Complex oscillator networks and applications in power systems

ETH-ITS lecture series "Collective dynamics, control and imaging"

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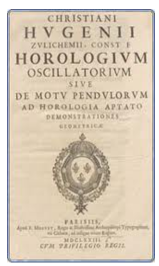
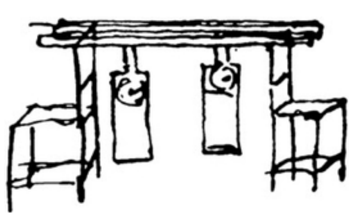
A brief history of sync

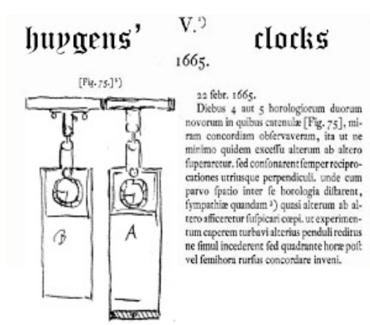
Christiaan Huygens (1629 – 1695)

- physicist & mathematician
- engineer & horologist

observed "*an odd kind of sympathy*"
[Letter to Royal Society of London, 1665]







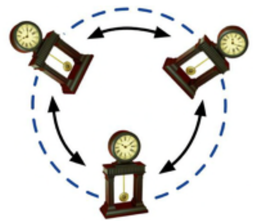
Recent reviews, experiments, & analysis
[M. Bennet et al. '02, M. Kapitaniak et al. '12]

A field was born

- sync in mathematical biology [A. Winfree '80, S.H. Strogatz '03, . . .]
- sync in physics and chemistry [Y. Kuramoto '83, M. Mézard et al. '87. . .]
- sync in neural networks [F.C. Hoppensteadt and E.M. Izhikevich '00, . . .]
- sync in complex networks [C.W. Wu '07, S. Bocaletti '08, . . .]
- . . . and numerous technological applications (reviewed later)

Coupled phase oscillators

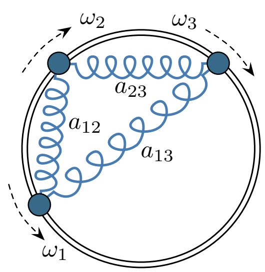
∃ various models of oscillators & interactions



canonical coupled **phase oscillator** model:
[A. Winfree '67, Y. Kuramoto '75]

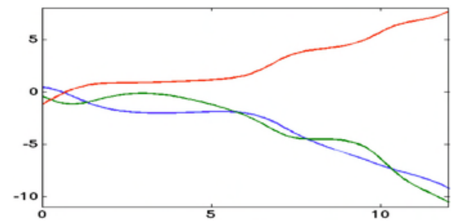
$$\dot{\theta}_i = \omega_i - \sum_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)$$

- n **oscillators** with phase $\theta_i \in \mathbb{S}^1$
- **non-identical** natural frequencies $\omega_i \in \mathbb{R}^1$
- elastic **coupling** with strength $a_{ij} = a_{ji}$
- undirected & connected **graph** $G = (\mathcal{V}, \mathcal{E}, A)$

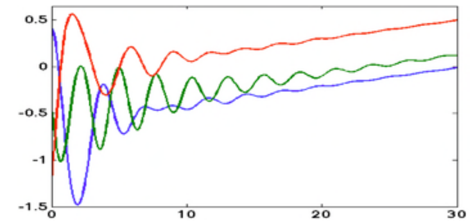


Phenomenology & challenges in synchronization

Transition to synchronization is a **trade-off**: coupling vs. heterogeneity



strong coupling & homogeneous



weak coupling & heterogeneous

"Surprisingly enough, this seemingly obvious fact seems difficult to prove."
(Y. Kuramoto's conclusion after proposing the model)

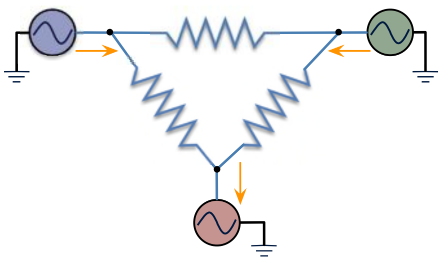
Two open **central questions:**
(still after 50 years of work)

- quantify "coupling" vs. "heterogeneity"
- basin of attraction for synchronization

. . . starting from here a theory talk would summarize many efforts leading to partial (or negative) results and conjectures . . .

