Boundary Value Problems and Multiscale Coupling Methods for Kinetic Equations

April 21-24, 2016

Department of Mathematics

University of Wisconsin-Madison

SCHEDULE

	Thursday, April 21	Friday, April 22	Saturday, April 23	Sunday, April 24
Venue	Van Vleck 911	Van Vleck 911	Van Vleck 911	Van Vleck 911
Chair	S. Jin	K. Aoki	I. Gamba	Q. Li
9:15 - 10:00	T-P. Liu	P. Degond	K. Aoki	F. Filbet
10:00 -10:45	T. Yang	M. Lemou	S. Takata	G. Dimarco
10:45 - 11:15	Coffee Break			
11:15 -12:00	C. Kim	N. Crouseilles	A. Klar	Z. Zhou
12:00 - 2:00	Lunch			
Chair	T-P. Liu	P. Degond	W. Sun	
2:00 - 2:45	J. Lu	I. Gamba	M. Tang	
2:45 - 3:30	W. Sun	L. Wang	C. Hauck	
3:30 - 4:00	Coffee Break			
4:00 - 4:45	J. Qiu	Colloquium	Y. Cheng	
4:45 - 5:30	L. Liu	(Van Vleck B239)	Y. Zhu	
6:00 - 8:00		Banquet (Soga)		

ABSTRACTS

DECAY OF A LINEAR OSCILLATOR IN A RAREFIED GAS: SPATIALLY ONE-DIMENSIONAL CASE

Kazuo Aoki
Kyoto University, Mechanical Engineering and Science

An infinitely wide plate, subject to an external force in its normal direction obeying Hooke's law, is placed in an infinite expanse of a rarefied gas. When the plate is displaced from its equilibrium position and released, it starts in general an oscillatory motion in its normal direction. This is the one-dimensional setting of a linear oscillator (or pendulum) considered previously for a collisionless gas and for a special Lorentz gas in our paper [T. Tsuji and K. Aoki, J. Stat. Phys. 146, 620 (2012)]. The motion decays as time proceeds because of the drag force on the plate exerted by the surrounding gas. The long-time behavior of the unsteady motion of the gas caused by the motion of the plate is investigated numerically on the basis of the BGK model of the Boltzmann equation, as well as the compressible Navier-Stokes equation (with the temperature-jump condition), with special interest in the rate of the decay of the oscillatory motion of the plate decays in proportion to an inverse power of time (power -3/2) for large time. This talk contains some results of the works in collaboration. The result provides numerical evidence that the displacement of the plate with Tetsuro Tsuji (Osaka University), Shingo Kosuge (Kyoto University, Japan), and Taiga Fujiwara (Kyoto University).

A SPARSE GRID DISCONTINUOUS GALERKIN METHOD FOR HIGH-DIMENSIONAL TRANSPORT EQUATIONS

Yingda Cheng
Michigan State University, Department of Mathematics

In this talk, we present a sparse grid discontinuous Galerkin (DG) scheme for transport equations and applied it to kinetic simulations. The method uses the weak formulations of traditional Runge-Kutta DG schemes for hyperbolic problems and is proven to be L^2 stable and convergent. A major advantage of the scheme lies in its low computational and storage cost due to the employed sparse finite element approximation space. This attractive feature is explored in simulating Vlasov and Boltzmann transport equations. Good performance in accuracy and conservation is verified by numerical tests in up to four dimensions.

NUMERICAL SCHEME FOR HIGHLY-OSCILLATORY KINETIC EQUATIONS

Nicolas Crouseilles
Inria, IRMAR

Numerical scheme for highly-oscillatory kinetic equations.

MODELS OF COLLECTIVE DYNAMICS WITH COMPLEX ORIENTATION MECHANISMS

Pierre Degond
Imperial College London, Department of Mathematics

We will report on recent developments on self-propelled particle models with complex orientation mechanisms. The most simple and widely studied orientation mechanism involves polar orientation and follows from the seminal work of Vicsek and co-authors. The macroscopic dynamics of this model gives rise to the SOH model (self-organized hydrodynamics) which has been shown to exhibit many surprising and unusual behavior. However, other orientation mechanisms can be found in nature such as nematic alignment or alignment of the full body configuration. We will present microscopic and macroscopic versions of these dynamics and outline some applications to bacterial colonies and flocking.

