Quantum, kinetic and hydrodynamic descriptions on spin transfer torque

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Joint work with
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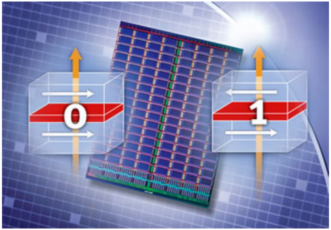
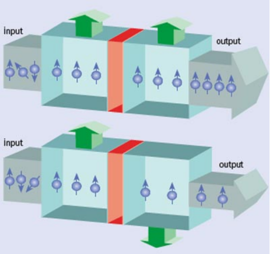
Outline

- Application[1]



- Magnetic recording devices: audio, video.
- Computer storage: floppy disks, hard disks.
- Magnetic Memory: Magnetoresistance Random Access Memory (MRAM).

- Methodology for detecting the orientation [2]



Julliere's model:
Constant tunneling matrix

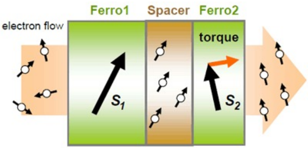
$$TMR \equiv \frac{G_{AP} - G_P}{G_{AP}} = \frac{2P_L P_R}{1 - P_L P_R}$$

$$P_L = \frac{n_L^{\uparrow} - n_L^{\downarrow}}{n_L^{\uparrow} + n_L^{\downarrow}} \qquad P_R = \frac{n_R^{\uparrow} - n_R^{\downarrow}}{n_R^{\uparrow} + n_R^{\downarrow}}$$

[1] Science@Berkeley Lab: The Current Spin on Spintronics
[2] http://ducthe.wordpress.com/category/spintronics/

The key is to control the relative orientation

- Spin transfer torque (STT) [3]



- Two layers of different thickness: different switching fields.
- The thin film is switched, and the resistance measured.

- Literature and references
  - Proposed independently by Berger (1996) and Slonczewski (1996).
  - Theoretical and experimental studies: Bazaliy et al., 1998; Tsoi et al., 1998; Myers et al., 1999; Sun, 2000; Waintal et al. 2000; Stiles and Zangwill, 2002; Zhang et al. 2002; Zutic et al. 2004 (Review); ...

[3] http://www.wpi-aimr.tohoku.ac.jp/miyazaki_ labo/spintorque.htm

Model equations – Magnetization

- Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma \boldsymbol{M} \times (\mathbf{h}_e + c\mathbf{s}) + \alpha \boldsymbol{M} \times \frac{\partial \boldsymbol{M}}{\partial t},$$

where the first term is precession, and the second is Gilbert damping term.

- The effective field $\mathbf{h}_e$ is defined as

$$\mathbf{h}_e = -\frac{2K_u}{M_s}(m_2\mathbf{e}_2 + m_3\mathbf{e}_3) + \frac{2C_{ex}}{M_s}\Delta \boldsymbol{M} + \boldsymbol{M}u_0(\mathbf{h}_s + \mathbf{h}_0),$$

- The stray field, $\mathbf{h}_s = -\nabla u$ is obtained by solving the magnetostatic equation:

$$\Delta u = \operatorname{div} \boldsymbol{M}, \quad \mathbf{x} \in \Omega, \quad \Delta u = 0, \quad \mathbf{x} \in \overline{\Omega}^c,$$

with jump boundary conditions

$$[u]_{\partial\Omega} = 0, \quad \left[\frac{\partial u}{\partial \nu}\right]_{\partial\Omega} = -\boldsymbol{M} \cdot \nu.$$

Model equations – Spin[4]

- Quantum (Schrödinger equation)

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \left(\left(-\frac{\hbar^2}{2m}\nabla_{\boldsymbol{x}}^2 + V(\boldsymbol{x})\right)\widehat{\boldsymbol{I}} + \frac{\mu_B}{2}\widehat{\boldsymbol{\sigma}}\cdot\boldsymbol{M}(\boldsymbol{x},t)\right)\psi.$$