Outline

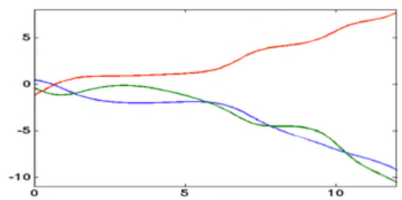
From coupled oscillators to AC power systems



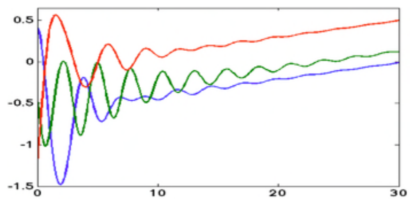
(simplified) **swing equation** model of interconnected synchronous generators:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = p_i^\star - \sum_j y_{ij}\sin(\theta_i - \theta_j)$$

where M_i, D_i, $p_i^\star$ are inertia, damping, power injection set-point of generator i

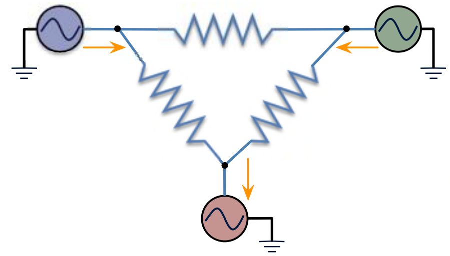


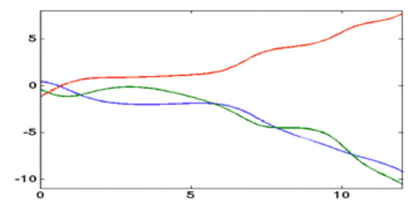
weak coupling & heterogeneous



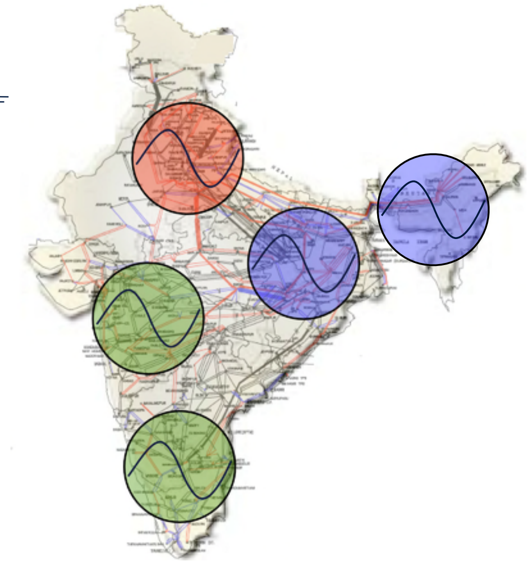
strong coupling & homogeneous

From coupled oscillators to AC power systems





weak coupling & heterogeneous



blackout India July 30/31 2012

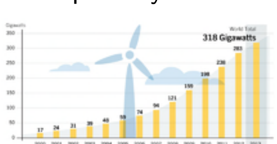
Renewable/distributed/inverter-based generation on the rise

synchronous generator

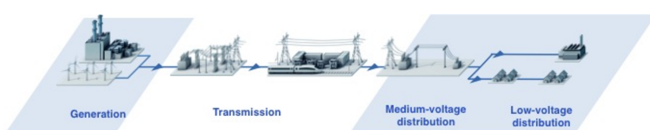
new workhorse

scaling

new primary sources



location & distributed implementation



focus today on **inverter-based** generation

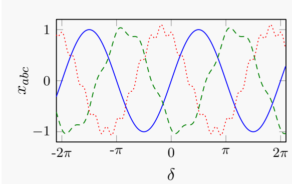
Modeling: signal space in three-phase AC power systems

three-phase AC

$$\begin{bmatrix} x_a(t)\\ x_b(t)\\ x_c(t)\end{bmatrix} = \begin{bmatrix} x_a(t+T)\\ x_b(t+T)\\ x_c(t+T)\end{bmatrix}$$

periodic with 0 average

$$\tfrac{1}{T}\int_0^T x_i(t)dt = 0$$

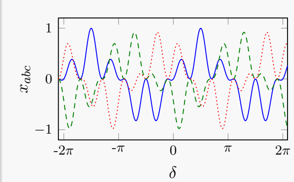


balanced (nearly true)

