrix} \mathbf{I} & -\mathbf{S}_{11} \\ & -\mathbf{S}_{21} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{S}_{12} & \\ \mathbf{S}_{22} & -\mathbf{I} \end{bmatrix} \begin{pmatrix} \mathbf{a}_2 \\ \mathbf{b}_2 \end{pmatrix} = e^{jk_x a} \begin{pmatrix} \mathbf{a}_2 \\ \mathbf{b}_2 \end{pmatrix}$$

*only for S_{12} and S_{21} do not have empty rows

PROPAGATION ALONG A LATTICE OF N-PATH FILTERS



NON-RECIPROCAL TRANSPORT



N-path filter params

$$f_s = 1 \text{ GHz}$$

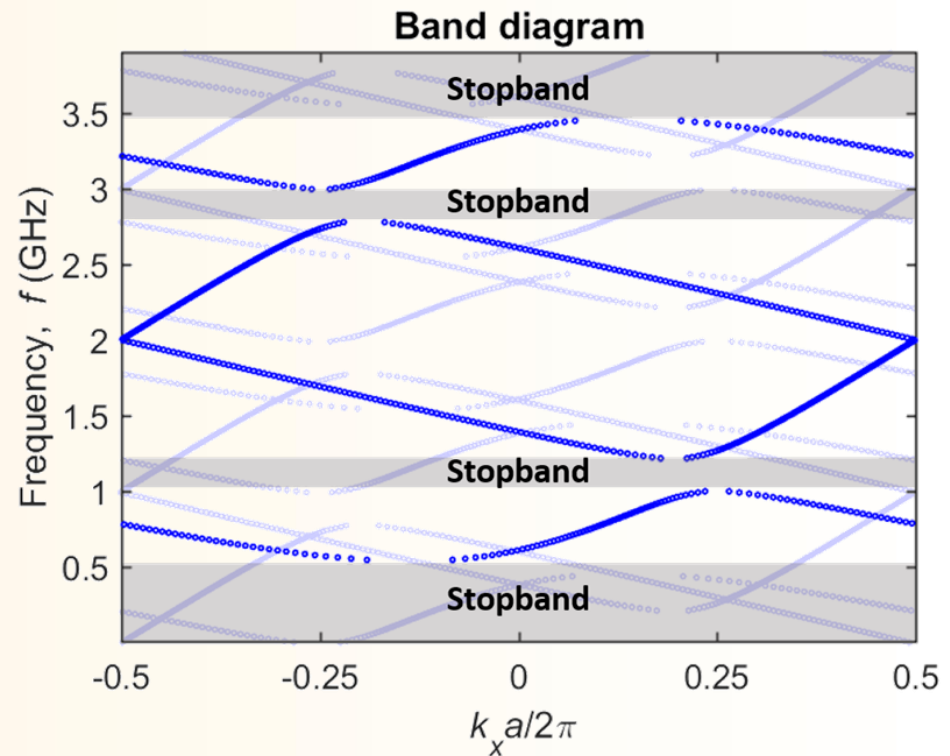
$$C = 25 \text{ ohm}$$

$N = 4$ (non-overlapping clocks)

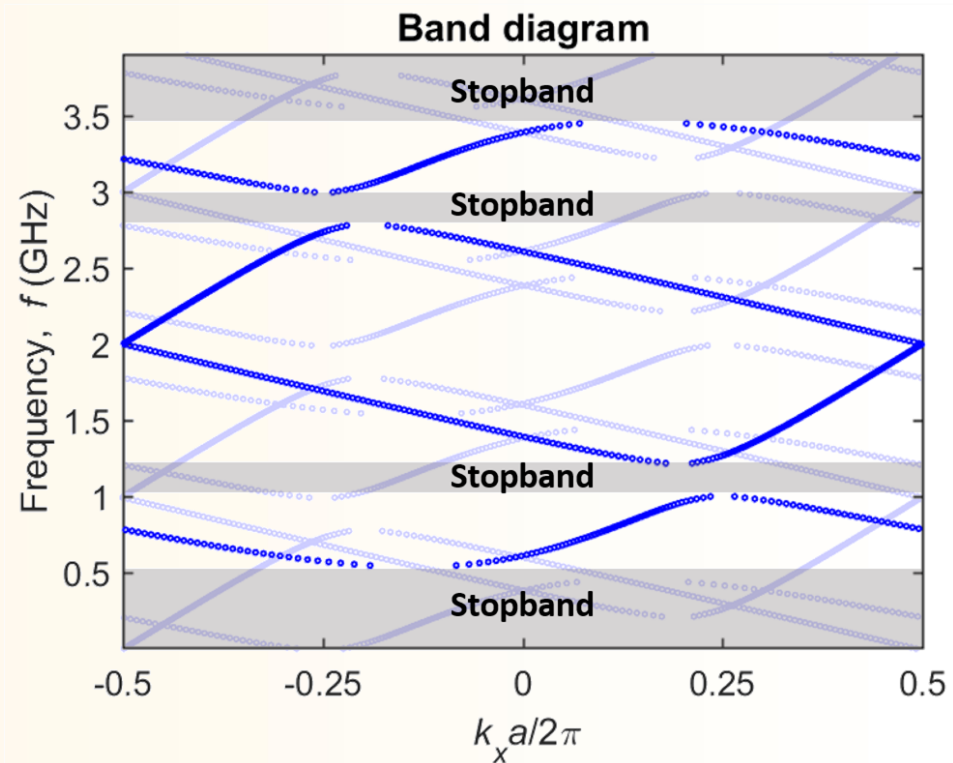
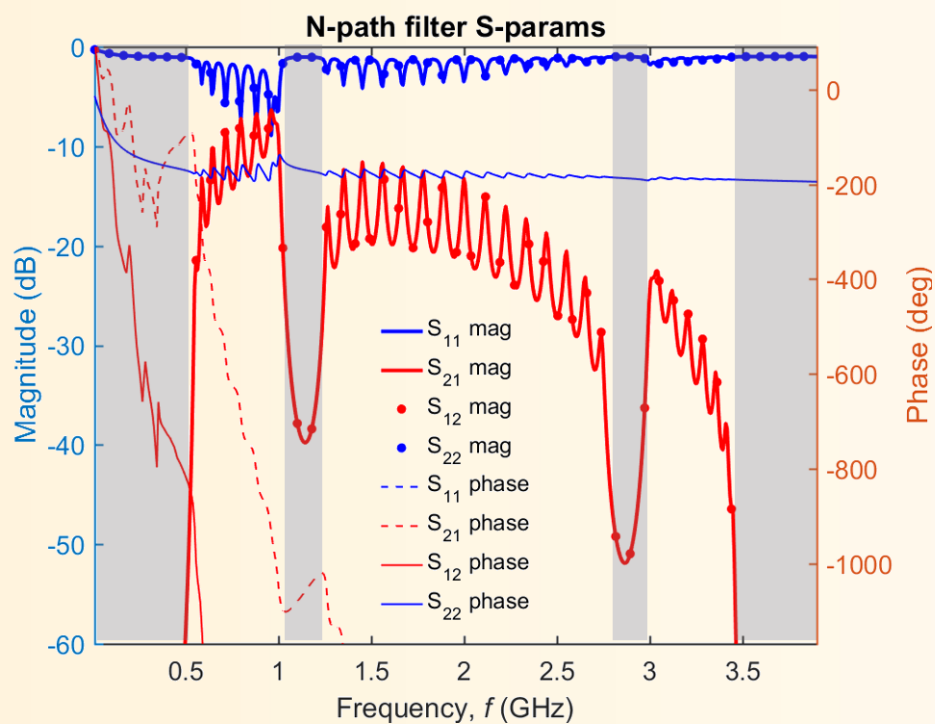
Transmission line params

$$C_1 = 50 \text{ ohm}$$

$$\beta = c_0/\sqrt{3}$$

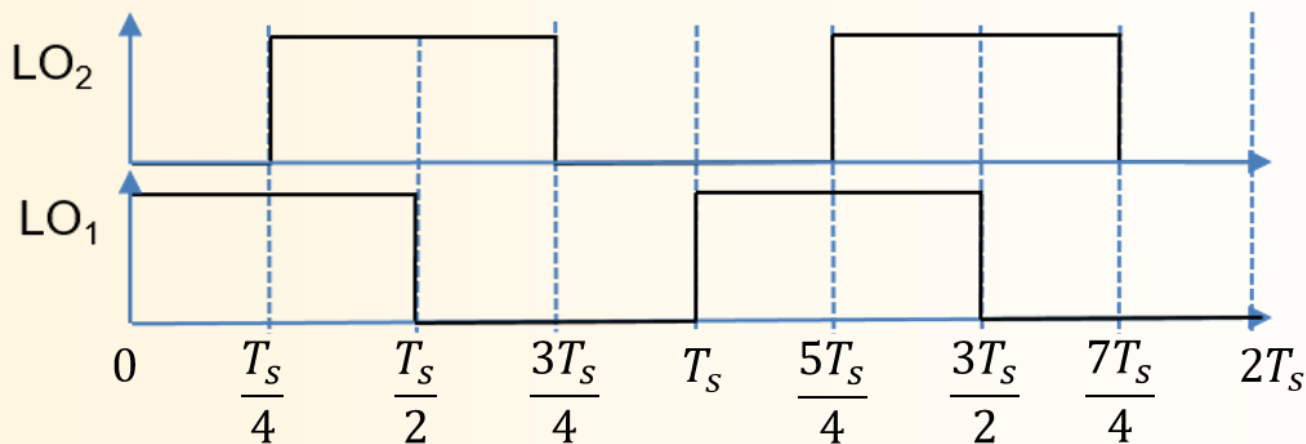
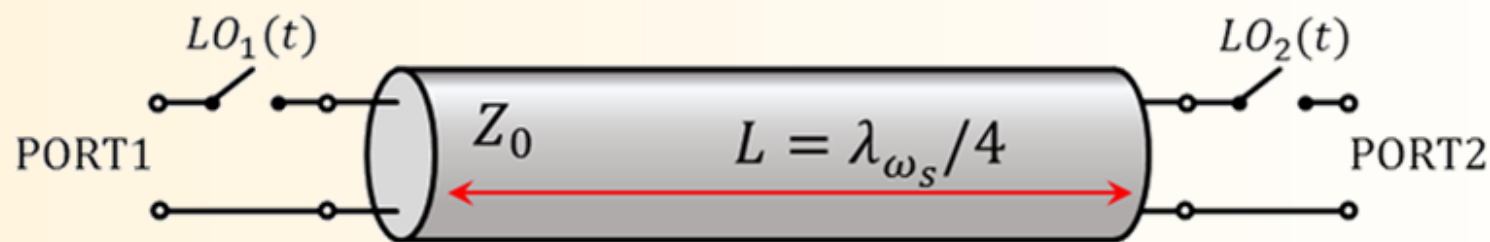