- Kinetic (Boltzmann equation)

$$\partial_t \boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t) - \frac{e}{m}\boldsymbol{E}\cdot\nabla_{\boldsymbol{v}}\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)$$
$$+\frac{\mathrm{i}}{2\hbar}[\mu_B\widehat{\boldsymbol{\sigma}}\cdot\boldsymbol{M}(\boldsymbol{x},t),\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)] = -\frac{\boldsymbol{W}-\bar{\boldsymbol{W}}}{\tau} - \frac{2}{\tau_{\mathrm{sf}}}\left(\bar{\boldsymbol{W}} - \frac{\widehat{\boldsymbol{I}}}{2}\mathrm{Tr}\,\bar{\boldsymbol{W}}\right)$$

- Hydrodynamic (Diffusion equation)

$$\frac{\partial\mathbf{s}}{\partial t} = -\mathrm{div}\,\mathbf{J}_s - 2D_0(\mathbf{x})\frac{\mathbf{s}}{\lambda_{sf}^2} - 2D_0(\mathbf{x})\frac{\mathbf{s}\times\boldsymbol{M}}{\lambda_J^2},$$
$$\mathbf{J}_s = \frac{\beta\mu_B}{e}\boldsymbol{M}\otimes\mathbf{J}_e - 2D_0(\mathbf{x})\left[\nabla\mathbf{s} - \beta\beta'\boldsymbol{M}\otimes(\nabla\mathbf{s}\cdot\boldsymbol{M})\right].$$

[4] Piechon and Thiaville, PRB 2007, Gaspari, PR 1966, Zhang, Levy and Fert, PRL 2002

Ambitious goals

- Set up the connections of the models at different levels of physics.
- Develop new models based on moment system.
- Design efficient numerical algorithms that can capture physical details at different scales.

Outline

Spin-orbital decomposition

Define $\boldsymbol{\sigma} = \boldsymbol{\sigma}_x \widehat{\boldsymbol{i}} + \boldsymbol{\sigma}_y \widehat{\boldsymbol{j}} + \boldsymbol{\sigma}_z \widehat{\boldsymbol{k}}$, where

$$\boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\boldsymbol{W} = w\boldsymbol{I} + \widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{w},$$

where w is the orbital part and $\boldsymbol{w}$ is the spin part.

Macroscopic quantities

$$\text{charge density} \quad n(\boldsymbol{x}, t) = \int_{\mathbb{R}^3} \mathrm{Tr}\,\boldsymbol{W} \mathrm{d}\boldsymbol{v},$$

$$\text{charge current density} \quad \boldsymbol{j}_n(\boldsymbol{x}, t) = \int_{\mathbb{R}^3} \boldsymbol{v} \mathrm{Tr}(\boldsymbol{W}) \mathrm{d}\boldsymbol{v},$$

$$\text{spin density} \quad \boldsymbol{m}(\boldsymbol{x}, t) = \int_{\mathbb{R}^3} \mathrm{Tr}(\widehat{\boldsymbol{\sigma}} \boldsymbol{W}) \mathrm{d}\boldsymbol{v},$$

$$\text{spin current density} \quad \boldsymbol{j}_{\boldsymbol{m}}(\boldsymbol{x}, t) = \int_{\mathbb{R}^3} \boldsymbol{v} \otimes \mathrm{Tr}(\widehat{\boldsymbol{\sigma}} \boldsymbol{W}) \mathrm{d}\boldsymbol{v}.$$

Equations for w and $\boldsymbol{w}$

$$\partial_t w + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} w - \frac{e}{m} \boldsymbol{E} \cdot \nabla_{\boldsymbol{v}} w = -\frac{w - \bar{w}}{\tau},$$

$$\partial_t (\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{w}) + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} (\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{w}) - \frac{e}{m} \boldsymbol{E} \cdot \nabla_{\boldsymbol{v}} (\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{w}) - \frac{\mu_B}{\hbar} \widehat{\boldsymbol{\sigma}} \cdot (\boldsymbol{M} \times \boldsymbol{w})$$
$$= -\frac{(\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{w}) - \overline{(\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{w})}}{\tau} - \frac{2}{\tau_{\mathrm{sf}}} \overline{(\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{w})},$$

where we have used the fact

$$\frac{\mathrm{i}}{2\hbar} [\mu_B \widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{M}(\boldsymbol{x}, t), \boldsymbol{W}(\boldsymbol{x}, \boldsymbol{v}, t)] = -\frac{\mu_B}{\hbar} \widehat{\boldsymbol{\sigma}} \cdot (\boldsymbol{M} \times \boldsymbol{w}).$$

Equations live in 6-dimension, which creates numerical difficulties.