$$= A(t)\begin{bmatrix} \sin(\delta(t))\\ \sin(\delta(t)-\frac{2\pi}{3})\\ \sin(\delta(t)+\frac{2\pi}{3})\end{bmatrix}$$

so that

$$x_a(t)+x_b(t)+x_c(t)=0$$

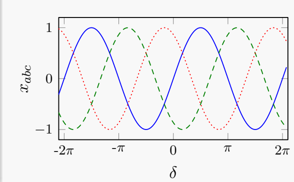


synchronous (desired)

$$= A\begin{bmatrix} \sin(\delta_0+\omega_0 t)\\ \sin(\delta_0+\omega_0 t-\frac{2\pi}{3})\\ \sin(\delta_0+\omega_0 t+\frac{2\pi}{3})\end{bmatrix}$$

const. freq & amp

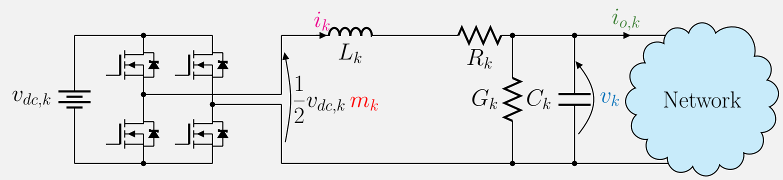
$\Rightarrow$ const. in rot. frame



assumption: signals are balanced $\Rightarrow$ 2d-coordinates $x(t) = [x_\alpha(t)\ x_\beta(t)]$

(equivalent representation: complex-valued polar/phasor coordinates)

Modeling: the inverter

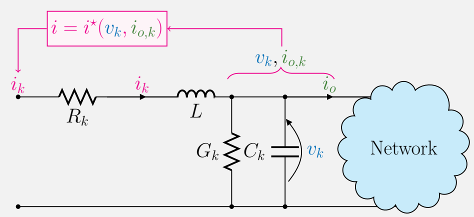


- **terminal signals**: voltage $v_k \in \mathbb{R}^2$ and output current $i_{o,k} \in \mathbb{R}^2$
- **controllable signal**: switching modulation signal m_k

⇒ common abstraction:
direct control of current i_k:

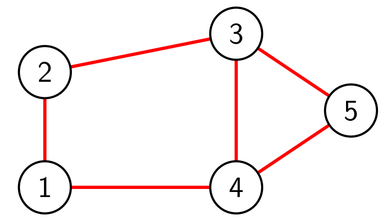
$$C_k \frac{\mathrm{d}}{\mathrm{d}t} v_k = -G_k v_k - i_{o,k} + i_k$$

∼ controllable voltage source



Modeling: the network equations

- **branch dynamics**: each branch is series of resistance r_{ij} & inductance ℓ_{ij}
- **time-scale separation**: all network signals are assumed to be in synchronous steady-state $\dot{x}(t) = \omega_0 J\, x(t)$ where $J = \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right] \sim \sqrt{-1}$
- **admittance matrix** $\mathcal{Y} = \mathcal{Y}^T$ with admittances $y_{ij} = (r_{ij} + \omega_0 \ell_{ij} J)^{-1}$



$$\mathcal{Y} = \begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -y_{k1} I_2 & \cdots & \sum_{j=1}^{n} y_{kj} I_2 & \cdots & -y_{kn} I_2 \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix}$$

∼ *generalized Laplacian matrix*

⇒ **balance equations**: $i_o = \mathcal{Y} v$ with terminal currents/voltages (i_o, v)

Objectives for decentralized control design

We aim to stabilize a target trajectory $(v(t), i_o(t))$ satisfying the following:

1. **frequency stability** at synchronous frequency ω_0:

$$\frac{\mathrm{d}}{\mathrm{d}t} v_k(t) = \omega_0 J\, v_k(t)$$

∼ synchronization to desired harmonic waveform

2. **voltage regulation** to desired voltage magnitudes $v_k^\star$:

$$\|v_k(t)\| = v_k^\star$$

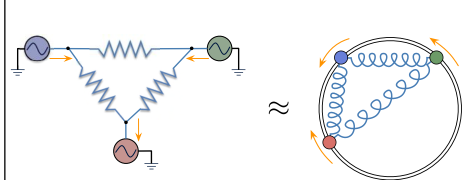
∼ stabilization of desired amplitudes (assume wlog that $v_k^\star = v^\star$)