ASYMPTOTIC PRESERVING TIME DIMINISHING METHODS FOR KINETIC EQUATIONS

Giacomo Dimarco
University of Ferrara, Mathematics and Computer Science

In this talk, we introduce a new class of numerical schemes for solving kinetic type equations. The idea consists in reformulating the problem using a micro-macro decomposition and successively in solving the microscopic part by using asymptotically stable Monte Carlo methods. The scheme is designed in such a way that it becomes computationally less expensive as the solution approaches the equilibrium state as opposite to standard Monte Carlo methods which computational cost increases with the number of interactions. At the same time the statistical error due to the Monte Carlo part of the solution decreases as the system approach the equilibrium state. This causes the method to degenerate to the sole solution of the macroscopic hydrodynamic equations in the limit of infinite number of collisions.

ON THE VLASOV-POISSON SYSTEM WITH A STRONG EXTERNAL MAGNETIC FIELD

Francis Filbet
Université Paul Sabatier, Toulouse III, Institut de Mathématiques de Toulouse

We consider the 3D Vlasov-Poisson system with a strong magnetic field and study different asymptotics to get reduced model.

APPROXIMATIONS TO BOUNDARY VALUE PROBLEMS FOR NONLINEAR COLLISIONAL KINETIC PLASMA MODELS

Irene Gamba
University of Texas at Austin, ICES

We will discuss recent approximations to boundary value problems to non-linear systems of Boltzmann or Landau (for Coulombic interactions) equations coupled to the Poisson equation. The proposed approximation methods involve hybrid schemes of spectral and Galerkin type, were conservation of flow invariants are achieved by a constrain minimization problem. We will discuss some analytical and computational issues related to these approximations.

IMPLICIT SOLUTION OF THE VLASOV-POISSON SYSTEM

Cory Hauck
Oak Ridge National Laboratory and University of Tennessee

Most Eulerian and semi-Lagrangian codes use a splitting in time approach for implicit, charged particle transport solvers. The reason for this is advection with respect to only position or only velocity can be solved explicitly and even exactly, but advection with respect to both position and velocity requires solving a linear system whose order is the size of the phase space discretization. This becomes unacceptable for higher dimensional problems. We propose a new approach which does not involve splitting in time, but splitting the phase space into subdomains. If each subdomain is appropriately chosen, the transport problem can be solved explicitly on each subdomain. A Krylov solver is then used to solve for the boundaries between subdomains, which involves a much smaller linear system than the one required for the no splitting approach.

NEW L^6 INTEGRABILITY OF THE HYDRODYNAMIC PART AND NAVIER-STOKES LIMIT

Chanwoo Kim
University of Wisconsin-Madison, Department of Mathematics

We present a new L^6 integrability of the hydrodynamic part. Then we prove a convergence of Boltzmann solution to the Navier-Stokes as the Knudsen number tends to zero.

A MULTI-SCALE PARTICLE METHOD FOR NON-LOCAL KINETIC EQUATIONS

Axel Klar
Technische Universität Kaiserslautern, Fachbereich Mathematik

The talk will present a mesh-free numerical method for non-local kinetic and hydrodynamic equations derived from interacting particle systems. We discuss the particle method, treatment of boundary conditions, non-local terms and an asymptotic-preserving property. Applications to granular flow, pedestrian flow and to interacting fibers are shown.

A MULTISCALE NUMERICAL APPROACH FOR A CLASS OF TIME-SPACE OSCILLATORY PROBLEMS

Mohammed Lemou
CNRS and University of Rennes 1, Mathematics, IRMAR

High oscillations may arise in many physical problems: Schrödinger equations, kinetic equations, or more generally high frequency waves. In this talk, we will present a general strategy that allows the construction of uniformly (with respect to the oscillation frequency) accurate numerical schemes in the following situations:

i) multi-frequency time oscillations with an application to multi-component Schrödinger equation.

ii) time-space oscillations with applications to some high frequency waves and semi-classical quantum models.

Some numerical tests will be presented to illustrate the efficiency of the strategy.

AN ASYMPTOTIC-PRESERVING STOCHASTIC GALERKIN METHOD FOR THE SEMICONDUCTOR BOLTZMANN EQUATION WITH RANDOM INPUTS AND DIFFUSIVE SCALINGS

Liu Liu

University of Wisconsin-Madison, Mathematics Department

In this talk, I will introduce the generalized polynomial chaos approach based stochastic Galerkin (gPC-SG) method for the linear semi-conductor Boltzmann equation with random inputs and diffusive scalings. The random inputs are due to uncertainties in the collision kernel or initial data. We study the regularity of the solution in the random space, and prove the spectral accuracy of the gPC-SG method. We then use the asymptotic-preserving framework for the deterministic counterpart developed in [Jin, Pareschi, 99] to come up with the stochastic asymptotic-preserving gPC-SG method for the problem under study which is efficient in the diffusive regime. Numerical experiments will be presented to validate the accuracy and asymptotic properties of the method. This is a joint work with Prof. Shi Jin.