Alternative: to develop and solve the moment system.

Hierarchical moment system

$$
\begin{aligned}
&\partial_t n(\boldsymbol{x},t) + \nabla\cdot \boldsymbol{j}_n(\boldsymbol{x},t) = 0,\\
&\partial_t \boldsymbol{j}_n(\boldsymbol{x},t) + \int_{\mathbb{R}^3} (\boldsymbol{v}\otimes\boldsymbol{v})\cdot\nabla w(\boldsymbol{x},\boldsymbol{v},t)\,\mathrm{d}\boldsymbol{v} + \frac{e}{m}\boldsymbol{E} n(\boldsymbol{x},t) = -\frac{\boldsymbol{j}_n(\boldsymbol{x},t)}{\tau},\\
&\partial_t \boldsymbol{m}(\boldsymbol{x},t) + \nabla\cdot \boldsymbol{j}_{\boldsymbol{m}}(\boldsymbol{x},t) - \frac{\mu_B}{\hbar}\boldsymbol{M}\times\boldsymbol{m} = -\frac{\boldsymbol{m}(\boldsymbol{x},t)}{\tau_{\text{sf}}},\\
&\partial_t \boldsymbol{j}_{\boldsymbol{m}}(\boldsymbol{x},t) + \int_{\mathbb{R}^3} (\boldsymbol{v}\otimes\boldsymbol{v})\cdot\nabla \boldsymbol{w}(\boldsymbol{x},\boldsymbol{v},t)\,\mathrm{d}\boldsymbol{v} + \frac{e}{m}\boldsymbol{E}\otimes\boldsymbol{m}(\boldsymbol{x},t)\\
&\qquad\qquad - \frac{\mu_B}{\hbar}\varepsilon_{jkl}\boldsymbol{M}_k(\boldsymbol{j}_{\boldsymbol{m}})_{il}(\boldsymbol{x},t) = -\frac{\boldsymbol{j}_{\boldsymbol{m}}(\boldsymbol{x},t)}{\tau}.
\end{aligned}
$$

Unclosed! Need equations of state for the second order moments.

Closure assumption

- Assumption I.

  $$w(\boldsymbol{x},\boldsymbol{v},t) \approx \beta_0 n(\boldsymbol{x},t) + \beta_0' \boldsymbol{M}\cdot\boldsymbol{m}(\boldsymbol{x},t) + \boldsymbol{\beta}_1\cdot\boldsymbol{v} n_1(\boldsymbol{x},t) + \beta_1' \boldsymbol{v}\cdot\boldsymbol{m}_1(\boldsymbol{x},t),$$

  then the charge current equation becomes

  $$\partial_t \boldsymbol{j}_n(\boldsymbol{x},t) + \beta_0 \overline{v^2} \nabla_{\boldsymbol{x}} n(\boldsymbol{x},t) + \beta_0' \overline{v^2} \nabla_{\boldsymbol{x}} \boldsymbol{m}(\boldsymbol{x},t)\boldsymbol{M} + \frac{e}{m}\boldsymbol{E} n(\boldsymbol{x},t) = -\frac{\boldsymbol{j}_n(\boldsymbol{x},t)}{\tau}.$$

- Assumption II.