3. **power injection set-points** for active & reactive power $\{p_k^\star, q_k^\star\}$:

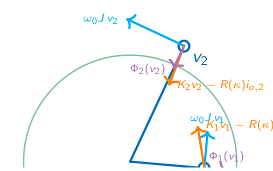
$$v_k^\top i_{o,k} = p_k^\star \quad , \quad v_k^\top J i_{o,k} = q_k^\star$$

∼ stabilization of desired angle set-points $\{\theta_{kj}^\star\}$

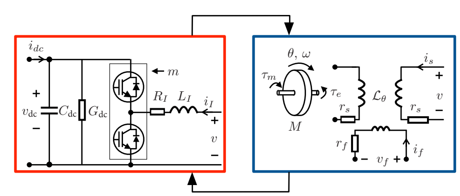
Overview of oscillator-based control strategies for inverters



classic droop control (brief review)

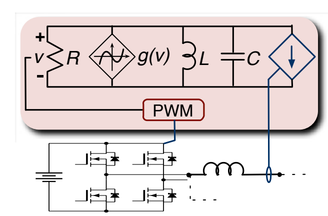


consensus-inspired approach (today)



synchronous generator emulation

[Zhong, Weiss, '11, D'Arco, Suul '13, Bevrani, Sie, Miura '14, Jouini, Arghir, FD '16]



virtual van-der-Pol oscillator control

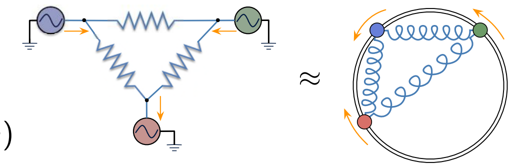
[Johnson, Dhople, Hamadeh, & Krein, '13, Sinha, Johnson, Ainsworth, FD, Dhople '15]

Outline

Droop control of power inverters

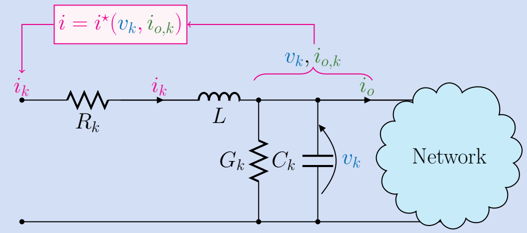
key idea: replicate the generator swing equation for the inverter:

$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = p_i^\star - \sum_j y_{ij}\sin(\theta_i - \theta_j)$



Standard implementation:

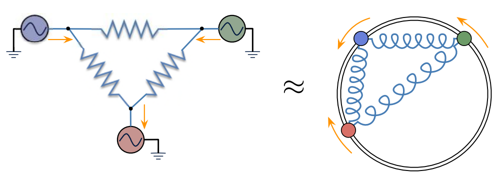
1. **measure** terminal currents and voltages $(v(t), i_o(t))$
2. **process** measurements:
   active power $p_k = v_k^\top i_{o,k}$
   reactive power $q_k = v_k^\top J i_{o,k}$
3. **proportional control** of terminal voltage waveform: $\frac{d}{dt}\begin{bmatrix}\theta_k \\ \|v\|_k\end{bmatrix} = (v_k^\star, p_k^\star, q_k^\star) - (\|v_k\|, p_k, q_k)$



Closer look at droop control for a lossless network

key idea: replicate the generator swing equation for the inverter:

$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = p_i^\star - \sum_j y_{ij}\sin(\theta_i - \theta_j)$



- $p - \omega$ **droop** mimicking a generator:

$$\frac{d}{dt}\theta_k = \omega_0 + k\,(p_k^\star - p_k) = \omega_0 + k p_k^\star - k \cdot \underbrace{\sum_j y_{kj}\,\|v_j\|\,\|v_k\|\,\sin(\theta_k - \theta_j)}_{\text{generator / Kuramoto oscillator}}$$

- analogous $q - \|v\|$ **droop** (many variations):

$$\frac{d}{dt}\|v_k\| = k\,(q_k^\star - q_k) + k\,(v^\star - \|v_k\|)\cdot\|v_k\|$$