NON-RECIPROcity AND TOPOLOGICAL TRANSPORT



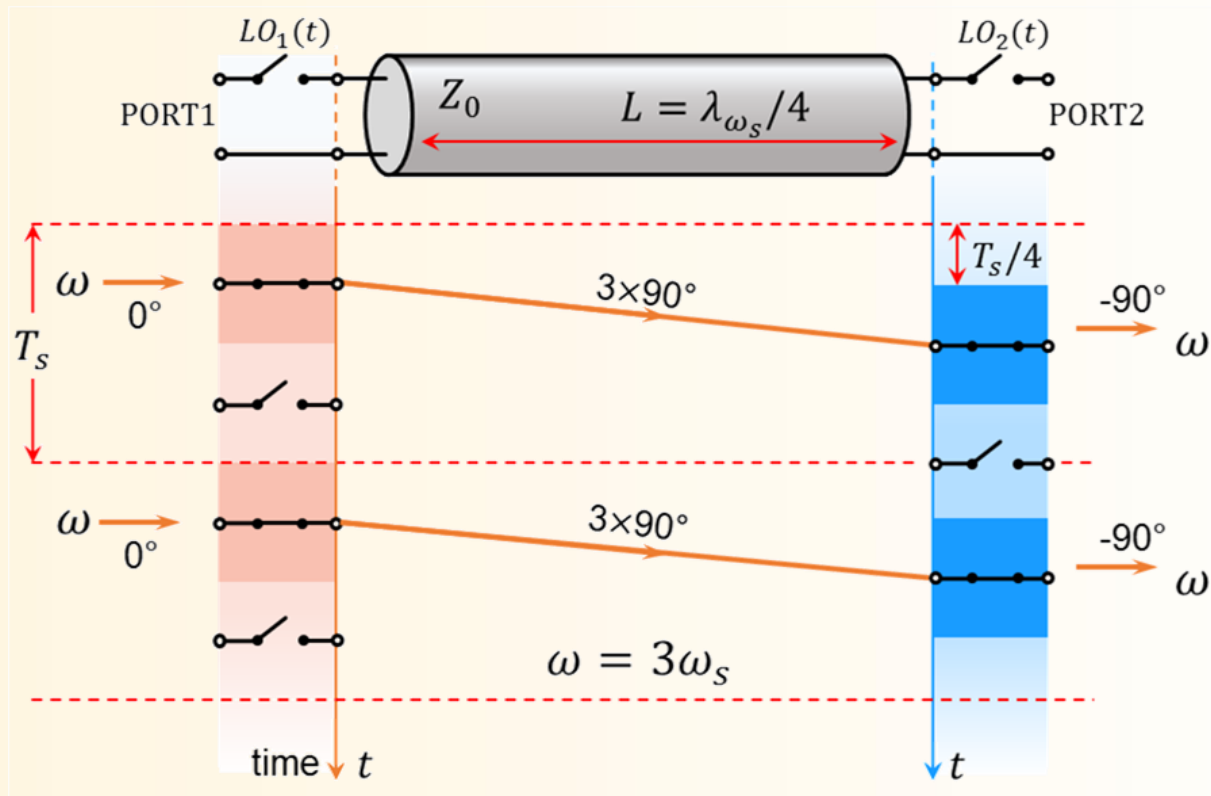
TOWARDS ULTRA-BROADBAND NON-RECIPROCAL ELEMENTS

Generalizing N-path filters: commutated delay lines



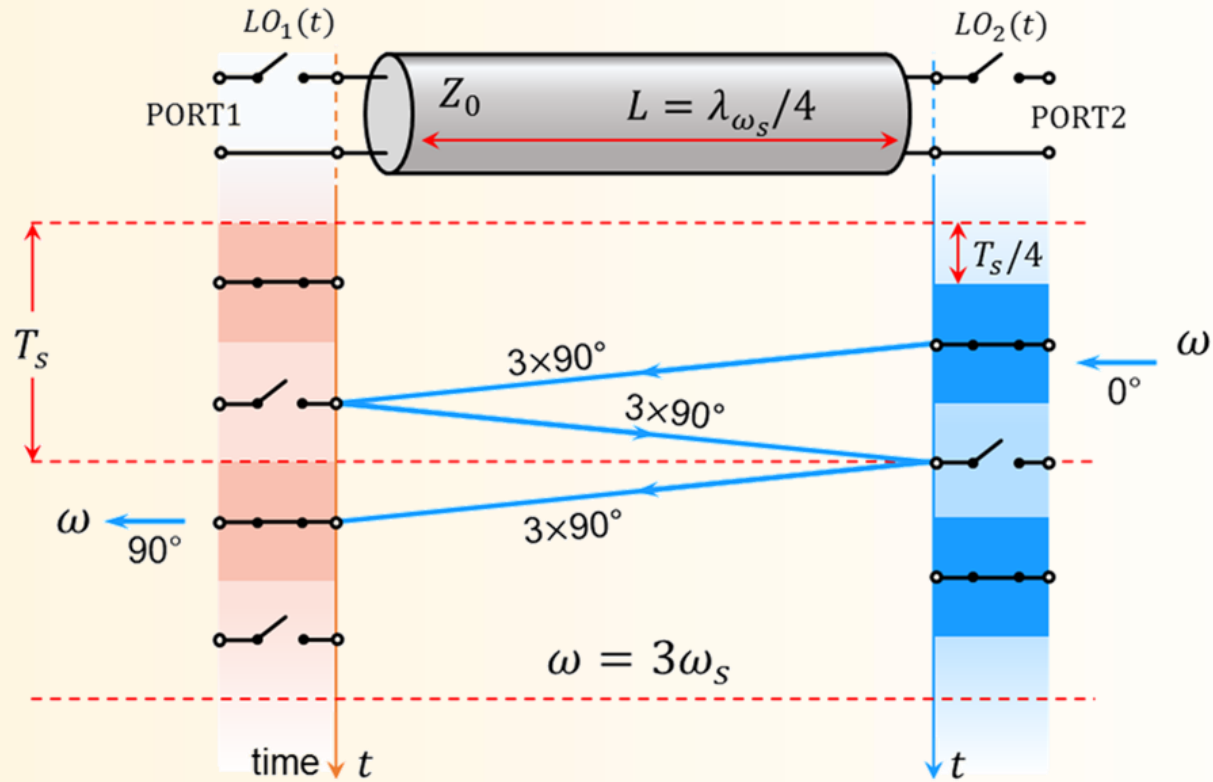
T. Dinc, M. Tymchenko, A. Nagulu, D. Sounas, A. Alù, H. Krishnaswamy, under review (2017)

FORWARD DIRECTION



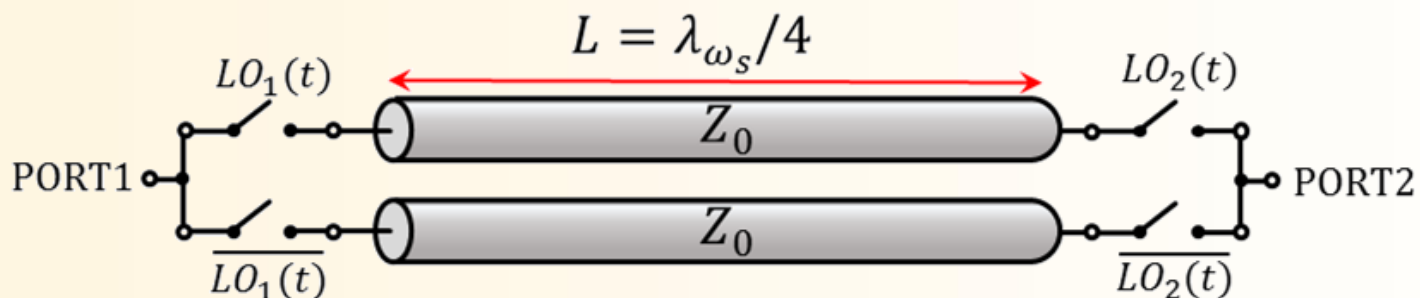
$$v_2^-(t) = v_1^+\left(t - \frac{T_s}{4}\right) LO_1\left(t - \frac{T_s}{4}\right) \rightarrow S_{21}(\omega) = \frac{1}{2} e^{-j\omega \frac{T_s}{4}}$$

BACKWARD DIRECTION



$$v_1^-(t) = v_2^+\left(t - \frac{3T_s}{4}\right) LO_2\left(t - \frac{3T_s}{4}\right) \rightarrow S_{12}(\omega) = \frac{1}{2} e^{-j\omega \frac{3T_s}{4}}$$

ULTRA-BROADBAND GYRATOR: SINGLE BALANCED



To avoid reflections, a parallel path with complementary switching can be added

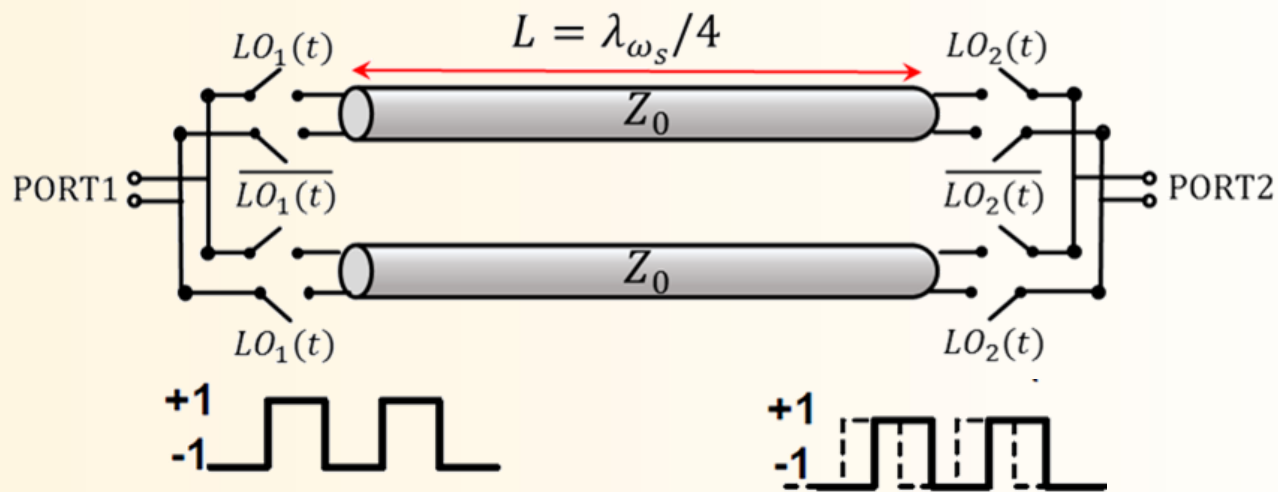
S-parameters

$$\begin{pmatrix} 0 & e^{-j\frac{3\pi}{2}\left(\frac{\omega}{\omega_s}\right)} \\ e^{-j\frac{\pi}{2}\left(\frac{\omega}{\omega_s}\right)} & 0 \end{pmatrix}$$

Non-Reciprocal Phase!

Non-reciprocal phase with no transmission loss

ULTRA-BROADBAND GYRATOR: DOUBLY BALANCED



S-parameters

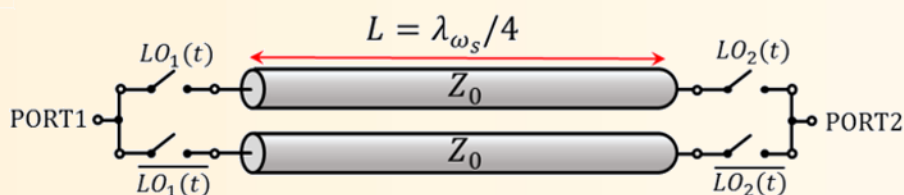
$$\begin{pmatrix} 0 & -e^{-j\frac{\pi}{2}\left(\frac{\omega}{\omega_s}\right)} \\ e^{-j\frac{\pi}{2}\left(\frac{\omega}{\omega_s}\right)} & 0 \end{pmatrix}$$

Non-Reciprocal
Phase

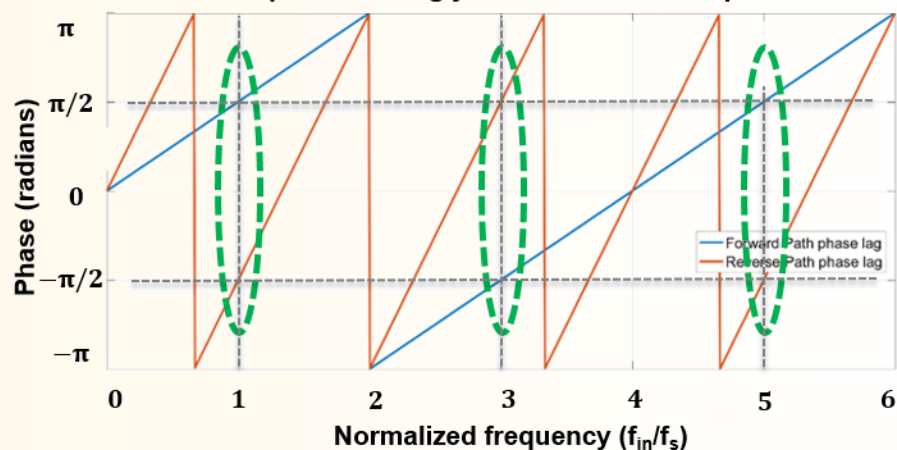
Non-reciprocal phase difference of 180° over a theoretically infinite BW

ULTRA-BROADBAND GYRATOR: DOUBLY BALANCED

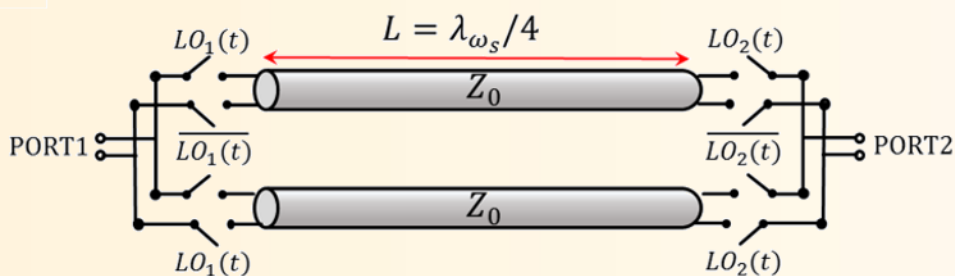
Singly – Balanced NR Element



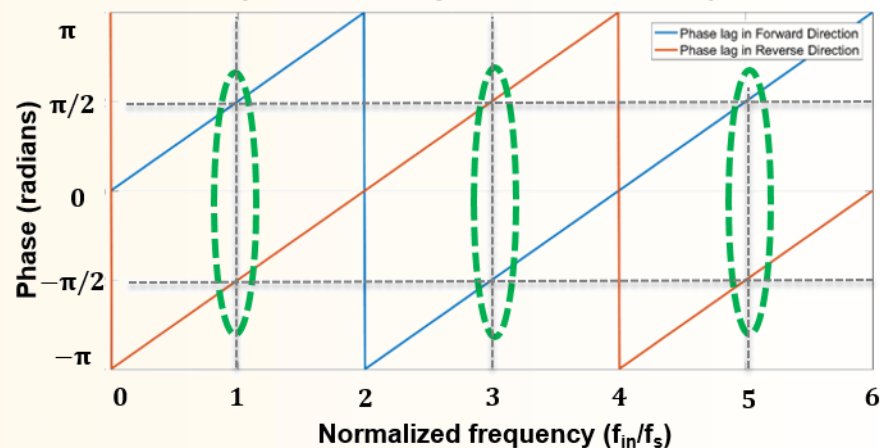
Phase Response of Singly-Balanced Non-Reciprocal Element