  $$\boldsymbol{w}(\boldsymbol{x},\boldsymbol{v},t) \approx \beta \boldsymbol{M} n(\boldsymbol{x},t) + \beta' \boldsymbol{m}(\boldsymbol{x},t) + \beta_2 \boldsymbol{v} n_1(\boldsymbol{x},t) + \beta_2' \boldsymbol{v}(\boldsymbol{M}\cdot\boldsymbol{m}_1(\boldsymbol{x},t)),$$

  then the spin current equation becomes

  $$\partial_t \boldsymbol{j}_{\boldsymbol{m}}(\boldsymbol{x},t) + \beta \overline{v^2} \nabla_{\boldsymbol{x}} n(\boldsymbol{x},t) \otimes \boldsymbol{M} + \beta' \overline{v^2} \nabla_{\boldsymbol{x}} \boldsymbol{m}(x,t) + \frac{e}{m}\boldsymbol{E} \otimes \boldsymbol{m}(\boldsymbol{x},t) - \frac{\mu_B}{\hbar}\varepsilon_{jkl} \boldsymbol{M}_k (\boldsymbol{j}_{\boldsymbol{m}})_{il}(\boldsymbol{x},t) = -\frac{\boldsymbol{j}_{\boldsymbol{m}}(\boldsymbol{x},t)}{\tau}.$$

Motivation for the assumption

- Introduce the spin-orbital coupling in the moment system.
- Quasistatic approximation for the current equations yields

$$\boldsymbol{j}_n(\boldsymbol{x},t) = -\frac{e}{m}\tau \boldsymbol{E} n(\boldsymbol{x},t) - \beta_0 \overline{v^2}\tau \nabla_{\boldsymbol{x}} n(\boldsymbol{x},t) - \beta_0' \overline{v^2}\tau \nabla_{\boldsymbol{x}} \boldsymbol{m}(\boldsymbol{x},t)\boldsymbol{M},$$

$$\boldsymbol{j}_{\boldsymbol{m}}(\boldsymbol{x},t) = -\frac{e}{m}\tau \boldsymbol{E}\otimes \boldsymbol{m}(x,t) - \beta \overline{v^2}\tau \nabla_{\boldsymbol{x}} n(\boldsymbol{x},t)\otimes \boldsymbol{M} + \frac{\mu_B}{\hbar}\varepsilon_{jkl}\boldsymbol{M}_k(\boldsymbol{j}_{\boldsymbol{m}})_{il}(\boldsymbol{x},t) - \beta' \overline{v^2}\tau \nabla_{\boldsymbol{x}} \boldsymbol{m}(\boldsymbol{x},t).$$

This is consistent with the linear response relation used in Zhang, Levy and Fert (PRL 2002) except the term in red.

Outline

Experiment setup

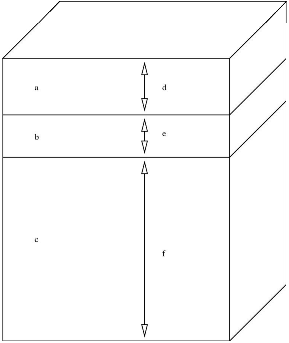


Figure: Device. From Bottom to Top: FM1 ($128 \times 64 \times 200\text{nm}^3$); Spacer ($128 \times 64 \times 20\text{nm}^3$); FM2 ($128 \times 64 \times 60\text{nm}^3$). $\boldsymbol{j}_n$ is applied from FM1 to FM2.