Standard analysis of lossless droop/generators/oscillators

energy function: inductive energy + quadratic amplitude error

$$V(v) = \sum_{j<k} y_{jk}\left(\|v_k\|^2 - \|v_k\|\|v_j\|\cos(\theta_k - \theta_j)\right) + \frac{1}{2}\sum_k (v^\star - \|v_k\|)^2$$

- $p - \omega$ **droop**: $\frac{d}{dt}(\theta_k - \omega_0\,t) = k p_k^\star - k\cdot\frac{\partial V}{\partial \theta}$
- $q - \|v\|$ **droop**: $\frac{d}{dt}\|v_k\| = k q_k^\star - k\cdot\|v_k\|\cdot\frac{\partial V}{\partial\|v_k\|}$

$\Rightarrow$ closed loop is weighted **gradient flow** if all $p_k^\star$ are identical and $q_k^\star = 0$

standard analysis: if equilibria exists, if $(p_k^\star, q_k^\star)$ are sufficiently small, if lossless, if $\nabla^2 V(v^\star)$ is positive definite, ... $\Longrightarrow$ local asymptotic stability

analysis has many limitations & droop has similar practical **limitations**!

Outline

Recall: objectives for decentralized control design

We aim to stabilize a target trajectory $(v(t), i_o(t))$ satisfying the following:

1. **frequency stability** at synchronous frequency ω_0 :

$$\frac{\mathrm{d}}{\mathrm{d}t} v_k(t) = \omega_0 J\, v_k(t)$$

$\sim$ synchronization to desired harmonic waveform

2. **voltage regulation** to desired voltage magnitudes $v_k^\star$:

$$\|v_k(t)\| = v^\star$$

$\sim$ stabilization of desired amplitudes

3. **power injection set-points** for active & reactive power $\{p_k^\star, q_k^\star\}$:

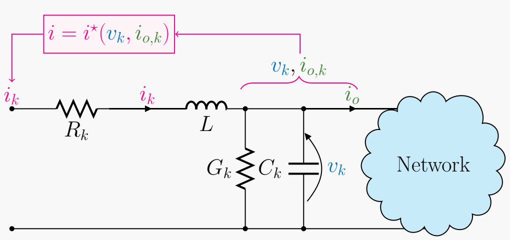
$$v_k^\top i_{o,k} = p_k^\star \quad , \quad v_k^\top J i_{o,k} = q_k^\star$$

$\sim$ stabilization of desired angle set-points $\{\theta_{kj}^\star\}$

Overview of design strategy

Step 1: construct **target dynamics** $f^\star(v(t))$ for the terminal voltages so that $\frac{\mathrm{d}}{\mathrm{d}t} v(t) = f^\star(v(t))$ is stable and satisfies the three control objectives.

Step 2: achieve desired target dynamics $\frac{\mathrm{d}}{\mathrm{d}t} v(t) = f^\star(v(t))$ via **fully decentralized** current controllers $i_k^\star(v_k, i_{k,o})$.



- ✓ local measurements $(v_k, i_{k,o})$
- ✓ local set-points $(v_k^\star, p_k^\star, q_k^\star)$
- ⚡ unknown: global set-points for angles $\{\theta_{kj}^\star\}$ and nonlocal measurements of $(v_j, i_{o,j})$

Step 3: implement control via **switching modulation** signal (not today).

Step 1: desirable closed-loop target dynamics

objectives: frequency, phase, and voltage stability

$$\frac{\mathrm{d}}{\mathrm{d}t} v = \underbrace{\omega_0 \mathcal{J}\, v}_{\text{rotation at } \omega = \omega_0} + \eta \cdot \underbrace{e_\theta(v)}_{\text{phase error}} + \alpha \cdot \underbrace{e_{\|v\|}(v)}_{\text{magnitude error}}$$

(i) synchronous rotation: $\frac{\mathrm{d}}{\mathrm{d}t} v = \omega_0 \mathcal{J}\, v = \omega_0 \begin{bmatrix} \ddots & & \\ & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & \\ & & \ddots \end{bmatrix} v$