Doubly – Balanced NR Element

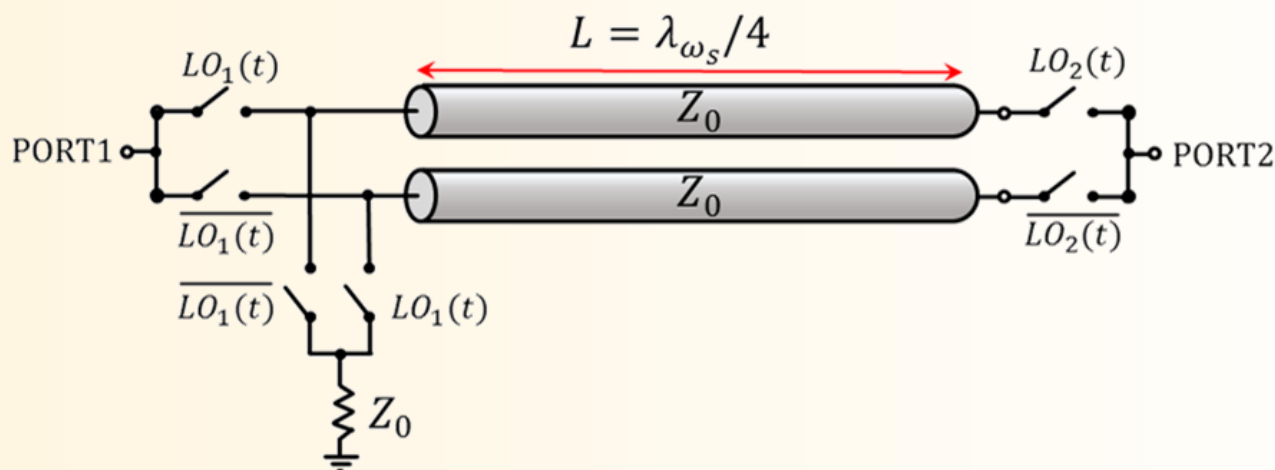


Phase Response of Doubly-Balanced Non-Reciprocal Element



Doubly – balanced structure has wider non-reciprocal phase bandwidth

ULTRA-BROADBAND ISOLATOR / CIRCULATOR

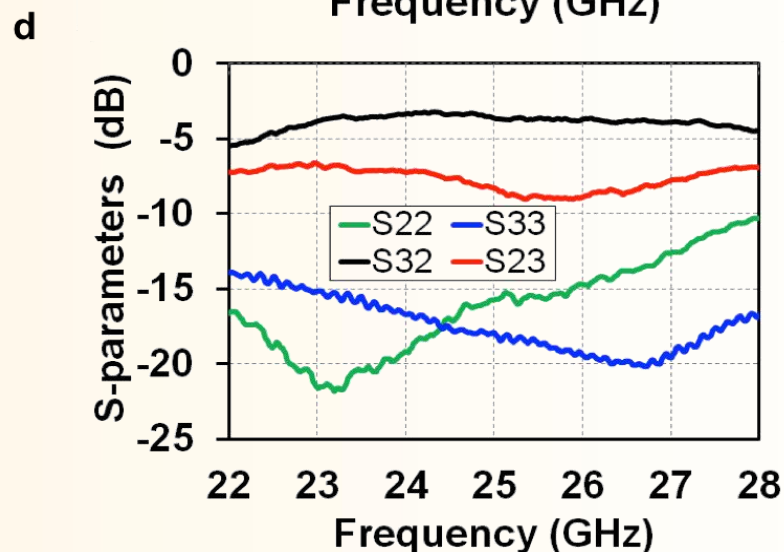
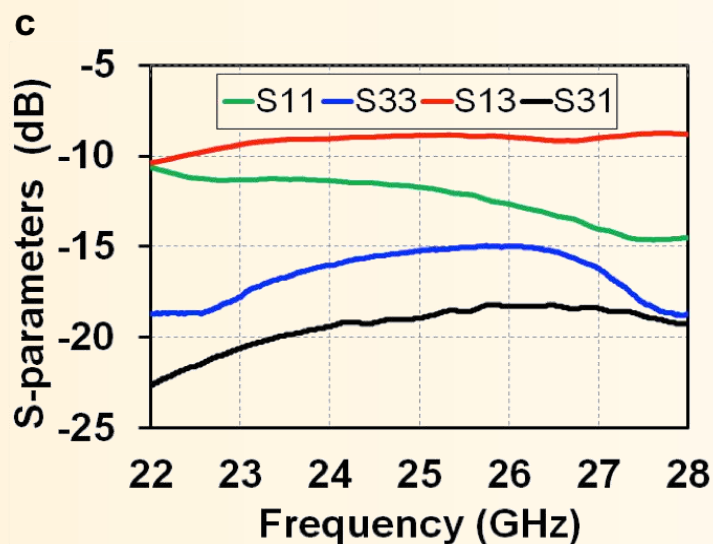
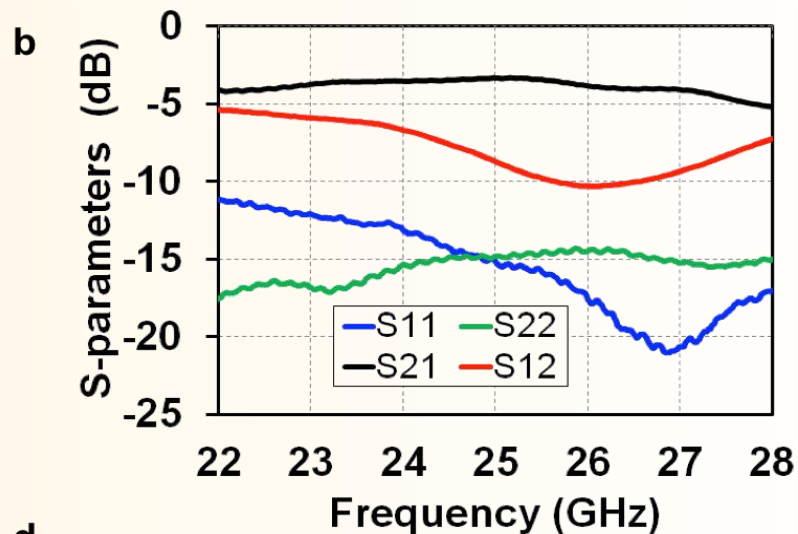
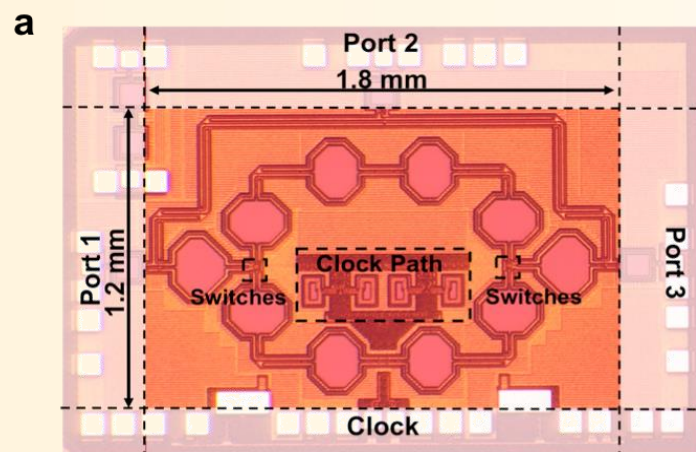


S-parameters

$$\begin{pmatrix} 0 & 0 \\ e^{-j\frac{\pi\omega}{2\omega_s}} & 0 \end{pmatrix}$$

Lossless transmission in forward direction and perfect isolation in reverse direction over a theoretically infinite BW

DOUBLY-BALANCED CIRCULATOR



T. Dinc, M. Tymchenko, A. Nagulu, D. Sounas, A. Alù, H. Krishnaswamy, under review (2017)

NON-RECIPROCALITY BASED ON NONLINEAR EFFECTS

Lorentz reciprocity theorem

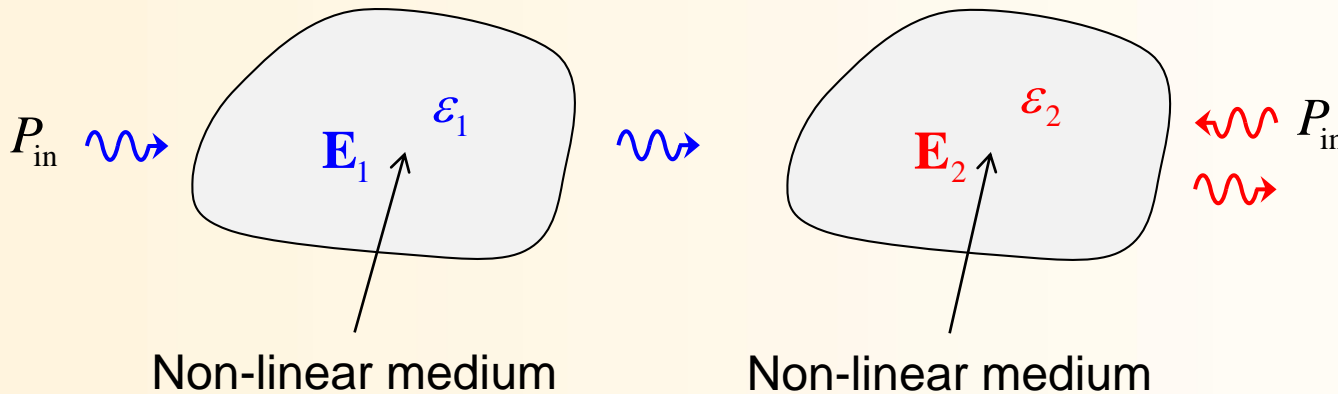
$$\iiint \mathbf{J}_1 \cdot \mathbf{E}_2 dV = \iiint \mathbf{J}_2 \cdot \mathbf{E}_1 dV$$

$$\bar{\epsilon} = \bar{\epsilon}^T$$

$$\bar{\mu} = \bar{\mu}^T$$

Time-invariant materials

Linear materials



Chi-3 non-linearity

$$\Delta\epsilon_{NL} = \chi^{(3)} |\mathbf{E}|^2$$

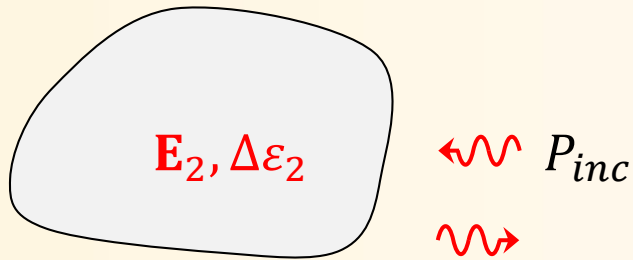
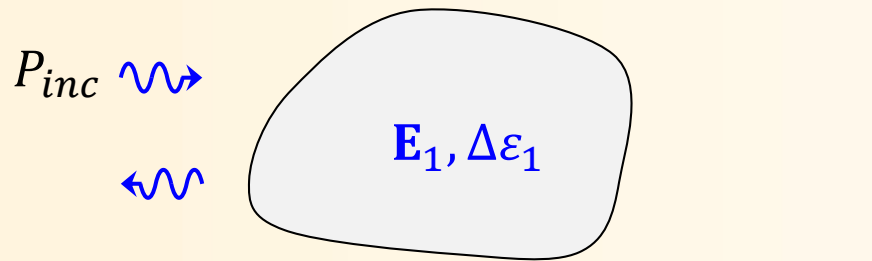
P. Saboo, J. Joseph, *Appl. Opt.* **52**, 8252–8257 (2013)

L. Fan, et al., *Opt. Lett.* **38**, 1259–1261 (2013)

Y. Shi, Z. Yu, S. Fan, *Nature Photon.* **9**, 388–392 (2015)

A. M. Mahmoud, A. Davoyan, N. Engheta, *Nature Comm.* **6**, 8359 (2015)

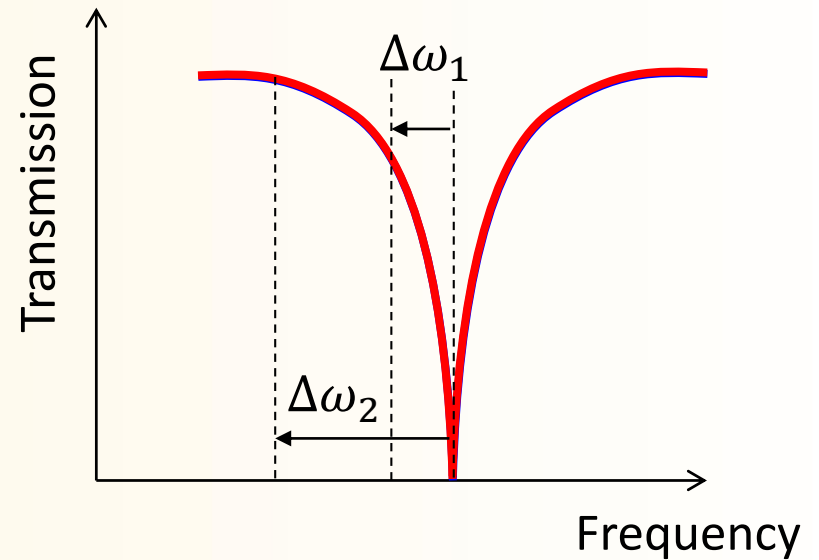
BASIC OPERATION OF NONLINEAR ISOLATORS



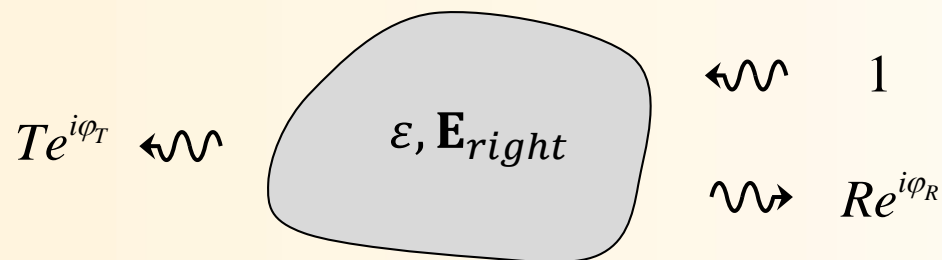
$$\kappa = \frac{|\mathbf{E}_2|^2}{|\mathbf{E}_1|^2} > 1$$

$$\varepsilon_{NL} = \varepsilon + \chi^{(3)}|\mathbf{E}|^2$$

$$\Delta\omega = -\omega_0 \frac{\int \Delta\varepsilon |\mathbf{E}_0|^2 dV}{\int \varepsilon |\mathbf{E}_0|^2 dV}$$

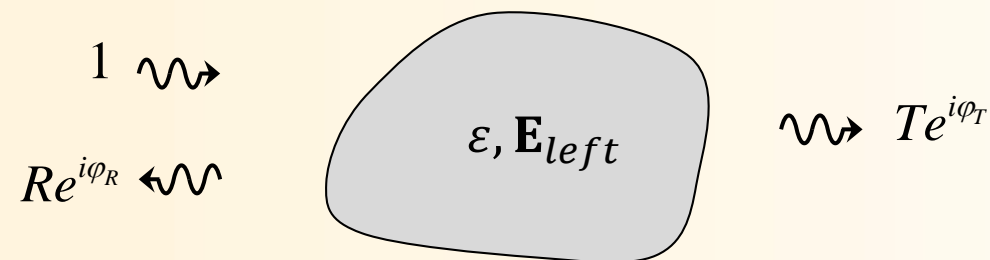


TRADE-OFF BETWEEN FIELD ASYMMETRY & INSERTION LOSS

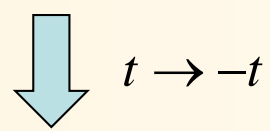


$$\mathbf{E}_{left}^* = Re^{-i\phi_R} \mathbf{E}_{left} + Te^{-i\phi_T} \mathbf{E}_{right}$$