Several locally stable states

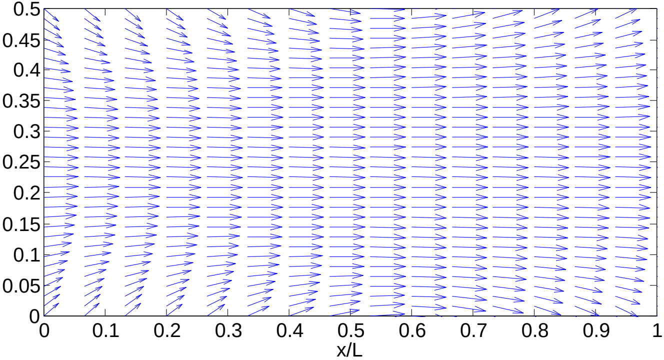


Figure: State 1

Several locally stable states

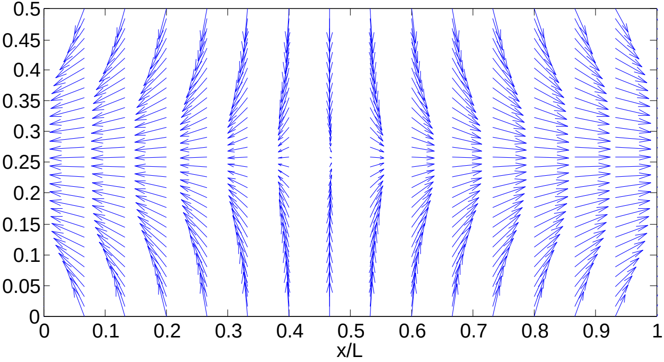


Figure: State 2

Several locally stable states

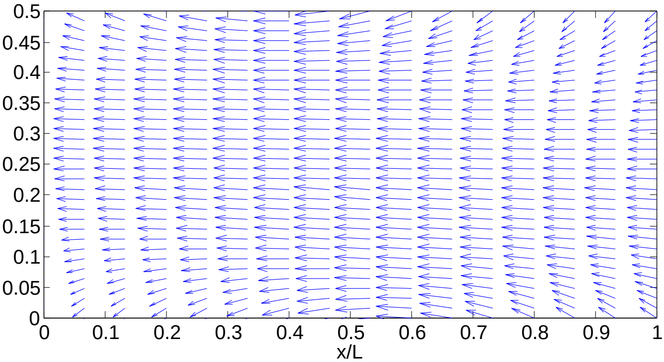


Figure: State 3

Comparison results

$$\boldsymbol{j_m} = \frac{\beta\mu_B}{e}\boldsymbol{j}_n \otimes \boldsymbol{M} - 2D_0(\boldsymbol{x})[\nabla \boldsymbol{m} - \beta\beta'(\nabla \boldsymbol{m} \cdot \boldsymbol{M}) \otimes \boldsymbol{M}]$$

$$\boldsymbol{j_m}\boldsymbol{A}(\boldsymbol{M}) = \frac{\beta\mu_B}{e}\boldsymbol{j}_n \otimes \boldsymbol{M} - 2D_0(\boldsymbol{x})[\nabla \boldsymbol{m} - \beta\beta'(\nabla \boldsymbol{m} \cdot \boldsymbol{M}) \otimes \boldsymbol{M}]$$

where

$$\boldsymbol{A}(\boldsymbol{M}) = \begin{pmatrix} 1 & -M_3 & M_2 \\ M_3 & 1 & -M_1 \\ -M_2 & M_1 & 1 \end{pmatrix}$$

with eigenvalues 1, $1 \pm i$.

Initial states: random in FM1 and uniform in FM2

Cont'd

Model	S1	S1→S2	S1→S3	S1→S2
Diffusion	≤ 7.4	$7.5 \sim 9.9$	10.0	≥ 10.1
Moment	≤ 9.0	$9.1 \sim 9.6$	$9.7 \sim 10.1$	≥ 10.2

Table: Critical applied current $\boldsymbol{j}_n$ for switching (unit: 10^{10}As)

A larger admissible window for effective switching by moment model

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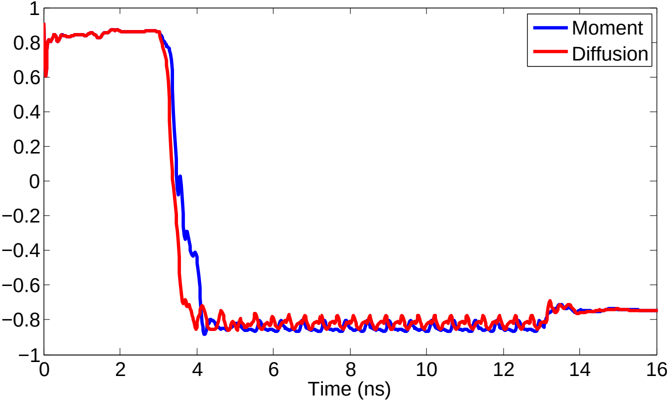