INVARIANT MANIFOLDS FOR STATIONARY BOLTZMANN EQUATION AND APPLICATIONS

Tai-Ping Liu

Stanford University, Mathematics Department

With Shih-Hsien Yu, we study the invariant manifolds for stationary Boltzmann equation and apply the theory to the construction of the boundary layers. Of particular interest is coupling of the Knudsen-type boundary layer and the interior fluid-like waves when the Mach number is around one or zero. Our Green's function approach allows for strong quantitative estimates required for the strongly nonlinear phenomena of resonance.

SOLVING LINEAR HALF-SPACE KINETIC EQUATIONS WITH GENERAL BOUNDARY CONDITIONS

Jianfeng Lu

Duke University, Department of Mathematics

In this talk, we will discuss some recent progress on efficient methods for solving linear / linearized half-space kinetic equations. The spectral method we recently developed relies on the ideas of even-odd decomposition and damping adding/removal. The method works for a large class of equations with general boundary conditions. If time permits, we will also discuss possible extensions and difficulties for higher dimensions. (Joint with Qin Li and Weiran Sun).

A HIERARCHICAL UNIFORMLY HIGH ORDER DG-IMEX SCHEME FOR THE BGK EQUATION

Jingmei Qiu
University of Houston, Department of Mathematics

A class of high order discontinuous Galerkin implicit-explicit (DG-IMEX) schemes with asymptotic preserving (AP) property have been developed for the BGK equation in Xiong et. al. (2015). The schemes are built based on a micro-macro reformulation of the equation, which properly describes the interplay between macroscopic and microscopic dynamics in the asymptotic limit of small Knudsen number. The nodal DG scheme (NDG) is coupled with globally stiffly accurate IMEX Runge-Kutta (RK) schemes based on such reformulation, leading to high order schemes for Knudsen number ranging from zero to order $\mathcal{O}(1)$ for scales between hydrodynamic and kinetic regimes. The schemes are theoretically analyzed and numerically verified to be asymptotically consistent and accurate with high order accuracy (up to third order) in both space and in time, in the limit as the Knudsen number becomes small or zero, corresponding to the compressible Navier-Stokes (CNS) or Euler limit respectively.

Motivated by the recent work in Filbet and Rey (2015) and references therein, it is natural to construct a hierarchical AP scheme under the NDG-IMEX framework without hybridization. In this paper, we propose to build a hierarchical high order AP method based on our previous work, by automatically applying kinetic, CNS and Euler solvers in regions where such models are appropriate. The numerical solvers for different regimes are coupled naturally by interface conditions across different regions, while preserving mass, momentum and energy at the discrete level. To the best of our knowledge, the proposed scheme is the very first hierarchical one in the literature, that enjoys AP property as well as uniform high order accuracy (up to third order for both space and time) for all three regimes (kinetic, CNS and Euler) with conservation of mass, momentum and energy. Numerical experiments demonstrate very efficient and effective performance of the proposed approach, with three different regimes dynamically identified and naturally coupled as time evolves. The use of hydrodynamic solvers, in place of a high-dimensional kinetic solver wherever such models are sufficient, leads to significant savings in terms of CPU time (more than 80% for some of our test problems).

REGULARIZATION OF HALF-SPACE EQUATIONS AND ITS APPLICATION

Weiran Sun
Simon Fraser University, Department of Mathematics

In this talk we show some recent results on approximating kinetic equations over bounded domains in 2D. In 2015 Wu-Guo proved an unexpected result which showed that the classical boundary layer analysis for the isotropic neutron transport equation fails over the unit disk $B(0,1)$ in 2D. In particular, if one uses the 1D half-space equation along the normal direction of the disk together with the Laplace equation in the interior, then this approximate solution produces an order $\mathcal{O}(1)$ error in the L^∞-norm compared with the true kinetic equation. However, it is not clear whether this $\mathcal{O}(1)$ error is restricted to the boundary layer or propagates to the interior. In this work with Qin Li and Jianfeng Lu, we show a systematic way to regularize solutions to the half-space equation and apply this technique to show that the $\mathcal{O}(1)$ error stays within the boundary layer. Thus the classical half-space equation still provides a valid boundary condition for the interior Laplace equation.