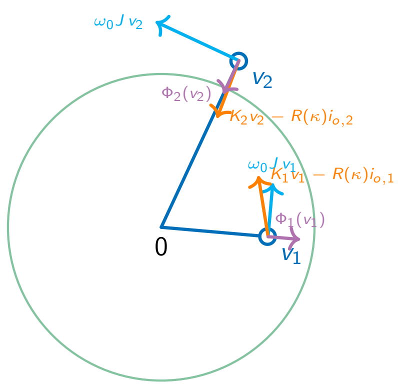
(ii) phase stabilization: $\frac{\mathrm{d}}{\mathrm{d}t} v = e_{\theta,k}(v) = \sum_{j=1}^{n} w_{jk}(v_j - R(\theta_{jk}^\star) v_k)$

(iii) magnitude stabilization: $\frac{\mathrm{d}}{\mathrm{d}t} v = e_{\|v\|,k}(v_k) = (v^\star - \|v_k\|) v_k$

gains: $\alpha_k, \eta_k > 0$ for each k and $w_{jk} \geq 0$ for relative phases amongst $\{j, k\}$

Illustration of desirable closed-loop dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t} v_k = \omega_0 \, J \, v_k + \eta \cdot e_\theta(v) + \alpha \cdot e_{\|v\|}(v)$$



phase error:

$$e_{\theta,1}(v) = v_2 - R(\theta^\star_{21}) v_1$$
$$e_{\theta,2}(v) = v_1 - R(\theta^\star_{12}) v_2$$

magnitude error:

$$e_{\|v\|,1}(v) = (v^\star - \|v_1\|) v_1$$
$$e_{\|v\|,2}(v) = (v^\star - \|v_2\|) v_2$$

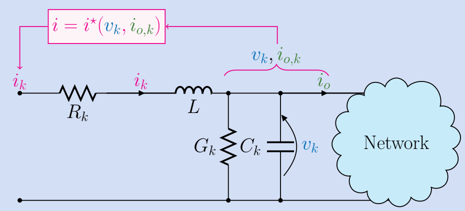
Step 2: decentralized implementation of target dynamics

open-loop system: controllable voltage sources + network coupling

$$C_k \frac{\mathrm{d}}{\mathrm{d}t} v_k = -G_k v_k - i_{o,k} + i_k, \quad k \in \{1, \ldots, n\} \qquad \text{inverter dynamics}$$

$$i_o = \mathcal{Y} v \qquad \text{network interconnection}$$

target dynamics: $$\frac{\mathrm{d}}{\mathrm{d}t} v = \underbrace{\omega_0 \, \mathcal{J} \, v}_{\text{rotation at } \omega = \omega_0} + \eta \cdot \underbrace{e_\theta(v)}_{\text{phase error}} + \alpha \cdot \underbrace{e_{\|v\|}(v)}_{\text{magnitude error}}$$



✓ **known:** local measurements $(v_k, i_{k,o})$ & set-points $(v_k^\star, p_k^\star, q_k^\star)$

↯ **unknown:** global set-points for relative angles $\{\theta^\star_{kj}\}$ and nonlocal measurements of $(v_j, i_{o,j})$

Decentralized implementation of phase error dynamics

$$e_{\theta,k}(v) = \underbrace{\sum\nolimits_j w_{jk}(v_j - R(\theta^\star_{jk}) v_k)}_{\text{need to know } v_j \text{ and } \theta^\star_{jk}} = \underbrace{\sum\nolimits_j w_{jk}(v_j - v_k)}_{\text{negative Laplacian:} -\mathcal{L}v} + \underbrace{\sum\nolimits_j w_{jk}(I - R(\theta^\star_{jk})) v_k}_{\text{local feedback: } K_k(\theta^\star) v_k}$$

insight I: non-local measurements from **communication through physics**: assume that all lines are homogeneous $r_{ij}/\ell_{ij} = \kappa = const.$, then

$$\underbrace{R(\kappa)\, i_o}_{\text{local feedback}} = \underbrace{R(\kappa)\, \mathcal{Y}}_{\text{Laplacian matrix}} v = \underbrace{\mathcal{L} v}_{\text{Laplacian feedback}}$$

insight II: angle set-points & line-parameters from **power flow equations**:

$$\left.\begin{aligned} p_k^\star &= v^{\star 2} \sum\nolimits_j \frac{R_{jk}(1-\cos(\theta^\star_{jk})) - \omega_0 L_{jk} \sin(\theta^\star_{jk})}{R_{jk}^2 + \omega_0^2 L_{jk}^2} \\ q_k^\star &= v^{\star 2} \sum\nolimits_j \frac{\omega_0 L_{jk}(1-\cos(\theta^\star_{jk})) + R_{jk} \sin(\theta^\star_{jk})}{R_{jk}^2 + \omega_0^2 L_{jk}^2} \end{aligned}\right\} \Rightarrow \underbrace{K_k(\theta^\star)}_{\text{global parameters}} = \underbrace{\frac{R(\kappa)}{v^{\star 2}} \begin{bmatrix} p_k^\star & q_k^\star \\ -q_k^\star & p_k^\star \end{bmatrix}}_{\text{local parameters}}$$

Surprising (?) connections revealed in polar coordinates

closed loop: $$\frac{\mathrm{d}}{\mathrm{d}t} v = \underbrace{\omega_0 \, \mathcal{J} \, v}_{\text{rotation at } \omega = \omega_0} + \eta \cdot \underbrace{(-\mathcal{L} + \mathcal{K})\, v)}_{\text{phase error}} + \alpha \cdot \underbrace{\mathrm{diag}(v^\star - \|v_k\|)\, v}_{\text{magnitude error}}$$

polar coordinates in **lossless** case $r_{ij} = 0$ & **near nominal voltage** $\|v_k\| \approx 1$:

$$\frac{d}{dt} \|v_k\| = \eta \left(\frac{q_k^\star}{v^{\star 2}} - \frac{q_k}{\|v_k\|^2} \right) \|v_k\| + \frac{\alpha}{v^\star} (v^\star - \|v_k\|) \|v_k\| \qquad \text{magnitude}$$

$$\approx \eta\, (q_k^\star - q_k) \|v_k\| + \alpha (v^\star - \|v_k\|) \|v_k\| \qquad q - \|v\| \text{ droop}$$

$$\frac{d}{dt} \theta_k = \omega_0 + \eta \left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2} \right) \qquad \text{phase}$$

$$\approx \omega_0 + \eta\, (p_k^\star - p_k) \qquad p - \omega \text{ droop}$$

$$= \omega_0 + p_k^\star + \eta \cdot \sum\nolimits_j w_{jk} \sin(\theta_j - \theta_k) \qquad \text{Kuramoto oscillator}$$

Closed-loop stability analysis

closed loop: $\frac{\mathrm{d}}{\mathrm{d}t} v = \underbrace{\cancel{\omega_0 \mathcal{J} v}}_{\text{in rotating frame}} + \eta \cdot (-\mathcal{L} + \mathcal{K})\, v) + \alpha \cdot \mathrm{diag}(v^\star - \|v_k\|)\, v$

stationary target sets in rotating coordinate frame:

$\mathcal{S} = \{v \in \mathbb{R}^n \mid v_k = R(\theta^\star_{k1}) v_1\}$ set of correct relative angles

$\mathcal{A} = \{v \in \mathbb{R}^n \mid \|v_k\| = v^\star\}$ set of correct magnitudes

main result: $\mathcal{T} = \mathcal{S} \cap \mathcal{A}$ is almost globally asymptotically stable if the grid parameters, angle set-points, and control gains satisfy a *mild condition*.

Discussion of stability condition

energy-like function centered at $\{\theta^\star_{kj}\}$ and with free-floating amplitudes:

$$V(v) = \sum_{j<k} w_{jk} \left(\|v_k\|^2 - \|v_k\|\|v_j\| \cos(\theta_k - \theta_j - \theta^\star_{kj})\right) = v^\top P v$$

$\Rightarrow$ closed loop is gradient flow $\dot{v} = -\nabla V(v)$ for $\alpha = \mathbb{0}$ and $\{\theta^\star_{kj}\} = \mathbb{0}$

condition $\Leftrightarrow$ energy function non-increasing despite $\alpha > \mathbb{0}$ & $\{\theta^\star_{kj}\} \neq \mathbb{0}$:

$$\eta \left((\mathcal{K} - \mathcal{L})^\top P + P(\mathcal{K} - \mathcal{L})\right) + 2\,\alpha P \preceq 0$$