Figure: $\boldsymbol{j}_n = 10.0 \times 10^{10}\text{As}$

Cont'd

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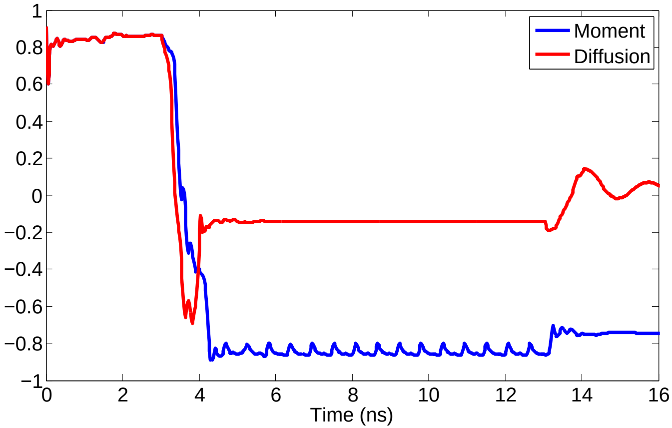

Figure: $\boldsymbol{j}_n = 9.8 \times 10^{10}$As

Outline

From moment system to diffusion[5]

- For a clearer illustration, forget the coupling term for the current moment, and rescale the moment system as follows,

$$\varepsilon \partial_t \boldsymbol{m}(x,t) + \partial_x \boldsymbol{j_m}(x,t) - \varepsilon \boldsymbol{M} \times \boldsymbol{m} = -\varepsilon \boldsymbol{m}(x,t),$$
$$\varepsilon \partial_t \boldsymbol{j_m}(x,t) + \overline{v^2} \partial_x \boldsymbol{m}(x,t) - E\boldsymbol{m}(x,t) - \frac{\boldsymbol{M} \times \boldsymbol{j_m}(x,t)}{\varepsilon} = -\frac{\boldsymbol{j_m}(x,t)}{\varepsilon}.$$

- Apply the following asymptotic expansion

$$\boldsymbol{m} = \boldsymbol{m}^0 + \varepsilon \boldsymbol{m}^1 + \varepsilon^2 \boldsymbol{m}^2 + \cdots,$$
$$\boldsymbol{j_m} = \boldsymbol{j}^0_{\boldsymbol{m}} + \varepsilon \boldsymbol{j}^1_{\boldsymbol{m}} + \varepsilon^2 \boldsymbol{j_m} + \cdots.$$

[5] A field with very rich literature: Bardos, Golse, Levermore, Degond, Ben Abdallah, Gamba, Jin, ...

To the leading order, we have the diffusion equation,

$$
\begin{aligned}
2\partial_t \boldsymbol{m} - (\boldsymbol{M} \cdot \partial_t \boldsymbol{m})\boldsymbol{M} = \overline{v^2}\partial_{x^2} \boldsymbol{m}(x,t) + \overline{v^2}\boldsymbol{M} \times \partial_{x^2} \boldsymbol{m}(x,t) \\
- E\partial_x \boldsymbol{m}(x,t) - E\boldsymbol{M} \times \partial_x \boldsymbol{m}(x,t) \\
+ \boldsymbol{M} \times \boldsymbol{m} - \boldsymbol{m},
\end{aligned}
$$

Assuming M is uniform and only nonzero in z-direction,

$$
\begin{aligned}
&\partial_t m^z(x,t) + \overline{v^2}\partial_{xx} m^z(x,t) \\
&\qquad - E\partial_x m^z(x,t) - Mm^z(x,t) = -m^z(x,t), \\
&\partial_t m^{\pm}(x,t) - \frac{1}{1 \mp iM}\overline{v^2}\partial_{xx} m^{\pm}(x,t) \\
&\qquad + \frac{1}{1 \mp iM} E\partial_x m^{\pm}(x,t) - Mm^{\pm}(x,t) = -m^{\pm}(x,t).
\end{aligned}
$$

This recovers the diffusion model in Zhang, Levy and Fert (PRL 2002) without the coupling term, which can be also included by a similar procedure on the coupling moment system.