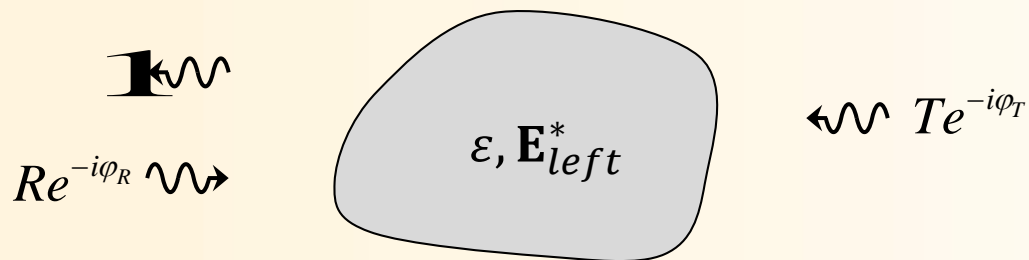
$$-|\mathbf{E}_{left}|^2 \leq \text{Re}\{e^{-i\phi_R} \mathbf{E}_{left}^2\} \leq |\mathbf{E}_{left}|^2$$



$$\kappa = |\mathbf{E}_{right}|^2 / |\mathbf{E}_{left}|^2$$

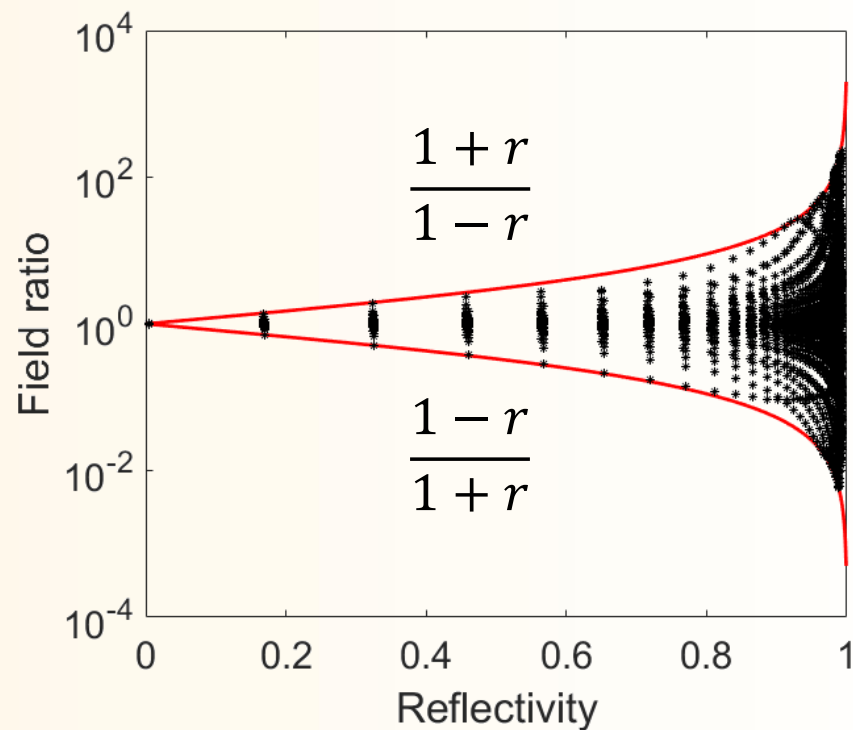
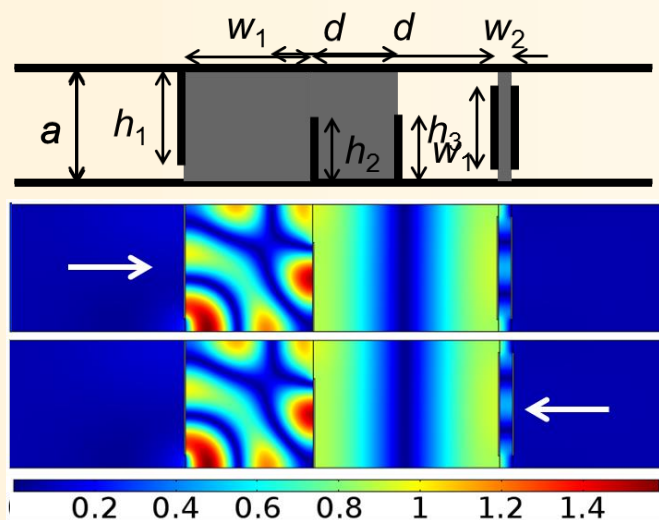


$$\frac{1-R}{1+R} \leq \kappa \leq \frac{1+R}{1-R}$$



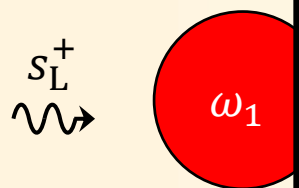
$$T^2 \leq \frac{4\kappa}{(\kappa + 1)^2}$$

NUMERICAL VALIDATION



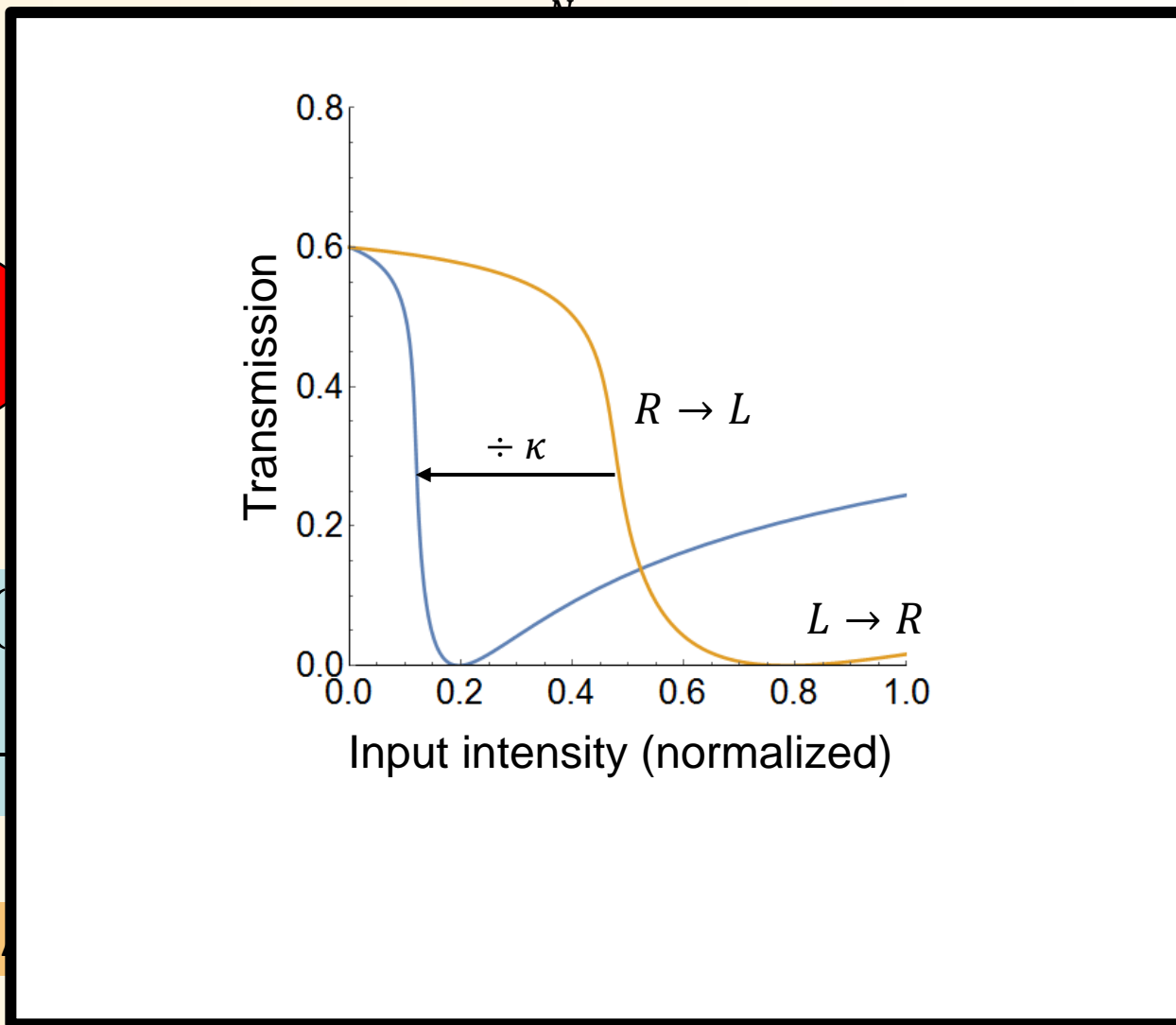
D. Sounas, A. Alù, *Phys. Rev. Lett.* **118**, 154302 (2017)

COUPLED-MODE ANALYSIS



$$\Delta\tilde{\omega}_{1L/R} \left[\left(\frac{\omega_1 - \omega}{\gamma} \right)^2 + \frac{\Gamma}{\gamma} \right]$$

$$\Delta\tilde{\omega}_{1L} =$$



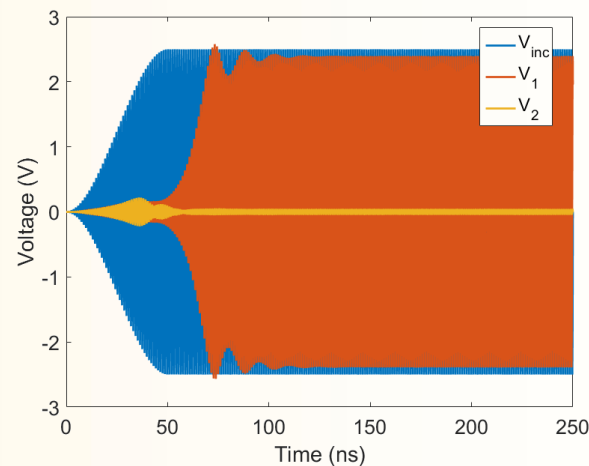
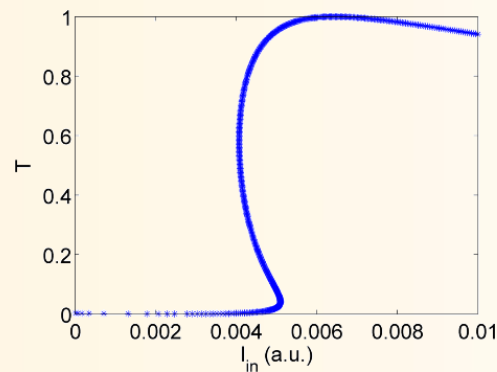
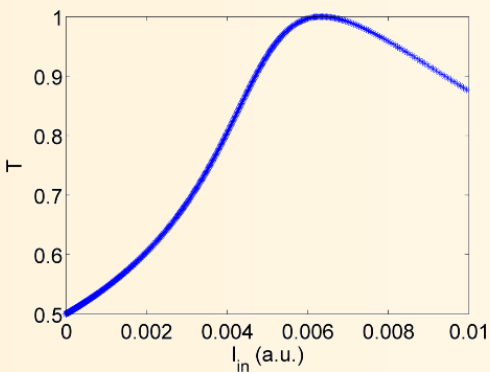
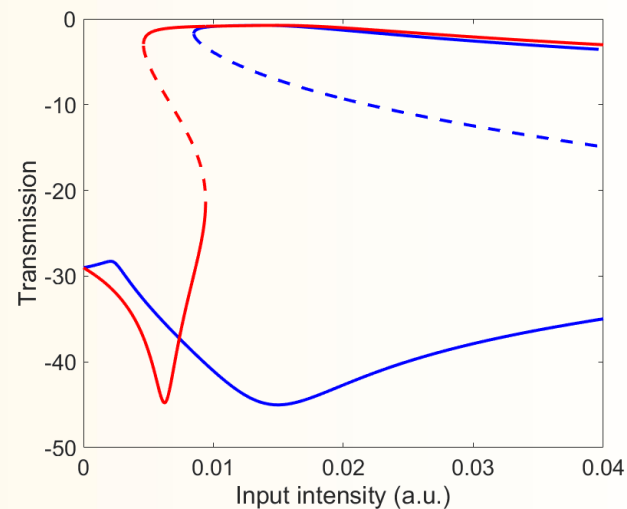
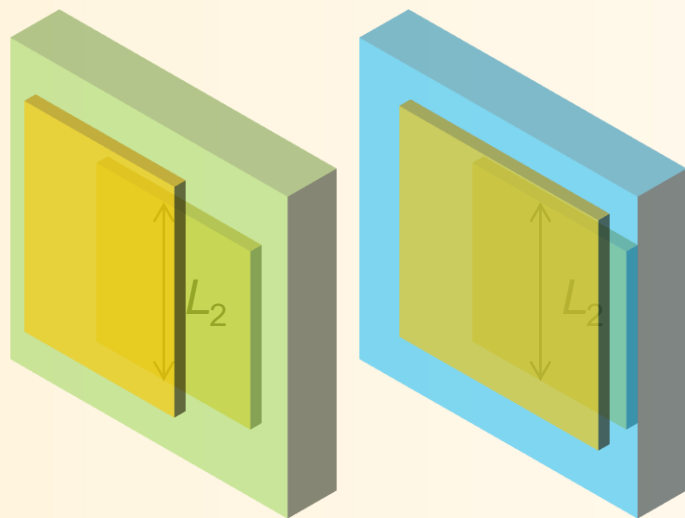
$$\frac{|a_1|^2}{|a_0|^2}$$