SOME RESULTS ON THE EFFECTS OF BOUNDARY GEOMETRY IN RAREFIED GASES

Shigeru Takata
Kyoto University, Department of Aeronautics and Astronautics

We will discuss the singular behavior of a rarefied gas near the boundary. We here focus on the time-independent behavior of the gas and study the possible singularities due to the effect of the boundary geometry. This talk is based on a joint work with Tatsuya Yoshida, Takashi Noguchi, Satoshi Taguchi, and Masanari Hattori.

UNIFORM CONVERGENT NUMERICAL METHOD FOR THE LINEAR TRANSPORT EQUATION VALID UP TO THE BOUNDARIES AND INTERFACES

Min Tang
Shanghai Jiao Tong University, Department of Mathematics

We discuss about the development of uniformly second order numerical methods for the discete-ordinate transport equation in one and two dimensions. The uniform convergence in the diffusive regime is valid up to the boundary and interface, even if the boundary or interface layers exist, so the boundary or interface layer does not need to be resolved numerically. Both isotropic and anisotropic scattering are investigated.

THE UNIFORM CONVERGENCE OF GENERALIZED POLYNOMIAL CHAOS-BASED NUMERICAL METHODS FOR TRANSPORT EQUATION WITH RANDOM INPUT

Li Wang
University at Buffalo, Mathematics Department

We discuss the convergence property of the generalized polynomial chaos based numerical methods for the linear transport equation with random scattering and initial data. For this standard treatment of the random media, the brute-force analysis only provides slow convergence rate in the diffusion regime, which is inconsistent with numerical evidences. The bypass is to employ the asymptotic convergence that entirely gets rid of epsilon dependence in that regime. For the stochastic collocation method, the proof relies on the regularity of the solution, whereas for the stochastic Galerkin method, we also need to take care of the position of the scattering term. This is a joint work with Qin Li.

EXTERIOR PROBLEM FOR THE VLASOV-POISSON-BOLTZMANN EQUATIONS

Tong Yang
City University of Hong Kong, Mathematics Department

In this talk, we will present the existence of stationary solutions to the Vlasov-Poisson-Boltzmann equations outside an obstacle. The proof is based on the spectrum analysis and a deposition of the solution operator following the approach introduced by Ukai-Asano for the Boltzmann equation. The stability of the stationary solution will also be discussed.

POSITIVITY-PRESERVING AND ASYMPTOTIC PRESERVING METHOD FOR 2D KELLER-SEGAL EQUATIONS

Zhennan Zhou
Duke University, Mathematics Department

We propose a semi-discrete scheme for 2D Keller-Segel equations based on a symmetrization reformation, which is equivalent to the convex splitting method and is free of any nonlinear solver. We show that, this new scheme is unconditionally stable as long as the initial condition does not exceed certain threshold, and it asymptotically preserves the quasi-static limit in the transient regime. Furthermore, we prove that the fully discrete scheme is conservative and positivity preserving, which makes it ideal for simulations. The analogical schemes for the radial symmetric cases and the subcritical degenerate cases are also presented and analyzed. With extensive numerical tests, we verify the claimed properties of the methods and demonstrate their superiority in various challenging applications. This is a joint work with Jian-Guo Liu and Li Wang.

AN ASYMPTOTIC PRESERVING METHOD FOR VLASOV-POISSON-FOKKER-PLANCK SYSTEM WITH UNCERTAINTY

Yuhua Zhu
University of Wisconsin-Madison, Mathematics Department

The Vlasov-Poisson-Fokker-Planck (VPFP) system is a mesoscopic model describing the Brownian Motion in Plasma. Due to inaccuracies in the experimental data in initial data or impurity in the surrounding fluid, the VPFP system will inevitably involve some uncertainty. In this paper, we develop a stochastic Asymptotic Preserving (s-AP) scheme for VPFP system with uncertainty based on generalized Polynomial Chaos-Stochastic Galerkin Method (gPC-SG). We show the regularity of initial data in random space is preserved by the analytical solution, which establishes the spectral convergence of the gPC-SG method. The gPC-SG framework gives a deterministic resulting system, which can be effectively solved by developed numerical methods. Moreover, we prove the asymptotic preserving property in full discretization, thus the scheme can be used with numerical parameters independent of the mean free path or other small parameters. Numerical examples are given to validate its the accuracy and the s-AP properties. This is a joint work with Shi Jin.