From Schrödinger to Liouville

In physical units,

$$i\hbar \frac{\partial}{\partial t}\psi(\boldsymbol{x}, t) = \widehat{\boldsymbol{H}}(\boldsymbol{x}, t)\psi(\boldsymbol{x}, t),$$

where

$$\widehat{\boldsymbol{H}}(\boldsymbol{x}, t) = \left(-\frac{\hbar^2}{2m}\nabla_{\boldsymbol{x}}^2 + V(\boldsymbol{x})\right)\widehat{\boldsymbol{I}} + \frac{\mu_B}{2}\widehat{\boldsymbol{\sigma}} \cdot \boldsymbol{M}(\boldsymbol{x}, t).$$

What is the correct nondimensional regime? $\hbar \to \varepsilon$?

Define the Wigner transformation of ψ as

$$\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)=\frac{m^{3/2}}{(2\pi\hbar)^{3/2}}\int_{\mathbb{R}^3}\psi(\boldsymbol{x}+\frac{\hbar}{2m}\boldsymbol{y},t)\otimes\psi^*(\boldsymbol{x}-\frac{\hbar}{2m}\boldsymbol{y},t)\mathrm{e}^{\mathrm{i}\boldsymbol{v}\cdot\boldsymbol{y}}\,\mathrm{d}\boldsymbol{y}.$$

To the leading order,

$$\begin{aligned}\partial_t\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)+\boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)-\frac{e}{m}\boldsymbol{E}\cdot\nabla_{\boldsymbol{v}}\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)\\+\frac{i}{2\hbar}\left[\mu_B\widehat{\boldsymbol{\sigma}}\cdot\boldsymbol{M}(\boldsymbol{x},t).\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)\right]=0.\end{aligned}$$

Compare the order of the last two terms in physical units[6],

$$\begin{aligned}\frac{e}{m}\frac{|\boldsymbol{E}|}{|\bar{\boldsymbol{v}}|}&\approx\frac{10^{-19}\times10^{5}}{10^{-30}\times5\times10^{5}}\approx10^{-12}s^{-1},\\ \frac{\mu_B}{\hbar}|\boldsymbol{M}|&\approx\frac{10^{-23}}{10^{-34}}\times0.1=10^{-12}s^{-1}.\end{aligned}$$

[6] Qi and Zhang, PRB 2003

Semiclassical regime

Weak spinor term:

$$\mathrm{i}\varepsilon\frac{\partial}{\partial t}\psi(\boldsymbol{x},t)=\left(\left(-\frac{\varepsilon^2}{2}\nabla_{\boldsymbol{x}}^2+V(\boldsymbol{x})\right)\widehat{\boldsymbol{I}}+\frac{\varepsilon}{2}\widehat{\boldsymbol{\sigma}}\cdot\boldsymbol{M}(\boldsymbol{x},t)\right)\psi(\boldsymbol{x},t).$$

The rescaled Wigner transform:

$$\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)=\frac{1}{(2\pi\varepsilon)^{3/2}}\int_{\mathbb{R}^3}\psi(\boldsymbol{x}+\frac{\varepsilon}{2}\boldsymbol{y},t)\otimes\psi^*(\boldsymbol{x}-\frac{\varepsilon}{2}\boldsymbol{y},t)\mathrm{e}^{\mathrm{i}\boldsymbol{v}\cdot\boldsymbol{y}}\,\mathrm{d}\boldsymbol{y},$$

which leads to the Liouville equation in the leading order,

$$\partial_t\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)+\boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)-\boldsymbol{E}\cdot\nabla_{\boldsymbol{v}}\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)+\frac{1}{2}\left[\widehat{\boldsymbol{\sigma}}\cdot\boldsymbol{M}(\boldsymbol{x},t),\boldsymbol{W}(\boldsymbol{x},\boldsymbol{v},t)\right]=0.$$

We still have the collision term missing in the picture.

Outline

- Summary
  - Develop STT model based on moment closure that introduces spin-orbital coupling.
  - Numerically study the qualitative difference of moment system model from the diffusion model.
  - Preliminary results on the connections of models at different scales.

- Future works
  - Further understand the connections of different models, e.g. the origin of collision term, the modeling accuracy of moment system in the long time and large space regime.
  - Develop multiscale numerical methods, e.g. Heterogeneous Multiscale Method (HMM), Asymptotic Preserving scheme (AP), and domain decomposition method (DD).
  - Simulating STT of magnetic devices in experiments.