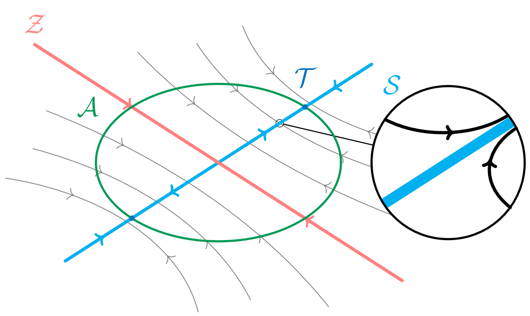
remark: the assumption can always be met in a connected grid if

- **slow/fast loops**: amplitude gain α < synchronizing gain η
- not too heavy **loading**: sufficiently small relative angle set-points $\{\theta^\star_{kj}\}$

$\Rightarrow$ stability condition is very reasonable (always met) for practical scenarios

Almost global synchronization

to trajectory with prescribed frequency, voltage amplitudes, & active/reactive power injections



exponential stability of $\mathcal{S}$:

under our assumption, $\|v\|^2_{\mathcal{S}}$ is a Lyapunov function for $\mathcal{S}$.

stability of $\mathcal{A}$ relative to $\mathcal{S}$:

provided $v(0) \in \mathcal{S} \setminus \{0\}$, the set $\mathcal{A}$ is asymptotically stable.

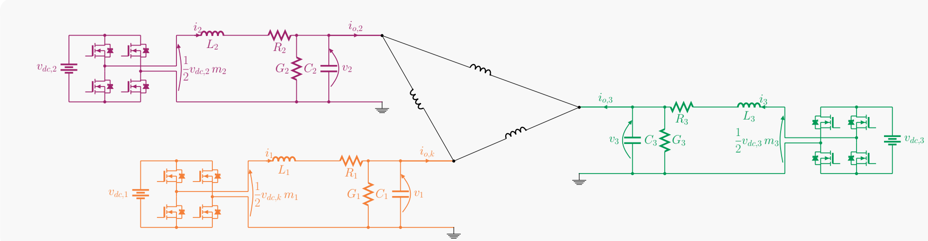
$\mathcal{Z}$ has measure zero:

the region of attraction of $\{0\}$, termed $\mathcal{Z}$, has measure zero.

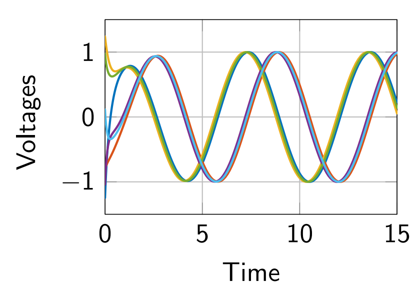
continuity argument:

almost all trajectories (not in $\mathcal{Z}$) approaching $\mathcal{S}$ must (by continuity) converge to $\mathcal{S} \cap \mathcal{A}$.

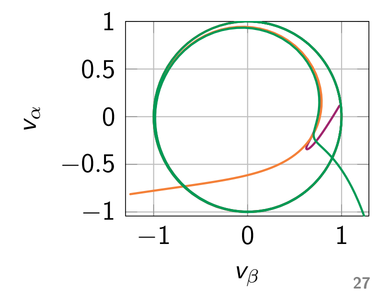
Simulation example



voltage time series



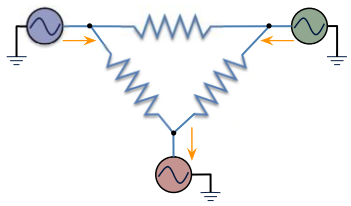
voltage phase space



Conclusions

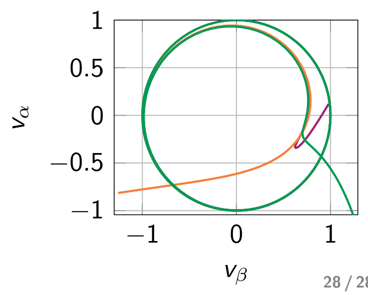
Summary:

- oscillator networks & power applications
- droop control & Kuramoto oscillators
- design of decentralized oscillator control



Outlook:

- robustness & performance analysis
- fair comparison of different approaches
- experimental validation



Insights for coupled oscillators:

- Kuramoto models rarely (almost never) admit almost global synchronization
- it pays off to work in $\mathbb{R}^2$ rather than $\mathbb{S}^1$