$$\frac{\text{Im}\{\overline{H}^{-1}\}_{11}}{\Gamma}$$

$$\text{Re}\{(\mathbf{H}^{-1})_{11}\}$$

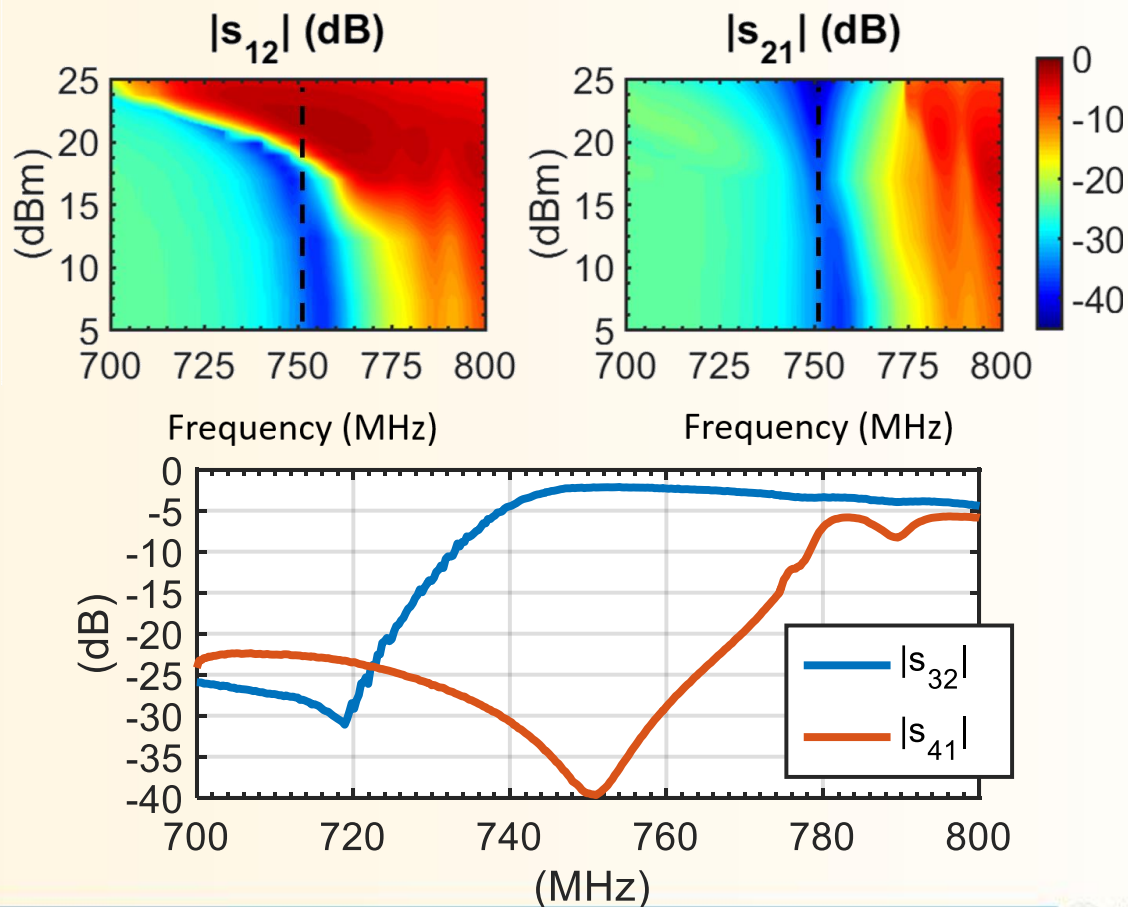
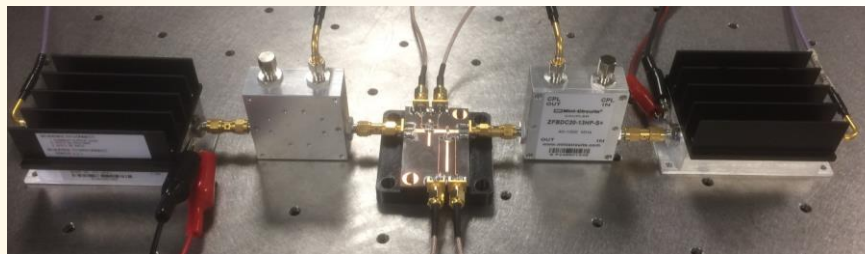
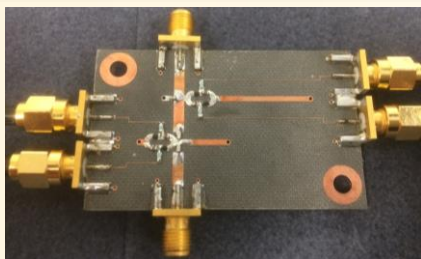
$$\frac{\omega_1 - \omega}{\gamma}$$

IDEAL ISOLATOR BASED ON TWO NON-LINEAR RESONATORS

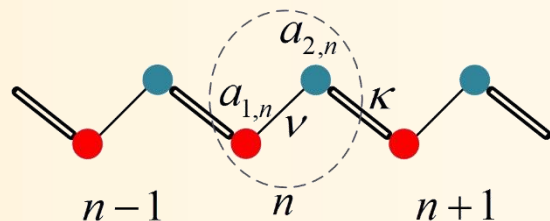


D. L. Sounas, J. Soric, and A. Alù, *under review* (2017)

EXPERIMENTAL RESULTS AT RF



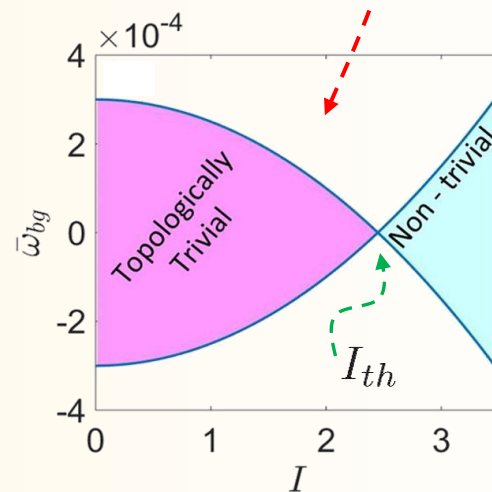
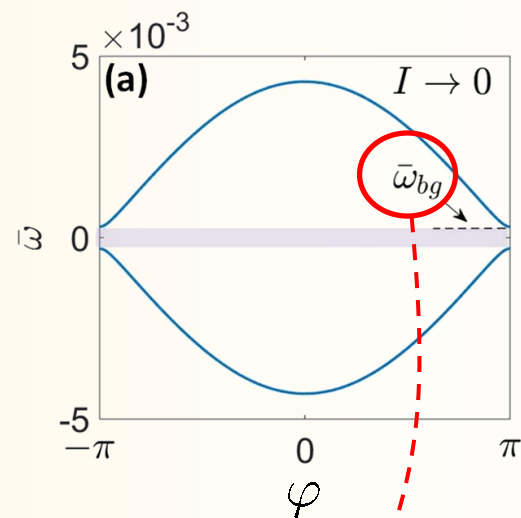
NONLINEARITY-INDUCED TOPOLOGICAL TRANSITIONS



$$i \frac{d\Psi_n}{dt} = \Omega \Psi_n + \mathbf{K}_m \Psi_{n-1} + \mathbf{K}_p \Psi_{n+1}$$

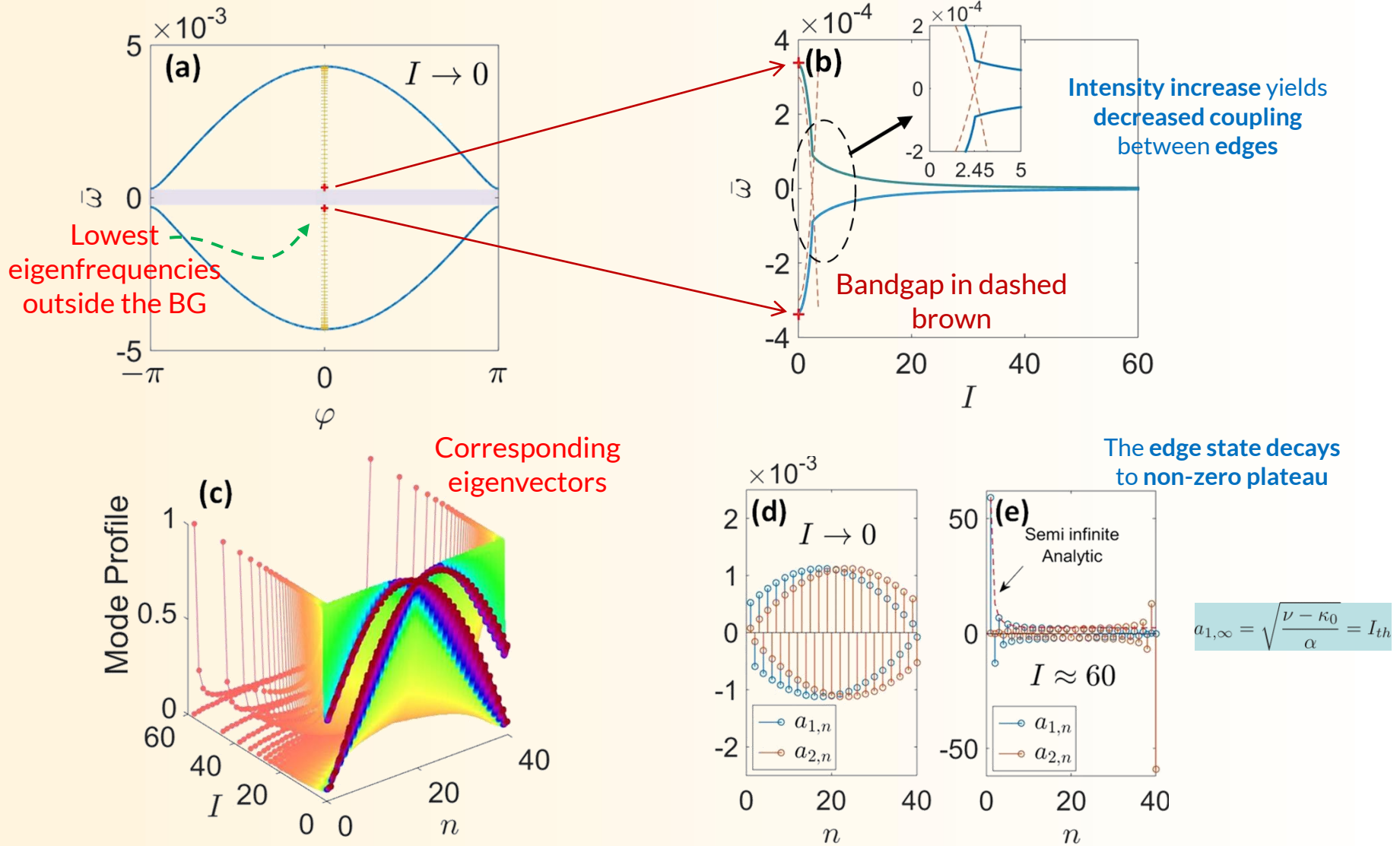
$$\nu > \kappa_0$$

$$\bar{\omega}_{bg} = \pm |\nu - \kappa_0 - \alpha I^2| \rightarrow I_{th} = \sqrt{\frac{\nu - \kappa_0}{\alpha}}$$



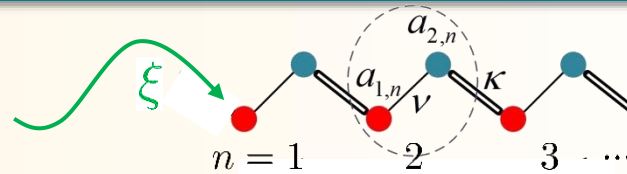
Y. Hadad, A. B. Khanikaev, and A. Alù, *Phys. Rev. B* **93**, 155112 (2016)

FINITE ARRAY: 40 ELEMENT CHAIN

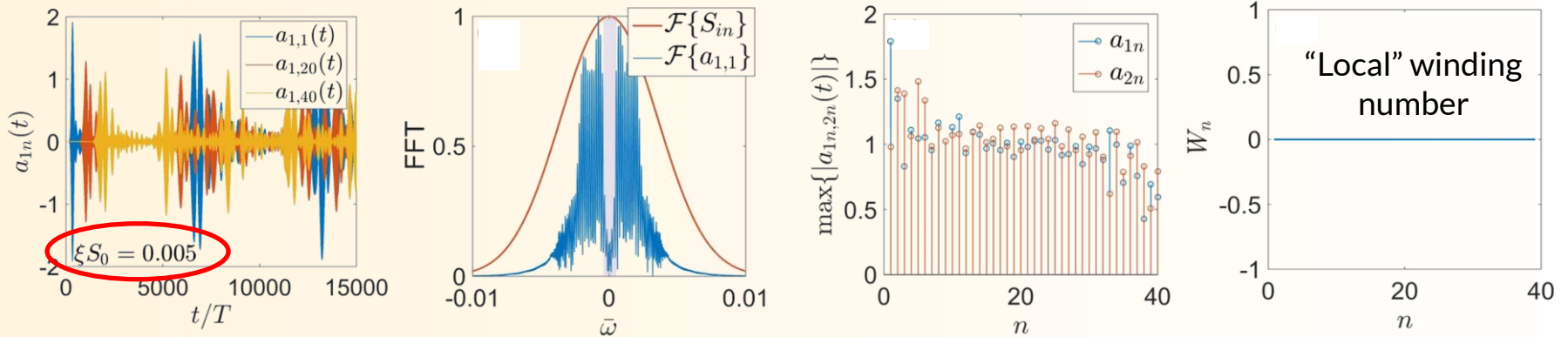


NONLINEARITY-INDUCED TOPOLOGICAL TRANSITIONS

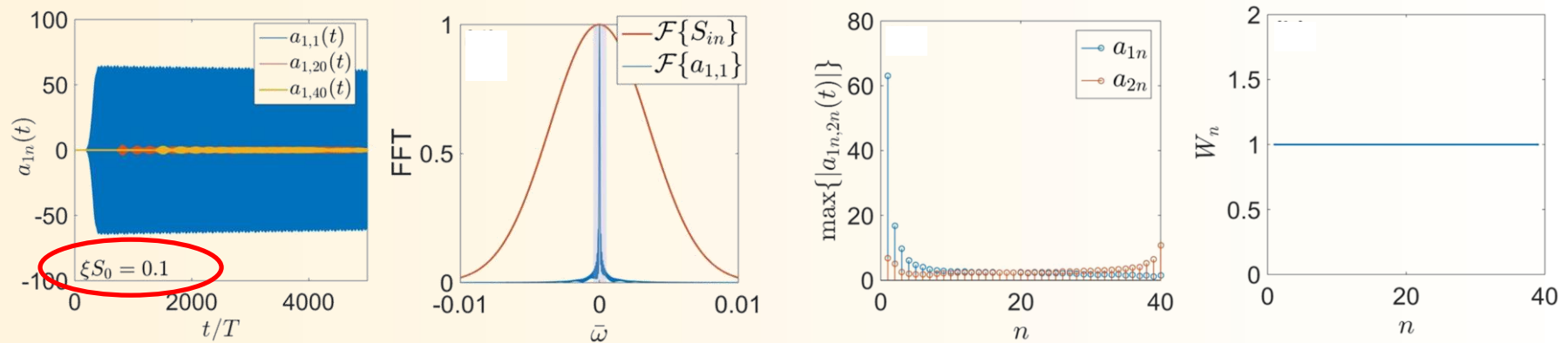
$$S_{in} = S_0 e^{i\omega t} e^{-(t-t_0)^2/T^2}$$



Low input intensity:

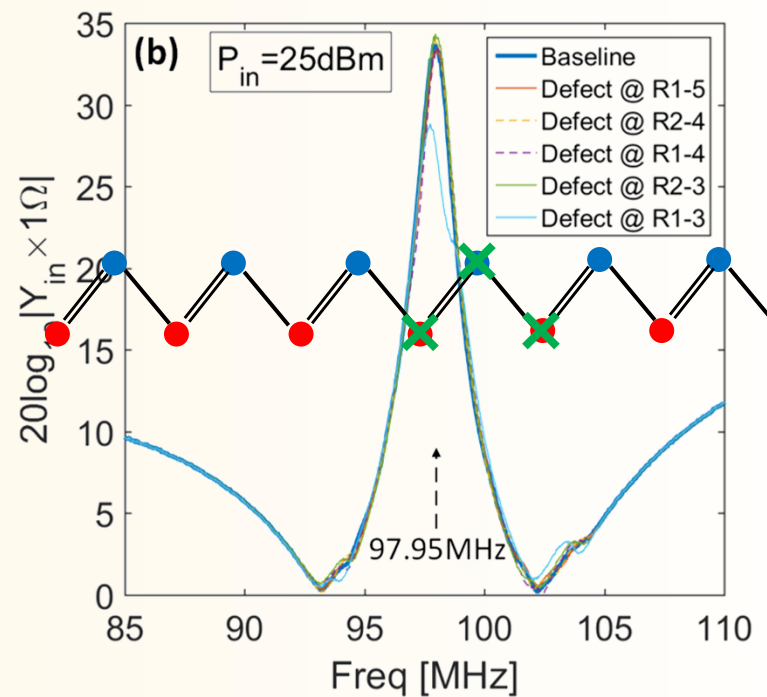
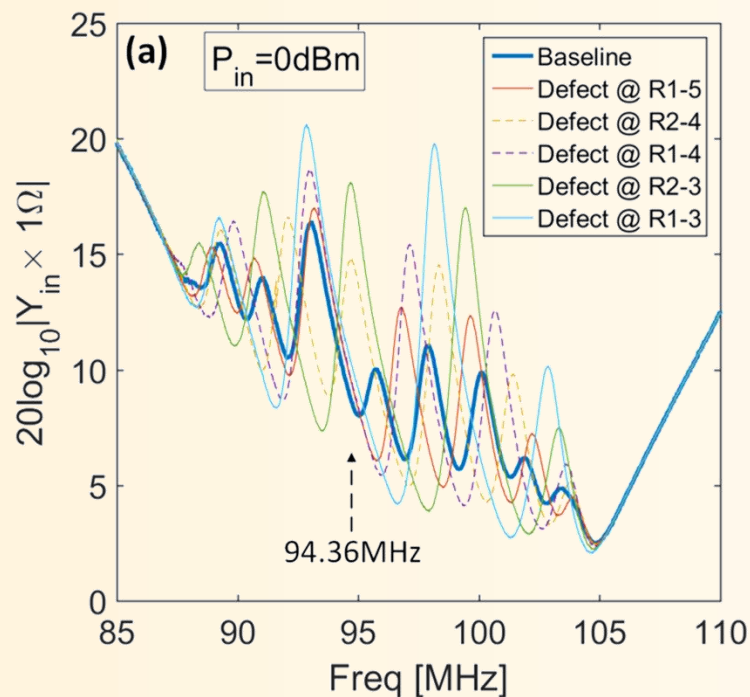
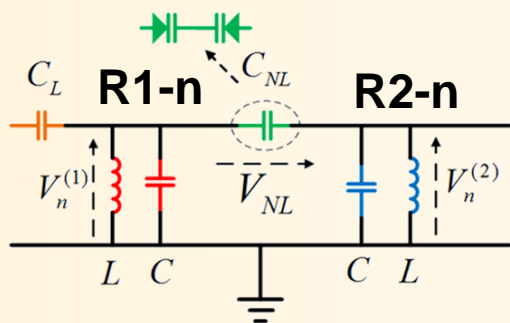


High input intensity:



Y. Hadad, A. B. Khanikaev, and A. Alù, *Phys. Rev. B* **93**, 155112 (2016)

NONLINEARITY-INDUCED TOPOLOGICAL TRANSITIONS



EQUIVALENT CONTINUUM MODEL

Stored energy in dimer $q_n(t) = \sqrt{|a_n^{(1)}|^2 + |a_n^{(2)}|^2}$

Relative phase in dimer $\alpha_n(t) = \tan^{-1} \left(j \frac{a_n^{(2)}}{a_n^{(1)}} \right)$



Continuum limit

Moving frame $\xi = x - ct$

$$q_n(t), \alpha_n(t) \mapsto q(x,t), \alpha(x,t) \mapsto q(\xi), \alpha(\xi)$$

Continuum model:

$$\frac{d}{d\xi} \begin{pmatrix} q \\ \alpha \end{pmatrix} = F(q) \begin{pmatrix} \kappa\Delta q \cos 2\alpha \\ -c - \kappa\Delta \sin 2\alpha \end{pmatrix}$$

Fixed points $\underline{x}^* = (q^*; \alpha^*)$

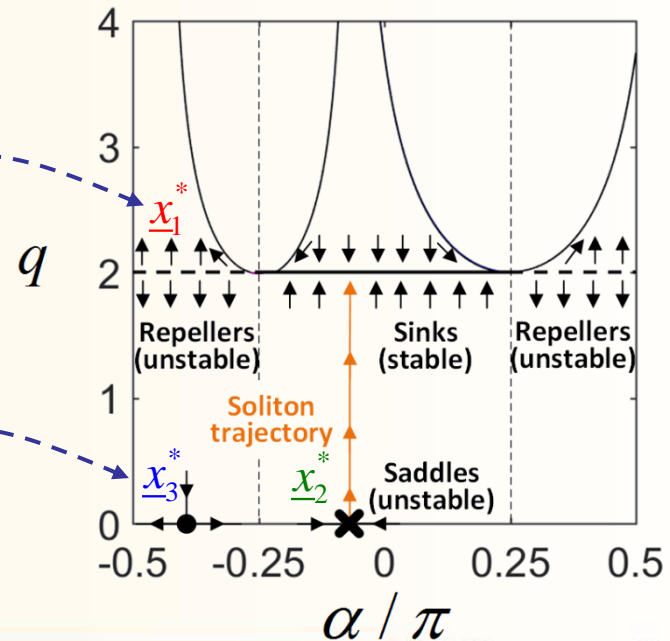
- line of unstable/stable nodes

$$\underline{x}_1^* \rightarrow q_1^* = (v_0 - \kappa) / (\kappa - v_\infty), \quad \forall \alpha_1^*$$

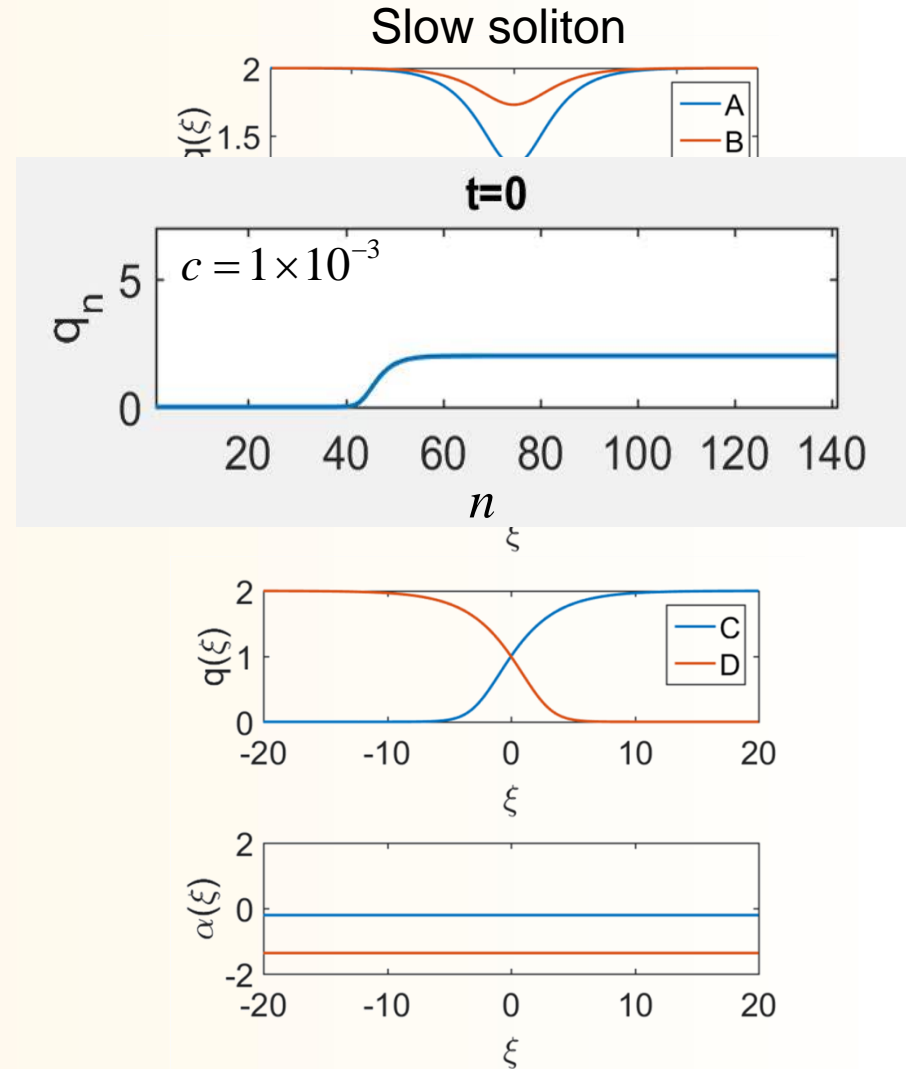
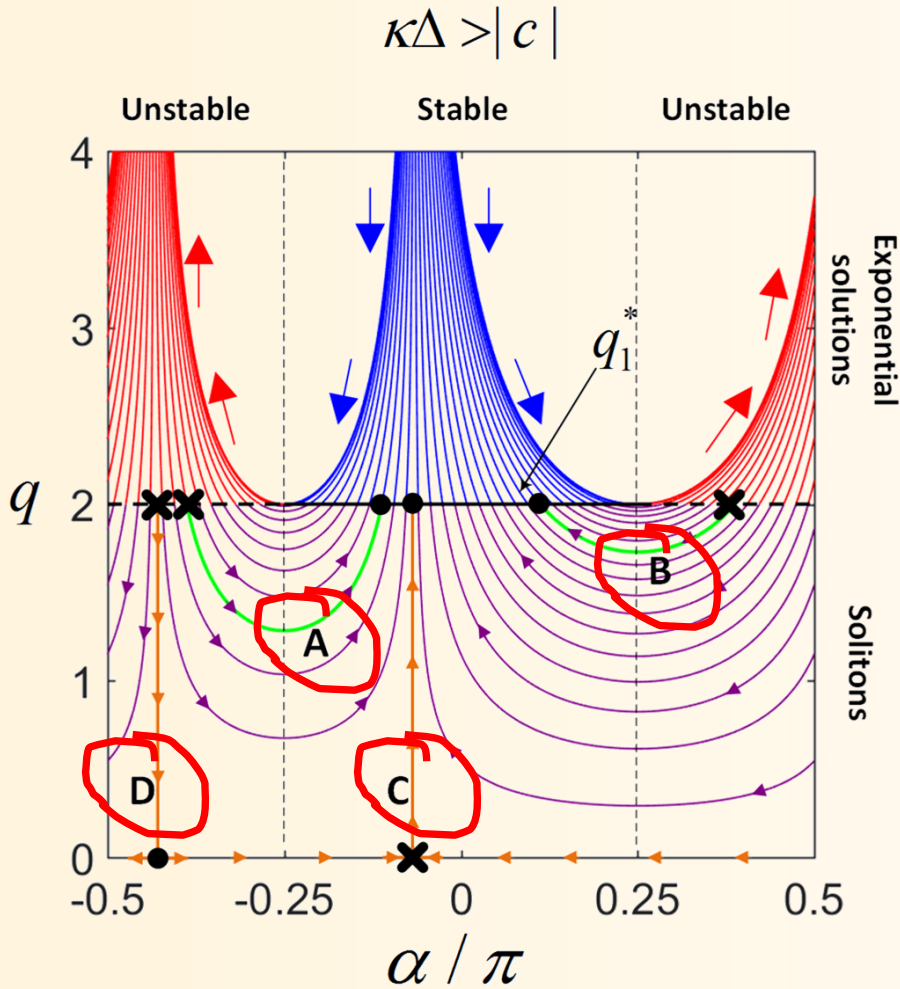
- Saddle (unstable) nodes (only if $|c| < \kappa\Delta$)

$$\underline{x}_2^* \rightarrow q_2^* = 0, \quad \alpha_2^* = -0.5 \sin^{-1} [c / \kappa\Delta],$$

$$\underline{x}_3^* \rightarrow q_3^* = 0, \quad \alpha_3^* = -\pi/2 - \alpha_2^*$$

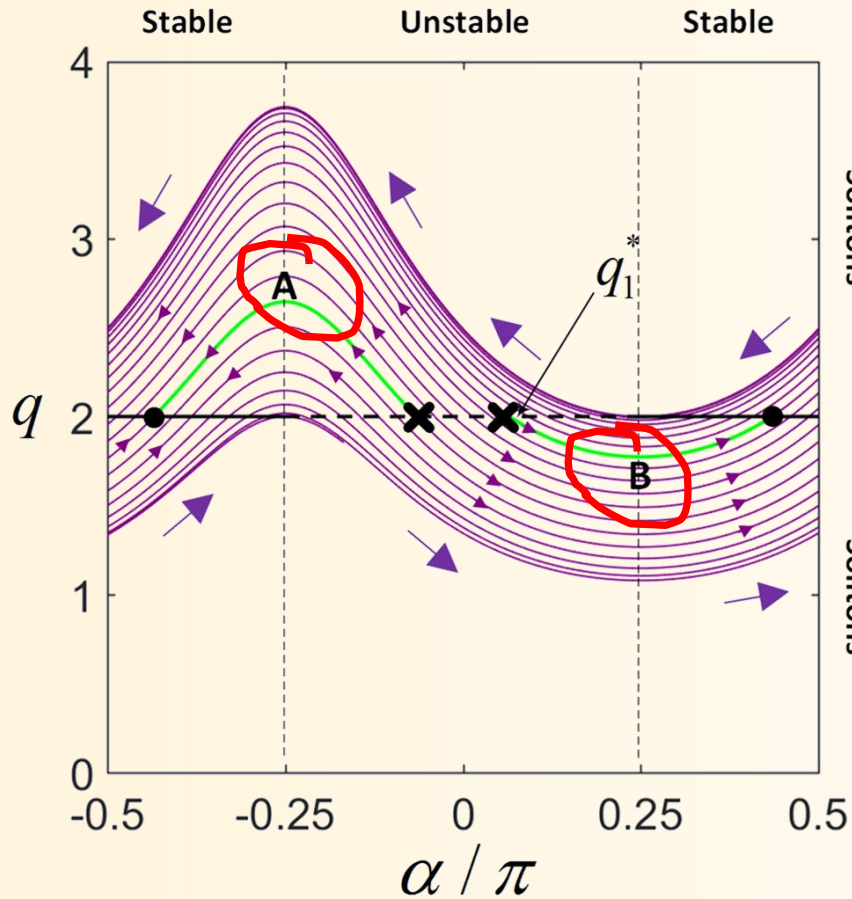


PHASE DIAGRAMS AND SOLITONS

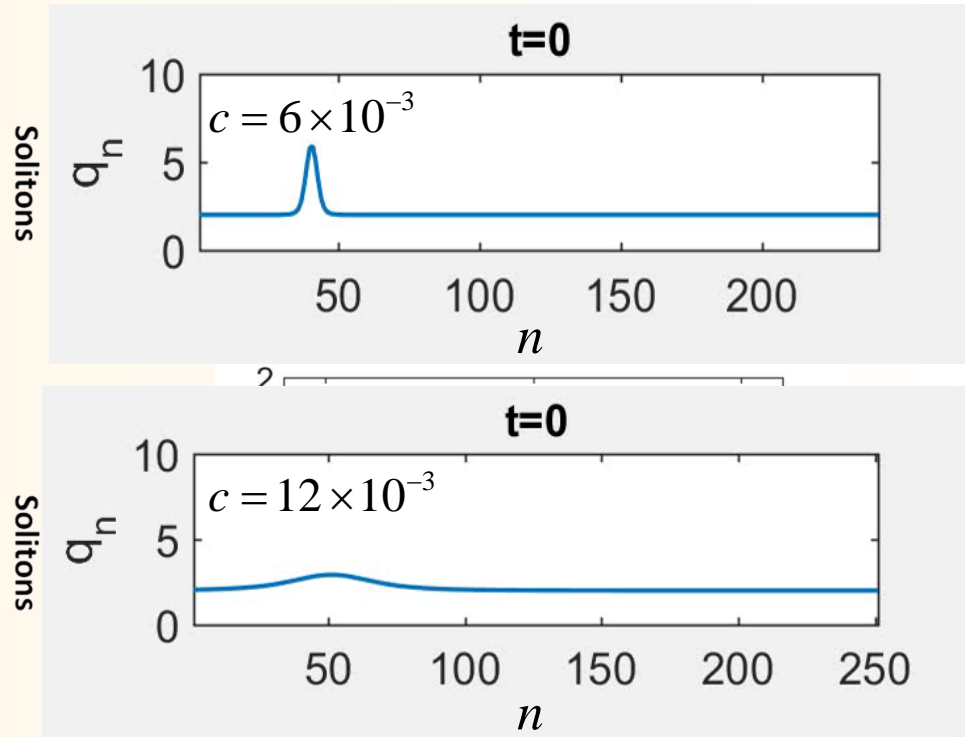


PHASE DIAGRAMS AND SOLITONS

$$\kappa\Delta < |c|$$

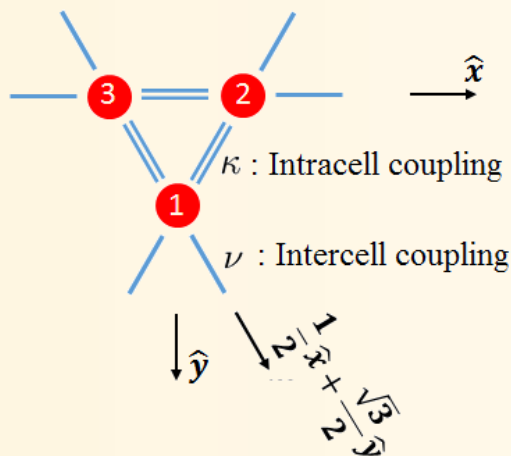


Fast soliton



2D NONLINEAR TOPOLOGICAL PHOTONIC INSULATOR

The Elementary Cell: Kagome Lattice



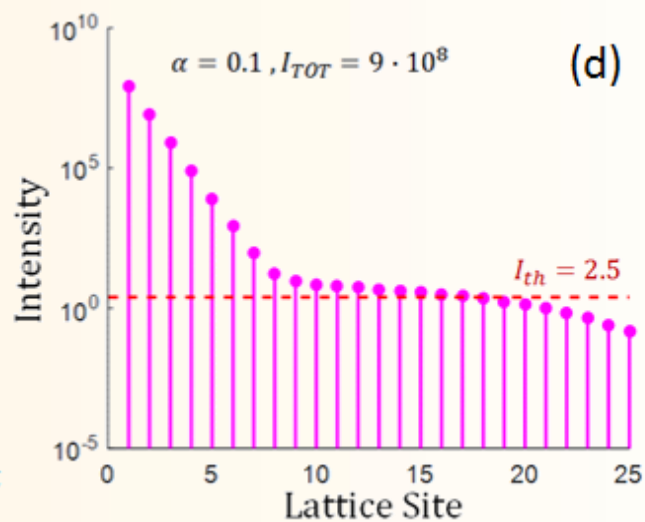
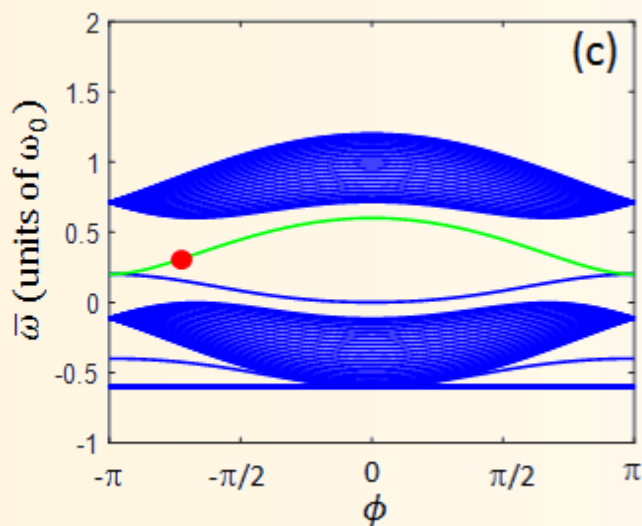
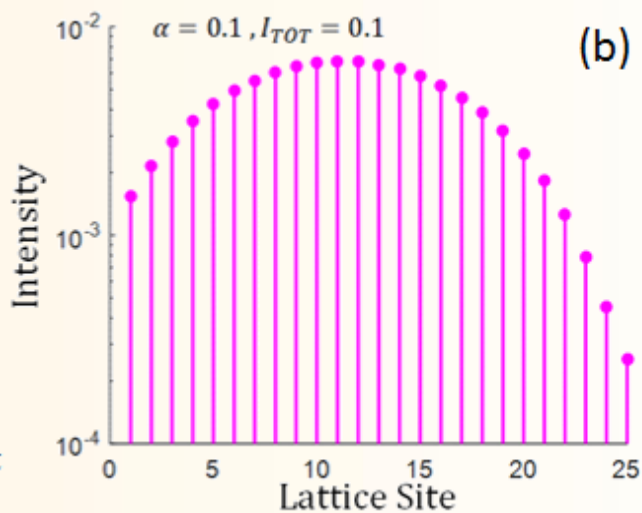
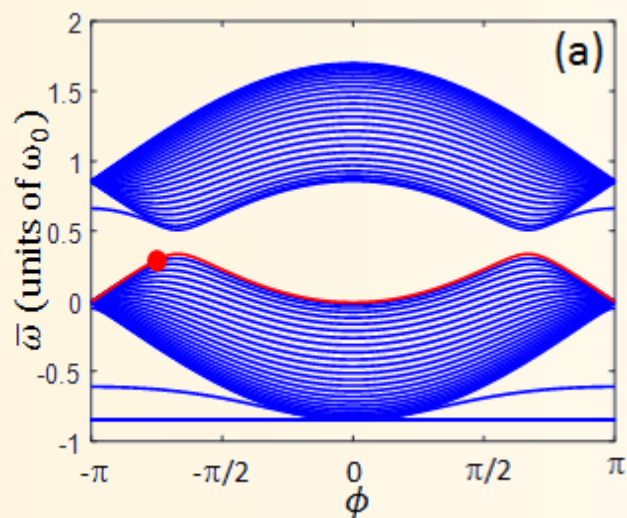
$$\kappa_{m,n}^{(j,k)} = \kappa_{\infty} + \frac{\kappa - \kappa_{\infty}}{1 + \alpha \left| a_{m,n}^{(j)} - a_{m,n}^{(k)} \right|^2}$$

$$i\dot{a}_{m,n}^{(1)} = \omega_0 a_{m,n}^{(1)} + \kappa_{m,n}^{(2,1)} a_{m,n}^{(2)} + \kappa_{m,n}^{(3,1)} a_{m,n}^{(3)} + \nu [a_{m+1,n-1}^{(2)} + a_{m+1,n}^{(3)}], \quad (1)$$

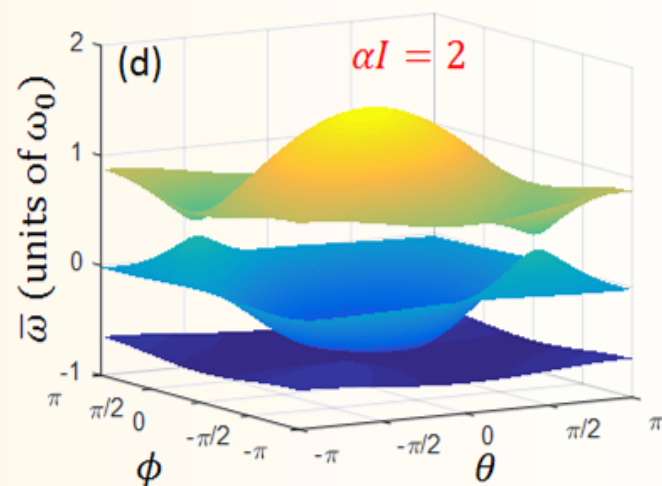
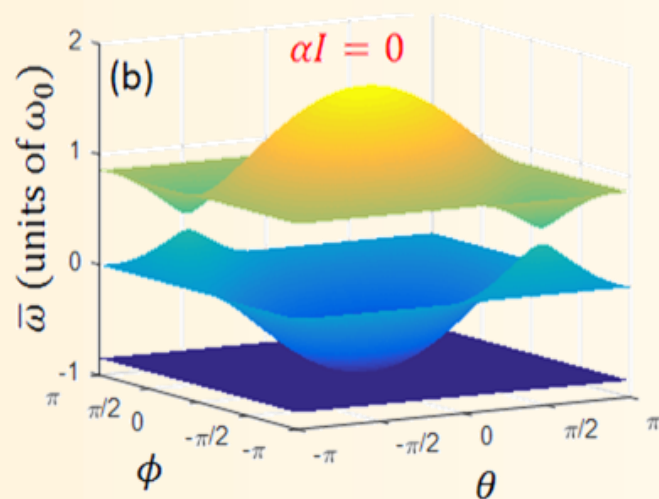
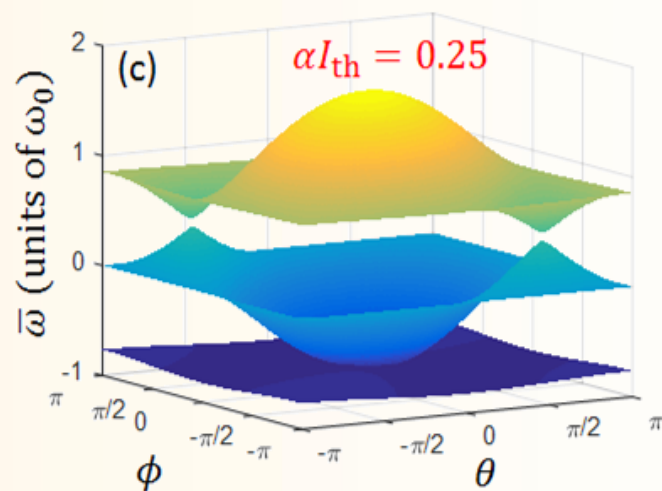
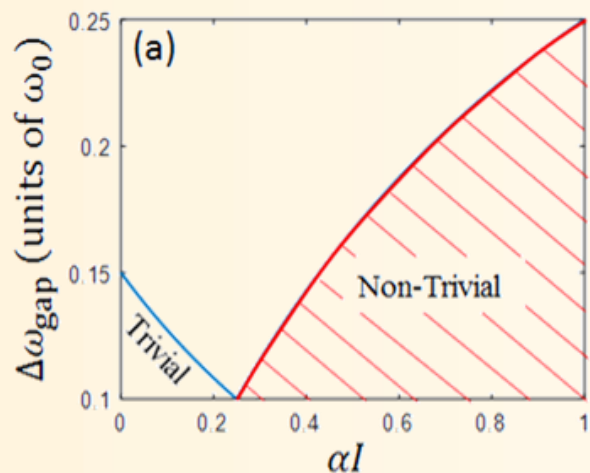
$$i\dot{a}_{m,n}^{(2)} = \omega_0 a_{m,n}^{(2)} + \kappa_{m,n}^{(1,2)} a_{m,n}^{(1)} + \kappa_{m,n}^{(3,2)} a_{m,n}^{(3)} + \nu [a_{m-1,n+1}^{(1)} + a_{m,n+1}^{(3)}], \quad (2)$$

$$i\dot{a}_{m,n}^{(3)} = \omega_0 a_{m,n}^{(3)} + \kappa_{m,n}^{(1,3)} a_{m,n}^{(1)} + \kappa_{m,n}^{(2,3)} a_{m,n}^{(2)} + \nu [a_{m-1,n}^{(1)} + a_{m,n-1}^{(2)}], \quad (3)$$

TOPOLOGICAL TRANSITION TRIGGERED BY INTENSITY

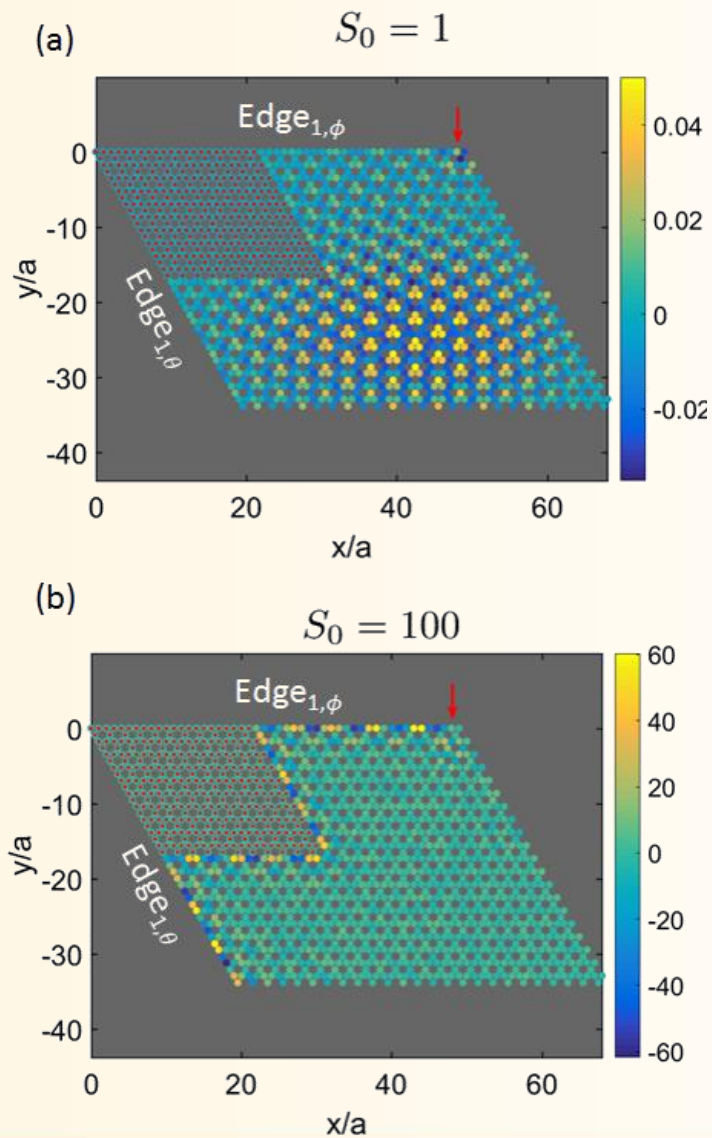


TOPOLOGICAL TRANSITION TRIGGERED BY INTENSITY

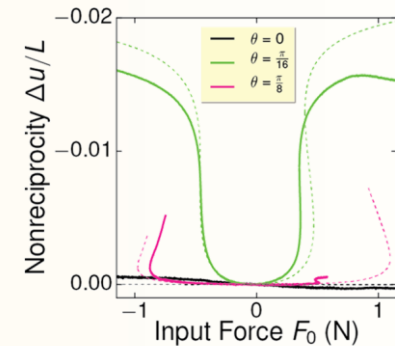
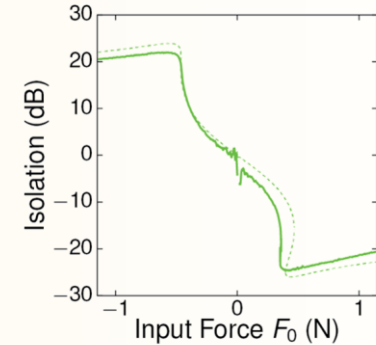
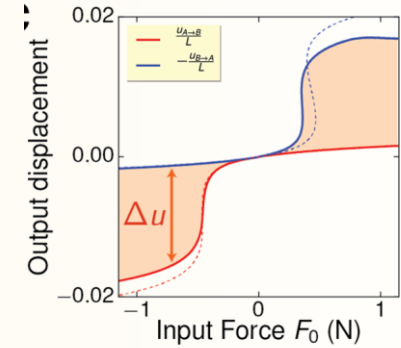
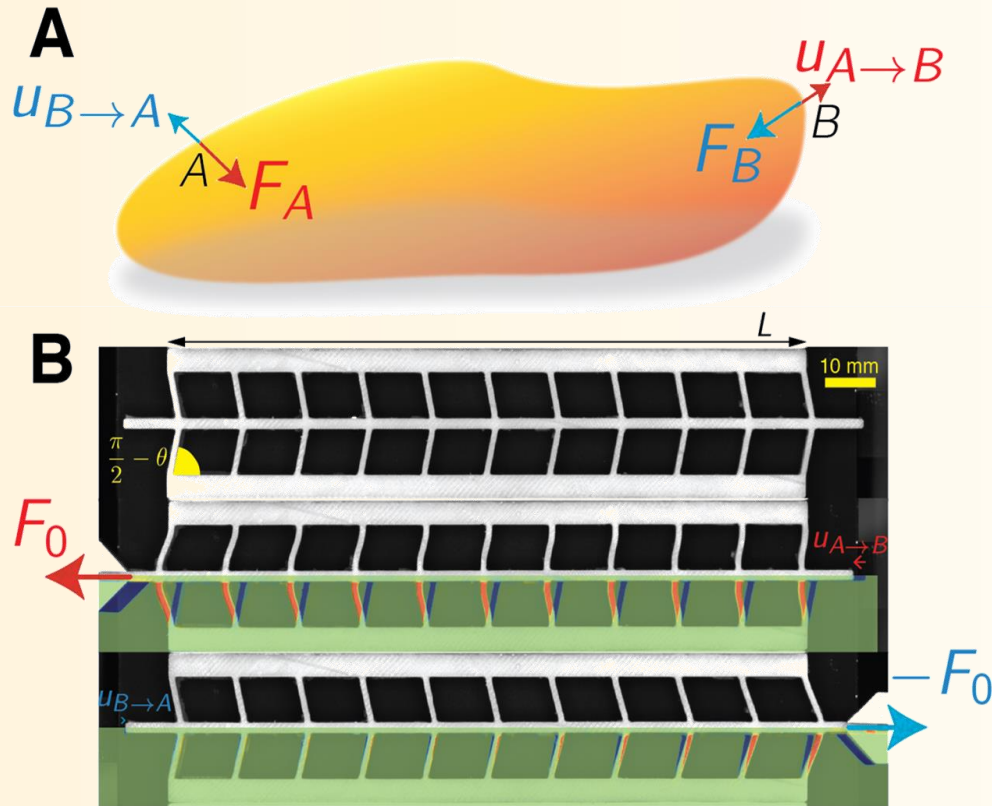


$$\kappa_{\infty} = 0.2\omega_0, \kappa = 0.45\omega_0, \nu = 0.4\omega_0$$

TOPOLOGICAL TRANSITION TRIGGERED BY INTENSITY

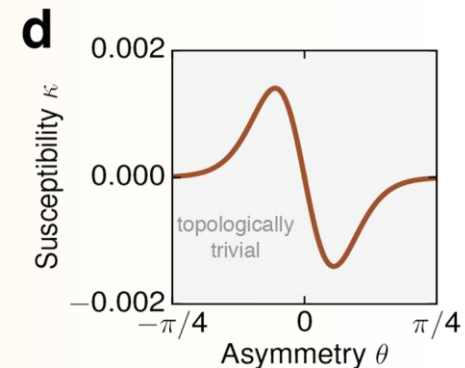
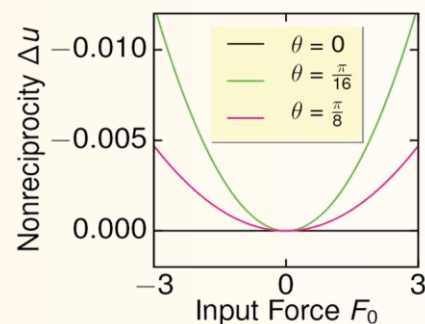
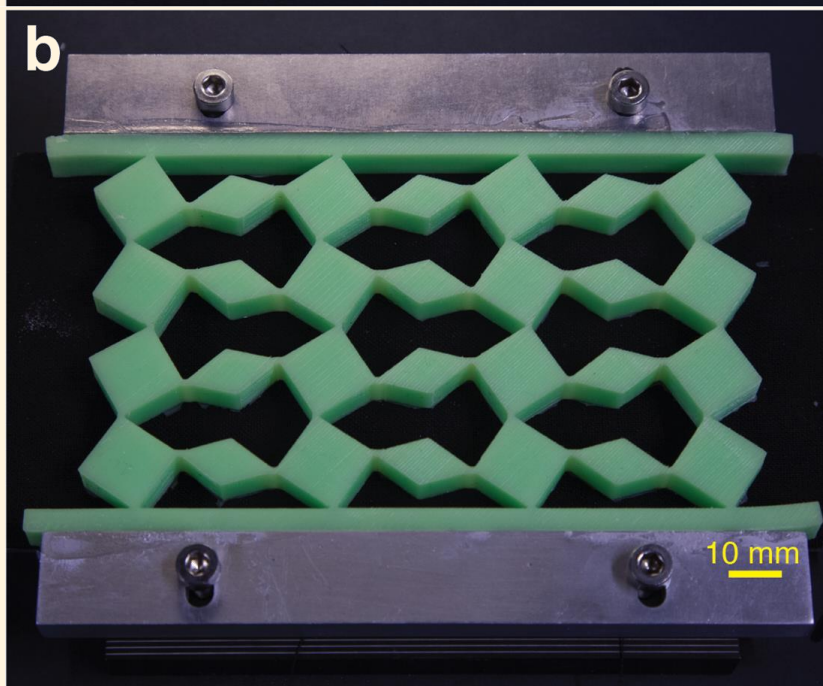
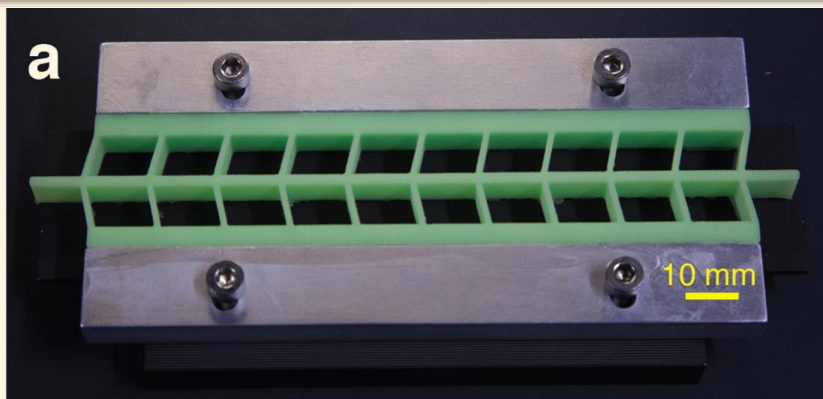


NON-RECIPROcity IN STATICS

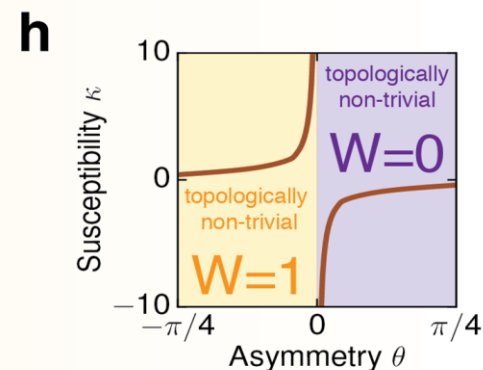
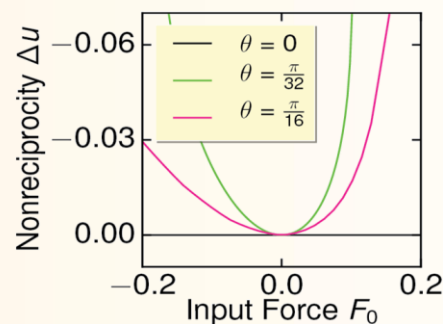


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NON-RECIPROCALITY IN STATIC TOPOLOGICAL METAMATERIALS

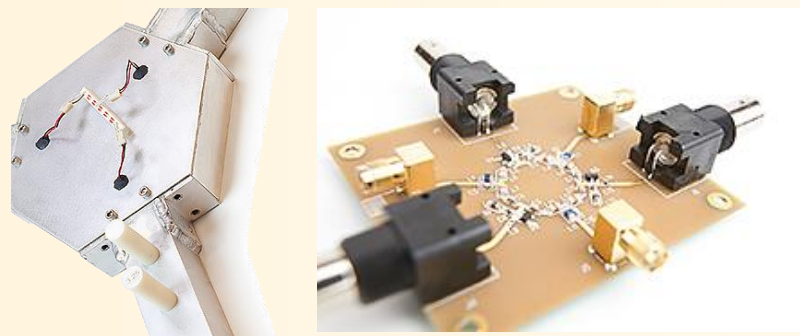


$$\Delta u = \kappa(\theta) F_0^2$$

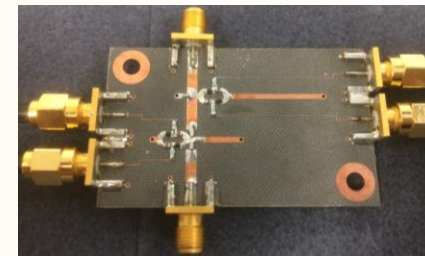
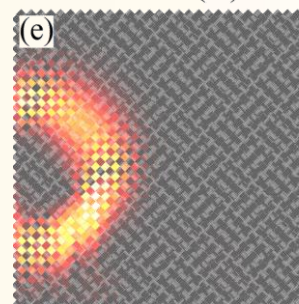


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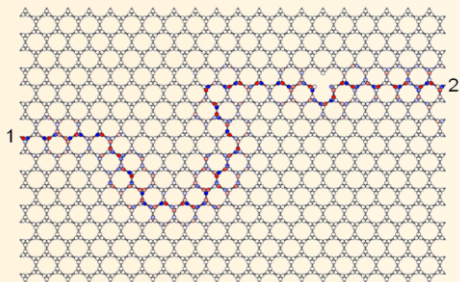
TOPOLOGICAL PHOTONICS AND PHONONICS IN METAMATERIALS



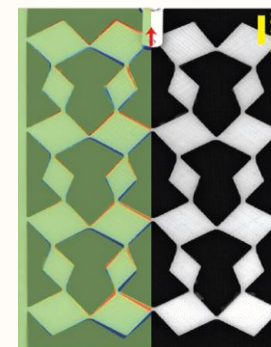
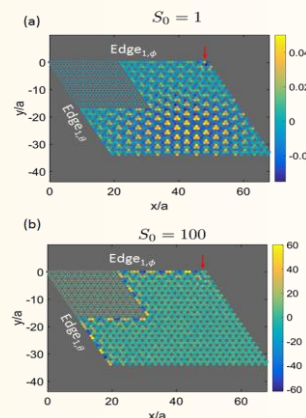
Magnetic-free, linear nonreciprocity at the subwavelength scale: angular-momentum biased meta-atoms



Nonlinearity and asymmetry to build optimal isolators and non-reciprocal devices



Topological metamaterials for broadband, one-way, reconfigurable signal transport



These concepts span a broad range of physical mechanisms, from acoustics to RF, mid-IR, optics, mechanics, statics, and more.